

Vectors

Pt.2: 3D Vectors

AS-Level

Pt. 1: 2D Vectors

A-Level

Pt. 2: 3D Vectors



1. In a triangle ABC , $\vec{AB} = 6\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\vec{AC} = 8\mathbf{i} - 5\mathbf{j} + 4\mathbf{k}$
 - a. Find the vector \vec{BC} (2)
 - b. Find the length of the line AB (2)

2. Three forces act on an object, $F_1 = -5\mathbf{i} + 7\mathbf{j} + 4\mathbf{k}$, $F_2 = 4\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$ and $F_3 = 3\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$. Find the resultant force. (2)

3. The points P , Q , R and S have position vectors, $p = -3\mathbf{i} + 10\mathbf{j} - 7\mathbf{k}$, $q = 5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, $r = -4\mathbf{i} + 6\mathbf{j} - 12\mathbf{k}$ and $s = 8\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$.
Show that \vec{PQ} is parallel to \vec{RS} . (5)

4. Relative to a fixed origin O , the point A has position vector $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and the point B has position vector $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$. The points A and B lie on a straight line l .
Find \vec{AB} (2)

5. Relative to a fixed origin O , the point A has position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$, the point B has position vector $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$, and the point C has position vector $(a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$, where a is a constant and $a < 0$. D is the point such that $\vec{AB} = \vec{BD}$.
 - a. Find the position vector of D .
Given that $|\vec{AC}| = 4$, (4)
 - b. Find the value of a . (4)

6. Given that $(b - a)\mathbf{i} - 2abc\mathbf{j} + 2\mathbf{k} = 10\mathbf{i} - 96\mathbf{j} + (7a + 5b)\mathbf{k}$, find the values of a , b and c . (5)

Mark Scheme

1a.

$\overrightarrow{BC} = \begin{pmatrix} 8 \\ -5 \\ 4 \end{pmatrix} - \begin{pmatrix} 6 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix}$	M1
$\overrightarrow{BC} = 2\mathbf{i} - 7\mathbf{j} + 5\mathbf{k}$	M1

1b.

$\sqrt{6^2 + 2^2 + 1^2}$	M1
$= \sqrt{41}$	M1

2.

$\begin{pmatrix} -5 \\ 7 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ -5 \\ 3 \end{pmatrix}$	M1
$= \begin{pmatrix} 2 \\ 8 \\ 5 \end{pmatrix}$ or $2\mathbf{i} + 8\mathbf{j} + 5\mathbf{k}$	M1

3.

$\overrightarrow{PQ} = \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix} - \begin{pmatrix} -3 \\ 10 \\ -7 \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \\ 10 \end{pmatrix}$	M1
$\overrightarrow{RS} = \begin{pmatrix} 8 \\ -3 \\ 3 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \\ -12 \end{pmatrix} = \begin{pmatrix} 12 \\ -9 \\ 15 \end{pmatrix}$	M1
$\overrightarrow{PQ} = 8\mathbf{i} - 6\mathbf{j} + 10\mathbf{k} = 2(4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$	M1
$\overrightarrow{RS} = 12\mathbf{i} - 9\mathbf{j} + 15\mathbf{k} = 3(4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$	M1
$\overrightarrow{PQ} = \frac{2}{3}\overrightarrow{RS}$ Therefore, same direction and parallel	M1

4.

$\overrightarrow{OA} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ $\overrightarrow{OB} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$	M1
$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$	M1

5a.

$\overrightarrow{OD} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ Then, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ $= 2\mathbf{i} - 5\mathbf{j} + 7\mathbf{k}$	M1
$\overrightarrow{BD} = \overrightarrow{OD} - \overrightarrow{OB}$ $= ((x-4)\mathbf{i} + (y+2)\mathbf{j} + (z-3)\mathbf{k})$	M1
As $\overrightarrow{AB} = \overrightarrow{BD}$, $x - 4 = 2$, $x = 6$ $y + 2 = -5$ $y = -7$ $z - 3 = 7$ $z = 10$	M1
$\overrightarrow{OD} = 6\mathbf{i} - 7\mathbf{j} + 10\mathbf{k}$	M1

5b.

$\vec{AC} = \vec{OC} - \vec{OA}$ $= ((a-2)\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$	M1
$ \vec{AC} = \sqrt{(a-2)^2 + 4 + 4}$ $= \sqrt{a^2 - 4a + 12}$	M1
As $ \vec{AC} = 4$, $a^2 - 4a + 12 = 16$ $a^2 - 4a - 4 = 0$	M1
$a = 2 - 2\sqrt{2}$	M1

6.

i coefficients: $(b-a)\mathbf{i} = 10\mathbf{i}$ $b - a = 10$	M1
j coefficients: $-2abc = -96$ $abc = 48$	M1
k coefficients: $7a + 5b = 2$	M1
Solving through substitution: $7b + 5b + 7a - 7a = 70 + 2$ $12b = 72$ $b = 6$	M1
$6 - a = 10$ $a = -4$	M1
$-4 \times 6c = 48$ $-24c = 48$ $c = -2$	M1

