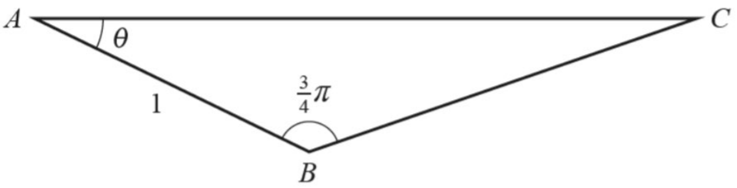
A-Level Unit Test: Trigonometry

Small Angle Approximations

1. Given that $\emptyset$ is small and is measured in radians, use the small angle approximations to find an approximate value of $\frac{1-cos4\emptyset}{2\emptyset\ sin3\emptyset}$ **(3)**

2. The diagram shows triangle *ABC* in which angle $A = \emptyset$ radians, angle $B = \frac{3}{4}\pi$ radians and $AB = 1$ unit.



a. Use the sine rule to show that $AC = \frac{1}{\cos\emptyset - \sin\emptyset}$ **(3)**

b. Given that $\emptyset$ is a small angle, use the result in part (i) to show that, $AC \approx 1 + p\emptyset + q\emptyset^2$, where p and q are constants to be determined. **(3)**

3. When $\emptyset$ is small, show that the equation $\frac{1+\sin\emptyset+\tan 2\emptyset}{2\cos 3\emptyset-1}$ can be written as $\frac{1}{1-3\emptyset}$ **(4)**

b. Hence write down the value of $\frac{1+\sin\emptyset+\tan 2\emptyset}{2\cos 3\emptyset-1}$ when $\emptyset$ is small. **(1)**

4a. When x is small, show that $\tan(3x)\cos(2x)$ can be approximated by $3x - 6x^3$ **(3)**
b. Hence, approximate the value of tan (0.3)cos(0.2) **(2)**
c. Calculate the percentage error in your approximation. **(1)**

Total marks: 20

Mark Scheme

1.

$cos4\emptyset = 1 - \frac{(4\emptyset)^2}{2}$	**M1**
$sin3\emptyset = 3\emptyset$	**M1**
$\frac{1-cos4\emptyset}{2\emptyset\ sin3\emptyset} \approx \frac{1-[1-\frac{(4\emptyset)^2}{2}]}{(2\emptyset)(3\emptyset)} \approx \frac{8\emptyset^2}{6\emptyset^2} \approx \frac{4}{3}$	**M1**

2a.

$\frac{AC}{sin\frac{3}{4}\pi} = \frac{1}{\sin(\pi-\frac{3}{4}\pi-\emptyset)}$	**M1**
$AC = \frac{sin\frac{3}{4}\pi}{sin\frac{1}{4}\pi\cos\emptyset - cos\frac{1}{4}\pi\sin\emptyset}$	**M1**
$sin\frac{3}{4}\pi = sin\frac{1}{4}\pi = cos\frac{1}{4}\pi$ $AC = \frac{1}{\cos\emptyset-\sin\emptyset}$	**M1**

2b.

$AC = (1 + (-\emptyset - \frac{1}{2}\emptyset^2))^{-1}$	**M1**
$AC = 1 + (-1)(-\emptyset - ½\emptyset^2) + \frac{(-1)(-2)}{2}(-\emptyset - ½\emptyset^2) + \ldots.$	**M1**
Therefore, $AC \approx 1 + \emptyset + \frac{3}{2}\emptyset^2$	**M1**

3a.

$2\cos 3\emptyset \approx 2(1 - \frac{9\emptyset^2}{2}) = 2 - 9\emptyset^2$	**M1**
$2\cos 3\emptyset - 1 \approx 1 - 9\emptyset^2 = (1 - 3\emptyset)(1 + 3\emptyset)$	**M1**
$1 + \sin\emptyset + \tan 2\emptyset = 1 + \emptyset + 2\emptyset = 1 + 3\emptyset$	**M1**
$\frac{1+\sin\emptyset+\tan 2\emptyset}{2\cos 3\emptyset-1} = \frac{1+3\emptyset}{(1-3\emptyset)(1+3\emptyset)} = \frac{1}{1-3\emptyset}$	**M1**

3b.

When $\emptyset$ is small, $\frac{1}{1-3\emptyset} \approx 1$	**M1**

4a.

