A-Level Unit Test: Trigonometry

Solving Trig Equations

Jethwa Maths

1. It is a given that $2\text{cosec}^2 x = 5 - 5\cot x$
a. Show that the equation that $2\text{cosec}^2 x = 5 - 5\cot x$ can be written in the form $2\cot^2 x + 5\cot x - 3 = 0$. **(2)**
b. Hence show that $\tan x = 2$ or $\tan x = -\frac{1}{3}$ **(2)**
c. Hence, or otherwise, solve the equation $2\text{cosec}^2 x = 5 - 5\cot x$, giving all values of x in radians to one decimal place in the interval $-\pi < x \leq \pi$ **(3)**

2a. Show that the equation $2\cot^2 x + 5\text{cosec}\, x = 10$ can be written in the form $2\text{cosec}^2 x + 5\text{cosec}\, x - 12 = 0$ **(2)**
b. Hence find the values of $\sin x$. **(3)**
c. Hence, or otherwise, solve the equation, $2\cot^2(\theta - 0.1) + 5\,\text{cosec}(\theta - 0.1) = 10$. Give all values of θ in radians to two decimal places in the interval $-\pi < x \leq \pi$. **(3)**

3a. Express $5\cos x - 3\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$ **(4)**
b. Hence, or otherwise, solve the equation $5\cos x - 3\sin x = 4$, for $0 < \alpha < 2\pi$, giving your answers to 2 decimal places. **(3)**

4a. Prove that for all value of x, $\sin x + \sin(60 - x) = \sin(60 + x)$ **(4)**
b. Given that $\sin 84 - \sin 36 = \sin \alpha$, deduce the exact value of the acute angle α **(2)**
c. Solve the equation $4\sin 2x + \sin(60 - 2x) = \sin(60 + 2x) - 1$, for values of x in the interval $0 \leq x < 360$, giving your answers to one decimal place **(5)**

5a. By writing $3x = 2x + x$, show that $3x = 3\sin x - 4\sin^3 x$ **(4)**
b. Hence, or otherwise, for $0 < x < \frac{\pi}{3}$, solve, $8\sin^3 x - 6\sin x + 1 = 0$ **(5)**
c. Using $\sin(x - \alpha) = \sin x\cos\alpha - \cos x\sin\alpha$, or otherwise show that $\sin 15 = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ **(4)**

6a. Show that $\frac{\sin 2x}{1+\cos 2x} = \tan x$ **(2)**
b. Hence find, for $-180 \leq x \leq 180$, all the solutions of $\frac{2\sin 2x}{1+\cos 2x} = 1$ **(3)**

7. Find all the solutions of $2\cos 2x = 1 - 2\sin x$ in the interval $0 \leq x < 360$ **(5)**

8a. By first expanding $(\cos 2x + x)$, prove that $\cos 3x = 4\cos^3 x - 3\cos x$ **(4)**
b. Hence prove that $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$ **(3)**
c. Show that the only solutions of the equation $1 + \cos 6x = 18\cos^2 x$ are odd multiples of 90. **(5)**

9a. Express $\cos 2x$ in terms of $\sin x$. **(1)**
b. Hence that $3\sin x - \cos 2x = 2\sin^2 x + 3\sin x - 1$ for all values of x **(2)**
c. Solve the equation $3\sin x - \cos 2x = 1$ for $0 < x < 360$ **(4)**

10a. Show that the equation $2\cot^2 x + 5\text{cosec}\, x = 10$ can be written in the form $2\text{cosec}^2 x + 5\,\text{cosec}\, x - 12 = 0$ **(2)**
b. Hence show that $\sin x = -\frac{1}{4}$ or $\sin x = \frac{2}{3}$ **(3)**
c. Hence, or otherwise, solve the equation, $2\cot^2(x - 0.1) + 5\,\text{cosec}(x - 0.1) = 10$, giving all values of x in radians to two decimal places in the interval $-\pi < x < \pi$ **(4)**

Total marks: 84

Mark Scheme

1a.

