A-Level Unit Test: Series and Sequences

Binomial Expansion 1b

1. Find the binomial expansion of $\sqrt[3]{1-x}$ in ascending powers of x up to and including the term in x^3. **(4)**

2a. Find the binomial expansion of $(1+\frac{1}{4}x)^{-4}$ in ascending powers of x up to and including the term in x^3. **(4)**
b. State the values of x for which the expansion is valid for. **(2)**

3. Expand $(3-x)^{-3}$ in ascending powers of x up to and including the term x^3 and the set of values of x for which the expansion is valid.
(4)

4. Find the first 4 terms of $\frac{1-x}{\sqrt{1+2x}}$ in ascending powers of x up to and including the term in x^3. State the set of values of x for which each expansion is valid.
(5)

5. Find the series expansion of f(x) in ascending powers of x up to and including the term in x^3 and state the set of value of x for which it is valid.

$$f(x) = \frac{3+5x}{(1+3x)(1+x)^2}$$

(12)

6. The first three terms in the expansion of $(1+ax)^b$, in ascending powers of x, for $|ax| < 1$, are $1 - 6x + 24x^2$
a. Find the values of the constants a and b **(4)**
b. Find the coefficient of x^3 in the expansion. **(1)**

7. $f(x) = \frac{4}{\sqrt{1+\frac{2}{3}x}}$ $\quad -\frac{3}{2} < x < \frac{3}{2}$
a. Show that $f(\frac{1}{10}) = \sqrt{15}$ **(2)**
b. Expand f(x) in ascending powers of x up to and including the term in x^2 **(1)**
c. Use your expansion to obtain an approximation for $\sqrt{15}$, giving your answers as an exact, simplified fraction. **(2)**
d. Show that $3\frac{55}{63}$ is more accurate approximation for $\sqrt{15}$ **(2)**

Total marks: 43

Mark Scheme

1.

$(1-x)^{1/3} = 1 + \frac{1}{3}(-x) + \frac{(\frac{1}{3})(-\frac{2}{3})}{2}(-x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3 \times 2}(-x)^3 + \ldots$	**M1 M1**
$= 1 - \frac{1}{3}x - \frac{1}{9}x^2 - \frac{5}{81}x^3$	**M1 M1**

2a.

$1 + (-4)(\frac{1}{4}x) + \frac{(-4)(-5)}{2}(\frac{1}{4}x)^2 + \frac{(-4)(-5)(-6)}{3 \times 2}(\frac{1}{4}x)^3 + \ldots$	**M1 M1**
$= 1 - x + \frac{5}{8}x^2 - \frac{5}{16}x^3$	**M1 M1**

2b.

$\left\lvert\frac{1}{4}x\right\rvert < 1,$	**M1**
Therefore, expansion is valid for $\lvert x\rvert < 4$	**M1**

3.

$3^{-3}(1 - \frac{1}{3}x)^{-3} = \frac{1}{27}(1 - \frac{1}{3}x)^{-3}$	**M1**
$= \frac{1}{27}[\,1 + (-3)(-\frac{1}{3}x) + \frac{(-3)(-4)}{2}(-\frac{1}{3}x)^2 + \frac{(-3)(-4)}{3\times2}(-\frac{1}{3}x)^3 + \ldots]$	**M1**
$= \frac{1}{27} + \frac{1}{27}x + \frac{2}{81}x^2 + \frac{10}{729}x^3 + \ldots$	**M1**
$\left\lvert-\frac{1}{3}x\right\rvert < 1$ Therefore expansion is valid for $\lvert x\rvert < 3$	**M1**

4.

$(1-x)(1+2x)^{-0.5} = (1-x)[1 + (-0.5)(2x) + \frac{(-0.5)(-1.5)}{2}(2x)^2 + \frac{(-0.5)(-1.5)(-2.5)}{3\times2}(2x)^3 + \ldots]$	**M1**
$(1-x)(1 - x + 1.5x^2 - 2.5x^3 + \ldots)$	**M1**
$1 - x + 1.5x^2 - 2.5x - 3 - x + x^2 - \frac{3}{2}x^3 + \ldots$	**M1**
$1 - 2x + 2.5x^2 - 4x^3 + \ldots$	**M1**
$\lvert 2x\rvert < 1$ Therefore, expansion is valid for $\lvert x\rvert < \frac{1}{2}$	**M1**

5.