$\tan(3x)\cos(2x) = 3x(1 - \frac{(2x)^2}{2})$	**M1**
$= 3x(1 - 2x^2)$	**M1**
$= 3x - 6x^3$	**M1**

4b.

$x = 0.1$, $3(0.1) - 6(0.1)^3 = 0.294$	**M1**

4c.

$\tan(0.3)\cos(0.2) = 0.3031701196$	**M1**
% error $= \frac{0.3031701196-0.294}{0.3031701196} \times 100$	**M1**
= 3.02% (to 2 decimal places)	**M1**

A-Level Unit Test: Trigonometry
Proof

1. Show that the equation tan $2x$ = 5 sin $2x$ can be written in the form (1 – 5cos $2x$)sin $2x$ = 0 **(3)**

2. Prove that tan x + cot x ≡ 2cosec $2x$ **(4)**

3. Prove $\frac{1-\cos 2x}{\sin 2x} = \tan x$ **(3)**

4. Prove cosec x + tan x = cot $\frac{x}{2}$ **(4)**

5. Show that cosec $2x$ + cot $2x$ = cot x **(4)**

6. By writing $3x = (2x + x)$, show that $3x = 3\sin x - 4\sin^3 x$ **(4)**

7. Use the identity $\cos^2 x + \sin^2 x = 1$ to prove that $\tan^2 x = \sec^2 x - 1$ **(2)**

8. Show that $(\sin x + \tan x)(\cos x + \cot x) \equiv (1 + \sin x)(1 + \cos x)$ **(3)**

9. Show that $\sec^2 x - \sin^2 x \equiv \tan^2 x + \cos^2 x$ **(2)**

10. Prove that $\frac{1-\sin 2x}{cosec\, x - 2\cos x} \equiv \sin x$ **(3)**

11. Prove that $\sin x + \sin 2x + \sin 3x \equiv \sin 2x\,(2\cos x + 1)$ **(2)**

12. Prove the identity $\frac{1-\cos x}{1+\cos x} = \ tan^2\ \frac{x}{2}$ **(2)**

Total marks: 36

Mark Scheme

1.

tan $2x$ = 5sin $2x$	**M1**
$\frac{\sin 2x}{\cos 2x} = 5\sin 2x$ sin $2x$ = 5sin $2x$ cos $2x$ sin $2x$ – 5sin $2x$ cos $2x$ = 0	**M1**
(sin $2x$)(1 – 5cos $2x$) = 0 (1 – 5 cos $2x$) sin $2x$ = 0	**M1**

2.

tan x + cot x = $\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{sin^2x + cos^2x}{cosx\ sinx}$	**M1**
$\frac{sin^2x + cos^2x}{cosx\ sinx} = \frac{1}{cosx\ sinx} \times \frac{2}{2}$	**M1**
$= \frac{2}{2\cos x \sin x}$	**M1**
$= \frac{2}{sin2x}$ = 2 cosec $2x$	**M1**

3.

$\frac{1-\cos 2x}{\sin 2x} = \frac{1-(1-2sin^2x)}{2sinxcosx}$	**M1**
$= \frac{1-1+2sin^2x}{2sinxcosx}$ $= \frac{\sin x}{\cos x}$	**M1**
= tan x	**M1**

4.

cosec x + tan x = $\frac{1}{\sin x} + \frac{\cos x}{\sin x}$	**M1**
$= \frac{1+\cos x}{\sin x}$	**M1**
$= \frac{1+2cos^2\frac{x}{2}-1}{2sin\frac{x}{2}cos\frac{x}{2}} = \frac{2cos^2\frac{x}{2}}{2sin\frac{x}{2}cos\frac{x}{2}}$	**M1**
$= \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} = \cot\frac{x}{2}$	**M1**

5.

cosec $2x$ + cot $2x$ = $\frac{1}{\sin 2x} + \frac{\cos 2x}{\sin 2x}$ $= \frac{1+\cos 2x}{\sin 2x}$	**M1**
$= \frac{1+(2cos^2x-1)}{2sin\,x\cos x}$	**M1**
$= \frac{2cos^2x}{2\sin x\cos x}$ $= \frac{\cos x}{\sin x}$	**M1**
= cot x	**M1**

6.

sin ($3x$) = sin ($2x + x$) = sin $2x$ cos x + sin x cos $2x$ = 2sin$x$ cos x (cos x) + sin x cos $2x$	**M1**
= 2sin x $\cos^2 x$ + sin x(1 – $2\sin^2 x$)	**M1**
= 2 sin x(1 – $\sin^2 x$) + sin x(1 – $2\sin^2 x$)	**M1**
= 2 sin x – $2\sin^3 x$ + sin x – $2\sin^3 x$ = 3sin x – $4\sin^3 x$	**M1**

7.