$2\ \text{cosec}^2 x = 5(1 - \cot x)$	**M1**
$2 + 2\cot^2 x = 5 - 5\cot x$ (use of $\text{cosec}^2 x = 1 + \cot^2 x$)	**M1**
$2\cot^2 x + 5\cot x = 3$	

1b.

$(2\cot x - 1)(\cot x + 3) = 0$	**M1**
$\cot x = \frac{1}{2} \rightarrow \tan x = 2$ $\cot x = -3 \rightarrow \tan x = -\frac{1}{3}$	**M1**

1b.

$\tan x = 2 \rightarrow x = 1.1, -2.0$	**M1**
$\tan x = -\frac{1}{3} \rightarrow x = -0.3, 2.8$	**M1**

2a.

$2(\text{cosec}^2 x - 1) + 5\ \text{cosec}\ x = 10$	**M1**
$2\text{cosec}^2 x - 2 + 5\text{cosec}\ x - 10 = 0$ $2\text{cosec}^2 x + 5\text{cosec}\ x - 12 = 0$	**M1**

2b.

$(2\text{cosec}\ x - 3)(\text{cosec}\ x + 4) = 0$	**M1**
$\text{cosec}\ x = \frac{3}{2} \rightarrow \sin x = \frac{2}{3}$	**M1**
$\text{cosec}\ x = -4 \rightarrow \sin x = -\frac{1}{4}$	**M1**

2c.

$\theta - 0.1 = 0.73, 2.41, -0.25, -2.89$	**M1**
$\theta = 0.83, 2.51, -0.15, -2.79$	**M1 M1**

3a.

$5\cos x - 3\sin x = \text{R}\cos x\cos\alpha - \text{R}\sin x\sin\alpha$	**M1**
Equating $\cos x$: $5 = \text{R}\cos\alpha$ Equating $\sin x$: $3 = \text{R}\sin\alpha$	**M1**
$\text{R} = \sqrt{5^2 + 3^2} = \sqrt{34}$	**M1**
$\tan\alpha = \frac{3}{5}$ $\alpha = 0.5404$	**M1**

3b.

$5\cos x - 3\sin x = 4 \rightarrow \sqrt{34}\cos(x + 0.5404) = 4$ $\cos(x + 0.5404) = \frac{4}{\sqrt{34}}$	**M1**
$x + 0.5404 = 0.81482\ldots$ $x = 0.2744\ldots$ $x = 0.27$	**M1**
$x + 0.5404 = 2\pi - 0.81482\ldots$ $x = 4.9279\ldots$ $x = 4.92$	**M1**

4a.

LHS = sin x + sin 60 – cos 60 sin x	**M1**
$= \sin x + \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$	**M1**
RHS = sin 60 cos x + cos 60 sin x	**M1**
$= \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x$	**M1**

4b.

From (a), sin (60 + x) – sin (60 – x) = sin x x = 24	**M1**
sin 84 – sin 36 = sin 24 α = 24	**M1**

4c.

3sin2x + sin 2x + sin(60 – 2x) = sin(60 + 2x) – 1	**M1**
Using (a), 3 sin 2x = -1	**M1**
2x = 199.47 or 340.53	**M1**
x = 99.7, 170.3	**M1**
x = 279.7, 350.3	**M1**

5a.

sin 3x = sin (2x + x) = sin 2x cos x + cos 2x sin x	**M1**
= 2 sin x cos x (cos x) + (1 – 2sin^2x)(sin x)	**M1**
= 2 sin x (1 – sin^2x) + sin x – 2 sin^3x	**M1**
= 3 sin x – 4 sin^3x	**M1**

5b.

