A-Level Unit Test: Trigonometry

Proof

1. Show that the equation $\tan 2x = 5 \sin 2x$ can be written in the form $(1 - 5\cos 2x)\sin 2x = 0$ **(3)**

2. Prove that $\tan x + \cot x \equiv 2\text{cosec } 2x$ **(4)**

3. Prove $\frac{1-\cos 2x}{\sin 2x} = \tan x$ **(3)**

4. Prove $\text{cosec } x + \tan x = \cot \frac{x}{2}$ **(4)**

5. Show that $\text{cosec } 2x + \cot 2x = \cot x$ **(4)**

6. By writing $3x = (2x + x)$, show that $3x = 3\sin x - 4\sin^3 x$ **(4)**

7. Use the identity $\cos^2 x + \sin^2 x = 1$ to prove that $\tan^2 x = \sec^2 x - 1$ **(2)**

8. Show that $(\sin x + \tan x)(\cos x + \cot x) \equiv (1 + \sin x)(1 + \cos x)$ **(3)**

9. Show that $\sec^2 x - \sin^2 x \equiv \tan^2 x + \cos^2 x$ **(2)**

10. Prove that $\frac{1-\sin 2x}{cosec\, x - 2\cos x} \equiv \sin x$ **(3)**

11. Prove that $\sin x + \sin 2x + \sin 3x \equiv \sin 2x\,(2\cos x + 1)$ **(2)**

12. Prove the identity $\frac{1-\cos x}{1+\cos x} = tan^2\, \frac{x}{2}$ **(2)**

Total marks: 36

Mark Scheme

1.

$\tan 2x = 5\sin 2x$	**M1**
$\frac{\sin 2x}{\cos 2x} = 5\sin 2x$ $\sin 2x = 5\sin 2x \cos 2x$ $\sin 2x - 5\sin 2x \cos 2x = 0$	**M1**
$(\sin 2x)(1 - 5\cos 2x) = 0$ $(1 - 5\cos 2x)\sin 2x = 0$	**M1**

2.

$\tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{sin^2x + cos^2x}{cosx\, sinx}$	**M1**
$\frac{sin^2x + cos^2x}{cosx\, sinx} = \frac{1}{cosx\, sinx} \times \frac{2}{2}$	**M1**
$= \frac{2}{2\cos x \sin x}$	**M1**
$= \frac{2}{sin2x}$ $= 2\,\text{cosec}\, 2x$	**M1**

3.

$\frac{1-\cos 2x}{\sin 2x} = \frac{1-(1-2sin^2x)}{2sinxcosx}$	**M1**
$= \frac{1-1+2sin^2x}{2sinxcosx}$ $= \frac{\sin x}{\cos x}$	**M1**
$= \tan x$	**M1**

4.

$\text{cosec}\, x + \tan x = \frac{1}{\sin x} + \frac{\cos x}{\sin x}$	**M1**
$= \frac{1+\cos x}{\sin x}$	**M1**
$= \frac{1+2cos^2\frac{x}{2}-1}{2sin\frac{x}{2}cos\frac{x}{2}} = \frac{2cos^2\frac{x}{2}}{2sin\frac{x}{2}cos\frac{x}{2}}$	**M1**
$= \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} = \cot\frac{x}{2}$	**M1**

5.

$\text{cosec}\, 2x + \cot 2x = \frac{1}{\sin 2x} + \frac{\cos 2x}{\sin 2x}$ $= \frac{1+\cos 2x}{\sin 2x}$	**M1**
$= \frac{1+(2cos^2x-1)}{2sin\,x\cos x}$	**M1**
$= \frac{2cos^2x}{2\sin x\cos x}$ $= \frac{\cos x}{\sin x}$	**M1**
$= \cot x$	**M1**

6.

$\sin(3x) = \sin(2x + x)$ $= \sin 2x \cos x + \sin x \cos 2x$ $= 2\sin x \cos x (\cos x) + \sin x \cos 2x$	**M1**
$= 2\sin x \cos^2 x + \sin x(1 - 2\sin^2 x)$	**M1**
$= 2\sin x(1 - \sin^2 x) + \sin x(1 - 2\sin^2 x)$	**M1**
$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$ $= 3\sin x - 4\sin^3 x$	**M1**

7.

$\cos^2 x + \sin^2 x = 1$ $\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$	**M1**
$1 + \tan^2 x = \sec^2 x$ Therefore, $\tan^2 x = \sec^2 x - 1$	**M1**

8.

$= \sin x \cos x + \sin x \cot x + \tan x \cos x + 1$ $= \sin x \cos x + \cos x + \sin x + 1$	**M1**
$= \sin x(\cos x + 1) + \cos x + 1$ $= (\cos x + 1)(\sin x + 1)$	**M1**

9.

(LHS) $1 + \tan^2 x - (1 - \cos^2 x)$	**M1**
$= \tan^2 x + \cos^2 x$	**M1**

10.

(LHS) $= \frac{\sin x(1-\sin 2x)}{\sin x(\operatorname{cosec} x - 2\cos x)}$	**M1**
$= \frac{\sin x(1-\sin 2x)}{1-2\sin x \cos x}$	**M1**
$= \frac{\sin x(1-\sin 2x)}{1-\sin 2x}$ $= \sin x$	**M1**

11.

(LHS) $= 2 \sin \frac{x+3x}{2} \cos \frac{x-3x}{2} + \sin 2x$	**M1**
$= 2 \sin 2x \cos(-x) + \sin 2x$ $= 2 \sin 2x \cos x + \sin 2x$ $= \sin 2x(2 \cos x + 1)$ (RHS)	**M1**

12.

(LHS) $= \frac{1-(1-2\sin^2\frac{x}{2})}{1+(2\cos^2\frac{x}{2}-1)}$ $= \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$	**M1**
$= \tan^2 \frac{x}{2}$	**M1**