$\cos^2 x + \sin^2 x = 1$ $\frac{cos^2 x}{cos^2 x} + \frac{sin^2 x}{cos^2 x} = \frac{1}{cos^2 x}$	**M1**
$1 + \tan^2 x = \sec^2 x$ Therefore, $\tan^2 x = \sec^2 x - 1$	**M1**

8.

$= \sin x \cos x + \sin x \cot x + \tan x \cos x + 1$ $= \sin x \cos x + \cos x + \sin x + 1$	**M1**
$= \sin x(\cos x + 1) + \cos x + 1$ $= (\cos x + 1)(\sin x + 1)$	**M1**

9.

(LHS) $1 + \tan^2 x - (1 - \cos^2 x)$	**M1**
$= \tan^2 x + \cos^2 x$	**M1**

10.

(LHS) $= \frac{\sin x(1-sin2x)}{\sin x(cosec\ x-2cosx)}$	**M1**
$= \frac{\sin x(1-\sin 2x)}{1-2\sin x\cos x}$	**M1**
$= \frac{\sin x(1-sin2x)}{1-\sin 2x}$ $= \sin x$	**M1**

11.

(LHS) $= 2\sin\frac{x+3x}{2}\cos\frac{x-3x}{2} + \sin 2x$	**M1**
$= 2\sin 2x\cos(-x) + \sin 2x$ $= 2\sin 2x\cos x + \sin 2x$ $= \sin 2x(2\cos x + 1)$ (RHS)	**M1**

12.

(LHS) $= \frac{1-(1-2sin^2\frac{x}{2})}{1+(2cos^2\frac{x}{2}-1)}$ $= \frac{2sin^2\frac{x}{2}}{2cos^2\frac{x}{2}}$	**M1**
$= \tan^2\frac{x}{2}$	**M1**

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Solving Trig Equations

1. It is a given that $2\text{cosec}^2 x = 5 - 5\cot x$
a. Show that the equation that $2\text{cosec}^2 x = 5 - 5\cot x$ can be written in the form $2\cot^2 x + 5\cot x - 3 = 0$. **(2)**
b. Hence show that $\tan x = 2$ or $\tan x = -\frac{1}{3}$ **(2)**
c. Hence, or otherwise, solve the equation $2\text{cosec}^2 x = 5 - 5\cot x$, giving all values of x in radians to one decimal place in the interval $-\pi < x \le \pi$ **(3)**

2a. Show that the equation $2\cot^2 x + 5\text{cosec}\, x = 10$ can be written in the form $2\text{cosec}^2 x + 5\text{cosec}\, x - 12 = 0$ **(2)**
b. Hence find the values of $\sin x$. **(3)**
c. Hence, or otherwise, solve the equation, $2\cot^2(\theta - 0.1) + 5\,\text{cosec}(\theta - 0.1) = 10$. Give all values of θ in radians to two decimal places in the interval $-\pi < x \le \pi$. **(3)**

3a. Express $5\cos x - 3\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$ **(4)**
b. Hence, or otherwise, solve the equation $5\cos x - 3\sin x = 4$, for $0 < \alpha < 2\pi$, giving your answers to 2 decimal places. **(3)**

4a. Prove that for all value of x, $\sin x + \sin(60 - x) = \sin(60 + x)$ **(4)**
b. Given that $\sin 84 - \sin 36 = \sin \alpha$, deduce the exact value of the acute angle α **(2)**
c. Solve the equation $4\sin 2x + \sin(60 - 2x) = \sin(60 + 2x) - 1$, for values of x in the interval $0 \le x < 360$, giving your answers to one decimal place **(5)**

