A-Level Unit Test: Trigonometry

Solving Trig Equations

1a. Express $3\cos x + 4\sin x$ in the form $R\cos(x - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90$. **(4)**
b. Hence find the maximum value of $3\cos x + 4\sin x$ and the smallest positive value of x for which this maximum occurs. **(3)**

The temperature, f(t), of a warehouse is modelling using the equation

$$f(t) = 10 + 3\cos(15t) + 4\sin(15t)$$

Where t is the time in hours from midday $0 \leq t < 24$.

c. Calculate the minimum temperature of the warehouse as given by this model. **(2)**
d. Find the value of t when this minimum temperature occurs. **(2)**

2a. Express $2\sin x$ $1.5\cos x$ in the form $4(\sin x - \alpha)$, where $R > 0$ and $0 < x < \frac{\pi}{2}$. Give the value of α to 4 decimal places. **(3)**
bi. Find the maximum value of $2\sin x - 1.5\cos x$. **(1)**
bii. Find the value of x, for $0 \leq x < \pi$, at which this maximum occurs. **(2)**

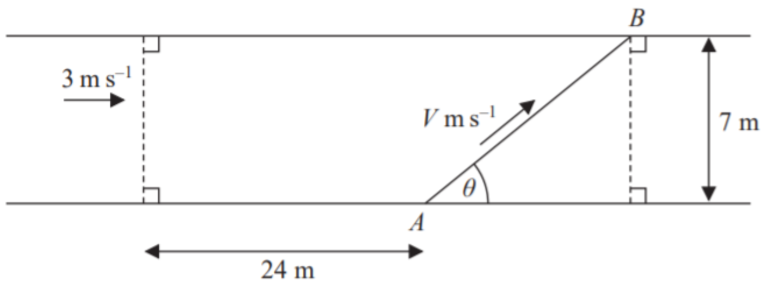
Tom models the height of the sea water, H metres, on a particular day by the equation

$$H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right),\ 0 \leq t < 12$$

Where t hours is the number of hours after midday.

c. Calculate the maximum value of H predicted by this model and the value of t to 2 decimal places, when this maximum occurs. **(3)**

3. Kate crosses a road, of constant width 7m in order to take a photograph of a marathon runner, John, approaching at 3ms^{-1}. Kate is 24 m ahead of John when she starts to cross the road from the fixed point A. John passes her as she reaches the other side of the road at a variable point B, as shown in Figure 2. Kate's speed is V ms^{-1} and she moves in a straight line, which makes an angle x, $0 < x < 150$, with the edge of the road, as shown in the figure below.



You may assume that V is given by the formula,

$$V = \frac{21}{24\sin x + 7\cos x},\ 0 < x < 150$$

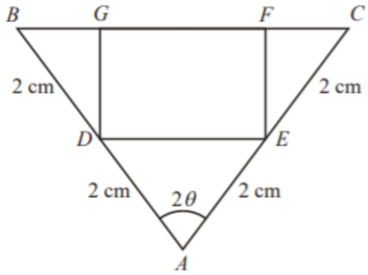
a. Express $24\sin x + 7\cos x$ in the form $R\cos(x - \alpha)$, where R and α are constants and where $R > 0$ and $0 < \alpha < 90$, giving the value of α to 2 decimal places. **(3)**

Given that x varies,
b. Find the minimum value of V **(2)**

Given that Kate's speed has the value found in part b,
c. Find the distance AB **(3)**

Given that Kate's speed in 1.68ms^{-1}
d. Find the two possible values of angle x, given that $0 < x < 150$ **(5)**

4. The diagram below shows an isosceles triangle ABC with $AB = AC = 4$cm and angle $BAC = 2x$. The midpoints of AB and AC are D and E respectively. Rectangle $DEFG$ is drawn, with F and G on BC. The perimeter of rectangle $DEFG$ is Pcm.



a. Show that $DE = 4 \sin x$ **(2)**
b. Show that $P = 8 \sin x + 4 \cos x$ **(2)**
c. Express P in the form $\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$ **(4)**
Given that $P = 8.5$
d. Find, to 3 significant figures, the possible values of x. **(4)**

Total marks: 84

Jelinva Maths

Mark Scheme

1a.

$R^2 = 3^2 + 4^2$	**M1**
$R = 5$	**M1**
$\tan\alpha = \frac{4}{3}$	**M1**
$\alpha = 53°$	**M1**

1b.

Maximum value is 5	**M1**
At maximum $\cos(x - \alpha) = 1$	**M1**
$x - \alpha = 0$ $x - 53 = 0$ $x = 53°$	**M1**

1c.

$f(t) = 10 + 5\cos(15t - \alpha)$ Minimum occurs when $\cos(15t - \alpha) = -1$	**M1**
The minimum temperature is (10 - 5) = 5	**M1**

1d.

$15t - \alpha = 180$	**M1**
$t = 15.5$	**M1**

2a.

$R = \sqrt{6.25} = 2.5$	**M1**
$\tan\alpha = \frac{1.5}{2}$	**M1**
$\alpha = 0.6435$	**M1**

2bi.

Max value = 2.5	**M1**

2bii

$\sin(x - 0.6435) = 1$	**M1**
$x = 2.21$	**M1**

2c.

$H_{max} = 8.5$	**M1**
$\sin(\frac{4\pi t}{25} - 0.6435) = 1$	**M1**
$t = 4.41$	**M1**

2d.

$6 + 2\sin(\frac{4\pi t}{25} - 0.6435) - 1.5\cos(\frac{4\pi t}{25} - 0.6435) = 7$	**M1 M1**
$\frac{4\pi t}{25} - 0.6435 = \sin^{-1}(0.4)$	**M1**
$t = 2.1$	**M1**
$\frac{4\pi t}{25} - 0.6435 = (\pi - 0.411517)$ $t = 14:06$	**M1**
$\frac{4\pi t}{25} - 0.6435 = 2.730076$ $t = 18:43$	**M1**

3a.

$R^2 = 7^2 + 24^2$ $R = 5$	**M1**
$\tan \alpha = \frac{24}{7}$	**M1**
$\alpha = 73.74°$	**M1**

3b.

Maximum value of 24sinx + 7cos x = 25	**M1**
Therefore Vmin = $\frac{21}{25}$ = 0.84	**M1**

3c.

Distance AB = $\frac{7}{\sin x}$ $x = \alpha$	**M1**
Therefore distance = 7.29	**M1**

3d.

$R\cos(x - \alpha) = \frac{21}{1.68}$	**M1**
$\cos(x - \alpha) = 0.5$	**M1**
$x - \alpha = 60$	**M1**
$x = 133.7$	**M1**
$x = 13.7$	**M1**

4a.

Complete method for DE [e.g. split triangle ADE and sin, or sine or cos rule]	**M1**
DE = $4 \sin x$	**M1**

4b.

P = 2 DE + 2EF or equivalent. With attempt at EF	**M1**
$= 8\sin x + 4\cos x$	**M1**

4c.

$8\sin x + 4\cos x = R \sin (x + \alpha) = R \sin x \cos \alpha + R \cos x \sin \alpha$	**M1**
$R^2 = 8^2 + 4^2$ $R = 4\sqrt{5}$	**M1**
$\tan \alpha = 0.5$	**M1**
$\alpha = 0.464$	**M1**

4d.

$4\sqrt{5} \sin(x + \alpha) = 8.5$	**M1**
$x + 0.464 = \sin^{-1} (\frac{8.5}{4\sqrt{5}})$	**M1**
$x = 0.791$	**M1**
$x = 1.42$	**M1**