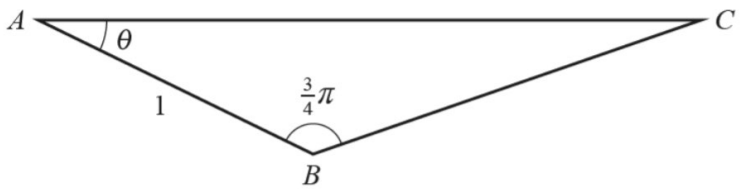
A-Level Unit Test: Trigonometry

Small Angle Approximations

1. Given that $\emptyset$ is small and is measured in radians, use the small angle approximations to find an approximate value of $\frac{1-cos4\emptyset}{2\emptyset\, sin3\emptyset}$ **(3)**

2. The diagram shows triangle ABC in which angle $A = \emptyset$ radians, angle $B = \frac{3}{4}\pi$ radians and $AB = 1$ unit.



a. Use the sine rule to show that $AC = \frac{1}{\cos\emptyset - \sin\emptyset}$ **(3)**

b. Given that $\emptyset$ is a small angle, use the result in part (i) to show that, $AC \approx 1 + p\emptyset + q\emptyset^2$, where p and q are constants to be determined. **(3)**

3. When $\emptyset$ is small, show that the equation $\frac{1+\sin\emptyset+\tan 2\emptyset}{2\cos 3\emptyset-1}$ can be written as $\frac{1}{1-3\emptyset}$ **(4)**

b. Hence write down the value of $\frac{1+\sin\emptyset+\tan 2\emptyset}{2\cos 3\emptyset-1}$ when $\emptyset$ is small. **(1)**

4a. When x is small, show that $\tan(3x)\cos(2x)$ can be approximated by $3x - 6x^3$ **(3)**

b. Hence, approximate the value of $\tan(0.3)\cos(0.2)$ **(2)**

c. Calculate the percentage error in your approximation. **(1)**

Total marks: 20

Mark Scheme

1.

$cos4\emptyset = 1 - \frac{(4\emptyset)^2}{2}$	**M1**
$sin3\emptyset = 3\emptyset$	**M1**
$\frac{1-cos4\emptyset}{2\emptyset\, sin3\emptyset} \approx \frac{1-[1-\frac{(4\emptyset)^2}{2}]}{(2\emptyset)(3\emptyset)} \approx \frac{8\emptyset^2}{6\emptyset^2} \approx \frac{4}{3}$	**M1**

2a.

$\frac{AC}{sin\frac{3}{4}\pi} = \frac{1}{\sin(\pi-\frac{3}{4}\pi-\emptyset)}$	**M1**
$AC = \frac{sin\frac{3}{4}\pi}{sin\frac{1}{4}\pi\cos\emptyset - cos\frac{1}{4}\pi\sin\emptyset}$	**M1**
$sin\frac{3}{4}\pi = sin\frac{1}{4}\pi = cos\frac{1}{4}\pi$ $AC = \frac{1}{\cos\emptyset-\sin\emptyset}$	**M1**

2b.

$AC = (1 + (-\emptyset - \frac{1}{2}\emptyset^2))^{-1}$	**M1**
$AC = 1 + (-1)(-\emptyset - ½\,\emptyset^2) + \frac{(-1)(-2)}{2}(-\emptyset - ½\,\emptyset^2) + \ldots.$	**M1**
Therefore, $AC \approx 1 + \emptyset + \frac{3}{2}\emptyset^2$	**M1**

3a.

$2\cos 3\emptyset \approx 2(1 - \frac{9\emptyset^2}{2}) = 2 - 9\emptyset^2$	**M1**
$2\cos 3\emptyset - 1 \approx 1 - 9\emptyset^2 = (1 - 3\emptyset)(1 + 3\emptyset)$	**M1**
$1 + \sin\emptyset + \tan 2\emptyset = 1 + \emptyset + 2\emptyset = 1 + 3\emptyset$	**M1**
$\frac{1+\sin\emptyset+\tan 2\emptyset}{2\cos 3\emptyset-1} = \frac{1+3\emptyset}{(1-3\emptyset)(1+3\emptyset)} = \frac{1}{1-3\emptyset}$	**M1**

3b.

When $\emptyset$ is small, $\frac{1}{1-3\emptyset} \approx 1$	**M1**

4a.

$\tan(3x)\cos(2x) = 3x(1 - \frac{(2x)^2}{2})$	**M1**
$= 3x(1 - 2x^2)$	**M1**
$= 3x - 6x^3$	**M1**

4b.

$x = 0.1$, $3(0.1) - 6(0.1)^3 = 0.294$	**M1**

4c.

$\tan(0.3)\cos(0.2) = 0.3031701196$	**M1**
% error $= \frac{0.3031701196-0.294}{0.3031701196} \times 100$	**M1**
= 3.02% (to 2 decimal places)	**M1**