5a. By writing $3x = 2x + x$, show that $3x = 3\sin x - 4\sin^3 x$ **(4)**
b. Hence, or otherwise, for $0 < x < \frac{\pi}{3}$, solve, $8\sin^3 x - 6\sin x + 1 = 0$ **(5)**
c. Using $\sin(x - \alpha) = \sin x\cos\alpha - \cos x\sin\alpha$, or otherwise show that $\sin 15 = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ **(4)**

6a. Show that $\frac{\sin 2x}{1+\cos 2x} = \tan x$ **(2)**
b. Hence find, for $-180 \le x \le 180$, all the solutions of $\frac{2\sin 2x}{1+\cos 2x} = 1$ **(3)**

7. Find all the solutions of $2\cos 2x = 1 - 2\sin x$ in the interval $0 \le x < 360$ **(5)**

8a. By first expanding $(\cos 2x + x)$, prove that $\cos 3x = 4\cos^3 x - 3\cos x$ **(4)**
b. Hence prove that $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$ **(3)**
c. Show that the only solutions of the equation $1 + \cos 6x = 18\cos^2 x$ are odd multiples of 90. **(5)**

9a. Express $\cos 2x$ in terms of $\sin x$. **(1)**
b. Hence that $3\sin x - \cos 2x = 2\sin^2 x + 3\sin x - 1$ for all values of x **(2)**
c. Solve the equation $3\sin x - \cos 2x = 1$ for $0 < x < 360$ **(4)**

10a. Show that the equation $2\cot^2 x + 5\text{cosec}\, x = 10$ can be written in the form $2\text{cosec}^2 x + 5\,\text{cosec}\, x - 12 = 0$ **(2)**
b. Hence show that $\sin x = -\frac{1}{4}$ or $\sin x = \frac{2}{3}$ **(3)**
c. Hence, or otherwise, solve the equation, $2\cot^2(x - 0.1) + 5\,\text{cosec}(x - 0.1) = 10$, giving all values of x in radians to two decimal places in the interval $-\pi < x < \pi$ **(4)**

Total marks: 84

Mark Scheme

1a.

$2\ \text{cosec}^2 x = 5(1 - \cot x)$	**M1**
$2 + 2\cot^2 x = 5 - 5\cot x$ (use of $\text{cosec}^2 x = 1 + \cot^2 x$)	**M1**
$2\cot^2 x + 5\cot x = 3$	

1b.

$(2\cot x - 1)(\cot x + 3) = 0$	**M1**
$\cot x = \frac{1}{2} \rightarrow \tan x = 2$ $\cot x = -3 \rightarrow \tan x = -\frac{1}{3}$	**M1**

1b.

$\tan x = 2 \rightarrow x = 1.1, -2.0$	**M1**
$\tan x = -\frac{1}{3} \rightarrow x = -0.3, 2.8$	**M1**

2a.

$2(\text{cosec}^2 x - 1) + 5\ \text{cosec}\ x = 10$	**M1**
$2\text{cosec}^2 x - 2 + 5\text{cosec}\ x - 10 = 0$ $2\text{cosec}^2 x + 5\text{cosec}\ x - 12 = 0$	**M1**

2b.

$(2\text{cosec}\ x - 3)(\text{cosec}\ x + 4) = 0$	**M1**
$\text{cosec}\ x = \frac{3}{2} \rightarrow \sin x = \frac{2}{3}$	**M1**
$\text{cosec}\ x = -4 \rightarrow \sin x = -\frac{1}{4}$	**M1**

2c.

$\theta - 0.1 = 0.73, 2.41, -0.25, -2.89$	**M1**
$\theta = 0.83, 2.51, -0.15, -2.79$	**M1 M1**

3a.

$5\cos x - 3\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$	**M1**
Equating $\cos x$: $5 = R\cos\alpha$ Equating $\sin x$: $3 = R\sin\alpha$	**M1**
$R = \sqrt{5^2 + 3^2} = \sqrt{34}$	**M1**
$\tan\alpha = \frac{3}{5}$ $\alpha = 0.5404$	**M1**

3b.

$5\cos x - 3\sin x = 4 \rightarrow \sqrt{34}\cos(x + 0.5404) = 4$ $\cos(x + 0.5404) = \frac{4}{\sqrt{34}}$	**M1**
$x + 0.5404 = 0.81482\ldots$ $x = 0.2744\ldots$ $x = 0.27$	**M1**
$x + 0.5404 = 2\pi - 0.81482\ldots$ $x = 4.9279\ldots$ $x = 4.92$	**M1**

4a.