8 sin^3 x – 6sin$x$ + 1 = 0 -2sin3x + 1 = 0	**M1**
sin 3x = $\frac{1}{2}$	**M1**
3x = $\frac{\pi}{6}, \frac{5\pi}{6}$	**M1**
$x = \frac{\pi}{18}, \frac{5\pi}{18}$	**M1 M1**

5c.

sin15 – sin(60 – 45) = sin 60 cos 45 – cos 60 sin 45	**M1**
$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$	**M1**
$= \frac{1}{4}\sqrt{6}$ - $\frac{1}{4}\sqrt{2}$	**M1**
= ¼ ($\sqrt{6}$ - $\sqrt{2}$)	**M1**

6a.

$\frac{2\sin x\cos x}{1+2cos^2x-1} = \frac{2\sin x\cos x}{2\cos x\cos x}$	**M1**
$\frac{2\sin x\cos x}{2\cos x\cos x}$ = tan x	**M1**

6b.

2 tan x = 1 tan x = $\frac{1}{2}$	**M1**
x = 26.6	**M1**
x = -153.4	**M1**

7.

$2\cos 2x = 1 - 2\sin x$ $2(1 - 2\sin^2 x) = 1 - 2\sin x$	**M1**
$2 - 4\sin^2 x = 1 - 2\sin x$ $4\sin^2 x - 2\sin x - 1 = 0$	**M1**
$\sin x = \frac{1 \pm \sqrt{5}}{4}$	**M1**
$x = 54, 126, 198, 342$	**M1 M1**

8a.

$\cos 2x \cos x - \sin 2x \sin x$	**M1**
$\sin 2x = 2\sin x \cos x$ $\cos 2x = 2\cos^2 x - 1$	**M1**
$2\cos^2 x - 1(\cos x) - 2\sin x \cos x\,(\sin x)$ $2\cos^3 x - \cos x - 2\sin^2 x \cos \mathrm{x}$ $2\cos^3 x - \cos x - 2\cos x\,(1 - \cos^2 x)$ $2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$	**M1**
$= 4\cos^3 \mathrm{x} - 3\cos \mathrm{x}$	**M1**

8b.

$\cos 6x = 2\cos^2 3x - 1$	**M1**
$32c^6 - 48c^4 + 18c^2 - 1$	**M1 M1**

8c.

$c = \cos 6x$	**M1**
$32c^6 - 48c^4 = 0$	**M1**
$c^2 = \frac{3}{2}$	**M1**
$c = 0, 90, 270, 540\ldots$	**M1**
Therefore c is odd multiples of 90.	**M1**

9a.

$\cos 2x = 1 - 2\sin^2 x$	**M1**

9b.

$3\sin x - \cos 2x = 3\sin x - (1 - 2\sin^2 x)$	**M1**
$= 3\sin x - 1 + 2\sin^2 x$	**M1**

9c.

$2\sin^2 x + 3\sin x - 2 = 0$ $(2\sin x - 1)(\sin x + 2) = 0$	**M1**
$2\sin x - 1 = 0$ $\sin x = \frac{1}{2}$ $x = 30, 150$	**M1** **M1**
$\sin x + 2 = 0$ $\sin x = -2$ no solutions	**M1**

10a.

$2(\mathrm{cosec}^2 x - 1) + 5\,\mathrm{cosec}\,x = 10$	**M1**
$2\mathrm{cosec}^2 x - 2 + 5\,\mathrm{cosec}\,x - 10 = 0$ $2\mathrm{cosec}^2 x + 5\,\mathrm{cosec}\,x - 12 = 0$	**M1**

10b.

$(2 \operatorname{cosec} x - 3)(\operatorname{cosec} x + 4) = 0$	**M1**
$\operatorname{cosec} x = \frac{3}{2}$ $\operatorname{cosec} x = 2$	**M1**
$\sin x = \frac{2}{3}$ $\sin x = -\frac{1}{4}$	**M1**

10c.

$(x - 0.1) = 0.73, 2.41, -0.25, -2.89$	**M1 M1**
$x = 0.83, 2.51, =0.15, -2.79$	**M1 M1**