Vectors

Pt.2: 3D Vectors

AS-Level
Pt. 1: 2D Vectors

A-Level
Pt. 2: 3D Vectors

1. In a triangle ABC, $\overrightarrow{AB} = 6\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\overrightarrow{AC} = 8\mathbf{i} - 5\mathbf{j} + 4\mathbf{k}$

a. Find the vector $\overrightarrow{BC}$ **(2)**

b. Find the length of the line AB **(2)**

2. Three forces act on an obkect, $F_1 = -5\mathbf{i} + 7\mathbf{j} + 4\mathbf{k}$, $F_2 = 4\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$ and $F_3 = 3\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$. Find the resultant force. **(2)**

3. The points P, Q, R and S have position vectors, $p = -3\mathbf{i} + 10\mathbf{j} - 7\mathbf{k}$, $q = 5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, $r = -4\mathbf{i} + 6\mathbf{j} - 12\mathbf{k}$ and $s = 8\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$.

Show that $\overrightarrow{PQ}$ is parallel to $\overrightarrow{RS}$. **(5)**

4. Relative to a fixed origin O, the point A has position vector $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and the point B has position vector $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$. The points A and B lie on a straight line l.

Find $\overrightarrow{AB}$ **(2)**

5. Relative to a fixed origin O, the point A has position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$, the point B has position vector $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$, and the point C has position vector $(a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$, where a is a constant and $a < 0$. D is the point such that $\overrightarrow{AB} = \overrightarrow{BD}$.

a. Find the position vector of D.

Given that $|\overrightarrow{AC}| = 4$, **(4)**

b. Find the value of a. **(4)**

6. Given that $(b - a)\mathbf{i} - 2abc\mathbf{j} + 2\mathbf{k} = 10\mathbf{i} - 96\mathbf{j} + (7a + 5b)\mathbf{k}$, find the values of a, b and c. **(5)**

Mark Scheme

1a.

$\overrightarrow{BC} = \begin{pmatrix} 8 \\ -5 \\ 4 \end{pmatrix} - \begin{pmatrix} 6 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix}$	**M1**
$\overrightarrow{BC}$ = 2**i** – 7**j** + 5**k**	**M1**

1b.

$\sqrt{6^2 + 2^2 + 1^2}$	**M1**
= $\sqrt{41}$	**M1**

2.

$\begin{pmatrix} -5 \\ 7 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ -5 \\ 3 \end{pmatrix}$	**M1**
= $\begin{pmatrix} 2 \\ 8 \\ 5 \end{pmatrix}$ or 2**i** + 8**j** + 5**k**	**M1**

3.

$\overrightarrow{PQ} = \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix} - \begin{pmatrix} -3 \\ 10 \\ -7 \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \\ 10 \end{pmatrix}$	**M1**
$\overrightarrow{RS} = \begin{pmatrix} 8 \\ -3 \\ 3 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \\ -12 \end{pmatrix} = \begin{pmatrix} 12 \\ -9 \\ 15 \end{pmatrix}$	**M1**
$\overrightarrow{PQ}$ = 8**i** – 6**j** + 10**k** = 2(4**i** – 3**j** + 5**k**)	**M1**
$\overrightarrow{RS}$ = 12**i** – 9**j** + 15**k** = 3(4**i** – 3**j** + 5**k**)	**M1**
$\overrightarrow{PQ} = \frac{2}{3}\overrightarrow{RS}$ Therefore, same direction and parllel	**M1**

4.

$\overrightarrow{OA}$ = **i** – 3**j** + 2**k** $\overrightarrow{OB}$ = -2**i** + 2**j** – **k**	**M1**
$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$	**M1**

5a.

$\overrightarrow{OD}$ = x**i** + y**j** + z**k** Then, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ = 2**i** – 5**j** + 7**k**	**M1**
$\overrightarrow{BD} = \overrightarrow{OD} - \overrightarrow{OB}$ =((x – 4)**i** + (y + 2)**j** + (z – 3)**k**)	**M1**
As $\overrightarrow{AB} = \overrightarrow{BD}$, x – 4 = 2, x = 6 y + 2 = -5 y = -7 z – 3 = 7 z = 10	**M1**
$\overrightarrow{OD}$ = 6**i** - 7**j** + 10**k**	**M1**

5b.

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$ = ((*a* – 2)**i** + 2**j** + 2**k**)	**M1**
$\left\|\overrightarrow{AC}\right\| = \sqrt{(a-2)^2 + 4 + 4}$ $= \sqrt{a^2 - 4a + 12}$	**M1**
As $\left\|\overrightarrow{AC}\right\| = 4$, $a^2 - 4a + 12 = 16$ $a^2 - 4a - 4 = 0$	**M1**
$a = 2 - 2\sqrt{2}$	**M1**

6.

i coefficients: (*b* – *a*)i = 10i *b* – *a* = 10	**M1**
j coefficients: -2*abc* = -96 *abc* = 48	**M1**
k coefficients: 7*a* + 5*b* = 2	**M1**
Solving through substitution: 7*b* + 5*b* + 7*a* – 7*a* = 70 + 2 12*b* = 72 *b* = 6	**M1**
6 – *a* = 10 *a* = -4	**M1**
-4 × 6*c* = 48 -24*c* = 48 *c* = -2	**M1**