LHS = $\sin x + \sin 60 - \cos 60 \sin x$	**M1**
$= \sin x + \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$	**M1**
RHS = $\sin 60 \cos x + \cos 60 \sin x$	**M1**
$= \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x$	**M1**

4b.

From (a), $\sin(60 + x) - \sin(60 - x) = \sin x$ $x = 24$	**M1**
$\sin 84 - \sin 36 = \sin 24$ $\alpha = 24$	**M1**

4c.

$3\sin 2x + \sin 2x + \sin(60 - 2x) = \sin(60 + 2x) - 1$	**M1**
Using (a), $3 \sin 2x = -1$	**M1**
$2x = 199.47$ or 340.53	**M1**
$x = 99.7, 170.3$	**M1**
$x = 279.7, 350.3$	**M1**

5a.

$\sin 3x = \sin(2x + x)$ $= \sin 2x \cos x + \cos 2x \sin x$	**M1**
$= 2 \sin x \cos x (\cos x) + (1 - 2\sin^2 x)(\sin x)$	**M1**
$= 2 \sin x (1 - \sin^2 x) + \sin x - 2 \sin^3 x$	**M1**
$= 3 \sin x - 4 \sin^3 x$	**M1**

5b.

$8 \sin^3 x - 6\sin x + 1 = 0$ $-2\sin 3x + 1 = 0$	**M1**
$\sin 3x = \frac{1}{2}$	**M1**
$3x = \frac{\pi}{6}, \frac{5\pi}{6}$	**M1**
$x = \frac{\pi}{18}, \frac{5\pi}{18}$	**M1 M1**

5c.

$\sin 15 - \sin(60 - 45) = \sin 60 \cos 45 - \cos 60 \sin 45$	**M1**
$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$	**M1**
$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2}$	**M1**
$= \frac{1}{4}(\sqrt{6} - \sqrt{2})$	**M1**

6a.

$\frac{2 \sin x \cos x}{1 + 2\cos^2 x - 1} = \frac{2 \sin x \cos x}{2 \cos x \cos x}$	**M1**
$\frac{2 \sin x \cos x}{2 \cos x \cos x} = \tan x$	**M1**

6b.

$2 \tan x = 1$ $\tan x = \frac{1}{2}$	**M1**
$x = 26.6$	**M1**
$x = -153.4$	**M1**

7.

$2 \cos 2x = 1 - 2 \sin x$ $2(1 - 2 \sin^2 x) = 1 - 2 \sin x$	**M1**
$2 - 4\sin^2 x = 1 - 2 \sin x$ $4\sin^2 x - 2 \sin x - 1 = 0$	**M1**
$\sin x = \frac{1 \pm \sqrt{5}}{4}$	**M1**
$x = 54, 126, 198, 342$	**M1 M1**

8a.

$\cos 2x \cos x - \sin 2x \sin x$	**M1**
$\sin 2x = 2 \sin x \cos x$ $\cos 2x = 2\cos^2 x - 1$	**M1**
$2\cos^2 x - 1(\cos x) - 2 \sin x \cos x (\sin x)$ $2\cos^3 x - \cos x - 2\sin^2 x \cos \text{x}$ $2\cos^3 x - \cos x - 2\cos x (1 - \cos^2 x)$ $2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$	**M1**
$= 4\cos^3 \text{x} - 3 \cos \text{x}$	**M1**

8b.

$\cos 6x = 2\cos^2 3x - 1$	**M1**
$32c^6 - 48c^4 + 18c^2 - 1$	**M1 M1**

8c.

$c = \cos 6x$	**M1**
$32c^6 - 48c^4 = 0$	**M1**
$c^2 = \frac{3}{2}$	**M1**
$c = 0, 90, 270, 540…$	**M1**
Therefore c is odd multiples of 90.	**M1**

9a.

$\cos 2x = 1 - 2\sin^2 x$	**M1**

9b.

$3\sin x - \cos 2x = 3\sin x - (1 - 2\sin^2 x)$	**M1**
$= 3\sin x - 1 + 2\sin^2 x$	**M1**

9c.