$f(x) = \frac{3+5x}{(1+3x)(1+x)^2} = \frac{A}{1+3x} + \frac{B}{1+x} + \frac{C}{(1+x)^2}$ $3 + 5x = A(1+x)^2 + B(1+3x)(1+x) + C(1+3x)$	**M1**
$x = -\frac{1}{3}, \frac{4}{3} = \frac{4}{9}A$ $A = 3$	**M1**
$x = -1, -2 = -2C$ $C = 1$	**M1**
Comparing coefficients of x^2: $0 = A + 3B$ $B = -1$	**M1**
$\frac{3+5x}{(1+3x)(1+x)^2} = \frac{3}{1+3x} - \frac{1}{1+x} + \frac{1}{(1+x)^2}$	
$\frac{3}{1+3x} = 3(1+3x)^{-1} = 3[1 + (-1)(3x) + \frac{(-1)(-2)}{2}(3x)^2 + \frac{(-1)(-2)(-3)}{3\times2}(3x)^3 + \ldots]$	**M1**
$= 3 - 9x + 27x^2 - 81x^3 + \ldots.$ $\lvert 3x\rvert < 1$ therefore, $\lvert x\rvert < \frac{1}{3}$	**M1**
$\frac{1}{1+x} = 1 + (-1)(x) + \frac{(-1)(-2)}{2}(x)^2 + \frac{(-1)(-2)(-3)}{3\times2}(x)^3 + \ldots$	**M1**

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$= 1 - x + x^2 - x^3 + \ldots.$ $\lvert x \rvert < 1$	**M1**
$\frac{1}{(1+x)^2} = (1+x)^{-2} = 1 + (-2)(x) + \frac{(-2)(-3)}{2}(x)^2 + \frac{(-2)(-3)(-4)}{3\times 2}(x)^3 + \ldots$	**M1**
$= 1 - 2x + 3x^2 - 4x^3 + \ldots$ $\lvert x \rvert < 1$	**M1**
$f(x) = (3 - 9x + 27x^2 - 81x^3 + \ldots) - (1 - x + x^2 - x^3 + \ldots) + (1 - 2x + 3x^2 - 4x^3 + \ldots)$	**M1**
$f(x) = 3 - 10x + 29x^2 - 84x^3 + \ldots$ As $\lvert x \rvert < \frac{1}{3}$ is the smallest restriction, $\lvert x \rvert < \frac{1}{3}$ for the expansion.	**M1**

6a.

$(1 + ax)^b = 1 + b(ax) + \frac{b(b-1)}{2}(ax)^2 + \ldots.$	**M1**
Therefore, $ab = -6$ and $\frac{1}{2}a^2b(b-1) = 24$	**M1**
$a = -\frac{6}{b}$ Therefore, $\frac{18}{b}(b-1) = 24$	**M1**
$18b - 18 = 24b$ $b = -3$, $a = 2$	**M1**

6b.

$\frac{(-3)(-4)(-5)}{3 \times 2}(2)^3 = -80$	**M1**

7a.

$f(\frac{1}{10}) = \frac{4}{\sqrt{1+\frac{1}{15}}}$	**M1**
$= \frac{4}{\sqrt{\frac{16}{15}}} = \sqrt{15}$	**M1**

7b.

$4(1 + \frac{2}{3}x)^{-0.5} = 4[1 + (-0.5)(\frac{2}{3}x) + \frac{(-0.5)(-1.5)}{2}(\frac{2}{3}x)^2 + \ldots]$ $= 4 - \frac{4}{3}x + \frac{2}{3}x^2 + \ldots$	**M1**

7c.

$\sqrt{15} = f(\frac{1}{10}) \approx 4 - \frac{4}{3} \times \frac{1}{10} + \frac{2}{3} \times (\frac{1}{10})^2 + \ldots$ $= 4 - \frac{2}{15} + \frac{1}{150}$ $= 3\frac{131}{150}$	**M1 M1**

7d.

$\sqrt{15} = 3.87298$ $3\frac{131}{150} = 3.87333$ $3\frac{55}{63} = 3.87301$	**M1**
Therefore, $\sqrt{15} < 3\frac{55}{63} < 3\frac{131}{150}$ Hence, $3\frac{55}{63}$ is a more accurate approximation.	**M1**

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