$2\sin^2 x + 3\sin x - 2 = 0$ $(2\sin x - 1)(\sin x + 2) = 0$	**M1**
$2\sin x - 1 = 0$ $\sin x = \frac{1}{2}$ $x = 30, 150$	**M1** **M1**
$\sin x + 2 = 0$ $\sin x = -2$ no solutions	**M1**

10a.

$2(\text{cosec}^2 x - 1) + 5 \text{ cosec } x = 10$	**M1**
$2\text{cosec}^2 x - 2 + 5 \text{ cosec } x - 10 = 0$ $2\text{cosec}^2 x + 5 \text{ cosec } x - 12 = 0$	**M1**

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10b.

(2 cosec x – 3)(cosec x + 4) = 0	**M1**
cosec x = $\frac{3}{2}$ cosec x =2	**M1**
sin x = $\frac{2}{3}$ sin x = -$\frac{1}{4}$	**M1**

10c.

(x – 0.1) = 0.73, 2.41, -0.25, -2.89	**M1 M1**
x = 0.83, 2.51, =0.15, -2.79	**M1 M1**

A-Level Unit Test: Trigonometry

Solving Trig Equations

1a. Express $3\cos x + 4\sin x$ in the form $R\cos(x - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90$. **(4)**
b. Hence find the maximum value of $3\cos x + 4\sin x$ and the smallest positive value of x for which this maximum occurs. **(3)**

The temperature, f(t), of a warehouse is modelling using the equation

$$f(t) = 10 + 3\cos(15t) + 4\sin(15t)$$

Where t is the time in hours from midday $0 \le t < 24$.

c. Calculate the minimum temperature of the warehouse as given by this model. **(2)**
d. Find the value of t when this minimum temperature occurs. **(2)**

2a. Express $2\sin x$ $1.5\cos x$ in the form $4(\sin x - \alpha)$, where $R > 0$ and $0 < x < \frac{\pi}{2}$. Give the value of α to 4 decimal places. **(3)**
bi. Find the maximum value of $2\sin x - 1.5\cos x$. **(1)**
bii. Find the value of x, for $0 \le x < \pi$, at which this maximum occurs. **(2)**

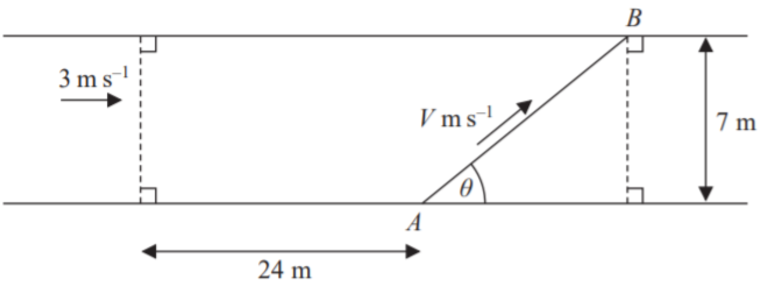
Tom models the height of the sea water, H metres, on a particular day by the equation

$$H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right), \; 0 \le t < 12$$

Where t hours is the number of hours after midday.

c. Calculate the maximum value of H predicted by this model and the value of t to 2 decimal places, when this maximum occurs. **(3)**

3. Kate crosses a road, of constant width 7m in order to take a photograph of a marathon runner, John, approaching at 3ms^{-1}. Kate is 24 m ahead of John when she starts to cross the road from the fixed point A. John passes her as she reaches the other side of the road at a variable point B, as shown in Figure 2. Kate's speed is V ms^{-1} and she moves in a straight line, which makes an angle x, $0 < x < 150$, with the edge of the road, as shown in the figure below.



You may assume that V is given by the formula,

$$V = \frac{21}{24\sin x + 7\cos x}, \; 0 < x < 150$$

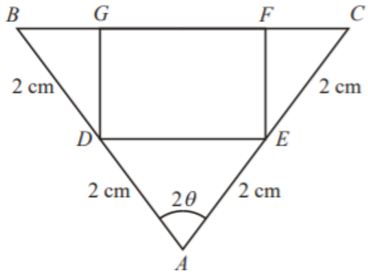
a. Express $24\sin x + 7\cos x$ in the form $R\cos(x - \alpha)$, where R and α are constants and where $R > 0$ and $0 < \alpha < 90$, giving the value of α to 2 decimal places. **(3)**

Given that x varies,
b. Find the minimum value of V **(2)**

Given that Kate's speed has the value found in part b,
c. Find the distance AB **(3)**

Given that Kate's speed in $1.68ms^{-1}$
d. Find the two possible values of angle x, given that $0 < x < 150$ **(5)**

4. The diagram below shows an isosceles triangle ABC with $AB = AC = 4$cm and angle $BAC = 2x$. The midpoints of AB and AC are D and E respectively. Rectangle $DEFG$ is drawn, with F and G on BC. The perimeter of rectangle $DEFG$ is Pcm.



a. Show that $DE = 4 \sin x$ **(2)**
b. Show that $P = 8 \sin x + 4 \cos x$ **(2)**
c. Express P in the form $\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$ **(4)**
Given that $P = 8.5$
d. Find, to 3 significant figures, the possible values of x. **(4)**

Total marks: 84

Mark Scheme

1a.

$R^2 = 3^2 + 4^2$	**M1**
$R = 5$	**M1**
$\tan \alpha = \frac{4}{3}$	**M1**
$\alpha = 53°$	**M1**

1b.

Maximum value is 5	**M1**
At maximum $\cos(x - \alpha) = 1$	**M1**
$x - \alpha = 0$ $x - 53 = 0$ $x = 53°$	**M1**

1c.

$f(t) = 10 + 5\cos(15t - \alpha)$ Minimum occurs when $\cos(15t - \alpha) = -1$	**M1**
The minimum temperature is (10 - 5) = 5	**M1**

1d.

$15t - \alpha = 180$	**M1**
$t = 15.5$	**M1**

2a.

$R = \sqrt{6.25} = 2.5$	**M1**
$\tan \alpha = \frac{1.5}{2}$	**M1**
$\alpha = 0.6435$	**M1**

2bi.

Max value = 2.5	**M1**

2bii

$\sin(x - 0.6435) = 1$	**M1**
$x = 2.21$	**M1**

2c.

$H_{max} = 8.5$	**M1**
$\sin(\frac{4\pi t}{25} - 0.6435) = 1$	**M1**
$t = 4.41$	**M1**

2d.

$6 + 2\sin(\frac{4\pi t}{25} - 0.6435) - 1.5\cos(\frac{4\pi t}{25} - 0.6435) = 7$	**M1 M1**
$\frac{4\pi t}{25} - 0.6435 = \sin^{-1}(0.4)$	**M1**
$t = 2.1$	**M1**
$\frac{4\pi t}{25} - 0.6435 = (\pi - 0.411517)$ $t = 14:06$	**M1**
$\frac{4\pi t}{25} - 0.6435 = 2.730076$ $t = 18:43$	**M1**

3a.

$R^2 = 7^2 + 24^2$ $R = 5$	**M1**
$\tan \alpha = \frac{24}{7}$	**M1**
$\alpha = 73.74°$	**M1**

3b.

Maximum value of 24sinx + 7cos x = 25	**M1**
Therefore Vmin = $\frac{21}{25}$ = 0.84	**M1**

3c.

Distance AB = $\frac{7}{\sin x}$ $x = \alpha$	**M1**
Therefore distance = 7.29	**M1**

3d.

$R\cos(x - \alpha) = \frac{21}{1.68}$	**M1**
$\cos(x - \alpha) = 0.5$	**M1**
$x - \alpha = 60$	**M1**
$x = 133.7$	**M1**
$x = 13.7$	**M1**

4a.

Complete method for DE [e.g. split triangle ADE and sin, or sine or cos rule]	**M1**
DE = 4 sin x	**M1**

4b.

P = 2 DE + 2EF or equivalent. With attempt at EF	**M1**
= 8sin x + 4cos x	**M1**

4c.

8sin x + 4cos x = R sin (x + α) = R sin x cos α + R cos x sin α	**M1**
$R^2 = 8^2 + 4^2$ $R = 4\sqrt{5}$	**M1**
$\tan \alpha = 0.5$	**M1**
$\alpha = 0.464$	**M1**

4d.

$4\sqrt{5}\ \sin(x + \alpha) = 8.5$	**M1**
$x + 0.464 = \sin^{-1}\left(\frac{8.5}{4\sqrt{5}}\right)$	**M1**
$x = 0.791$	**M1**
$x = 1.42$	**M1**