A-Level Unit Test: Series and Sequences

Recurrence Relations

1. For the sequence $U_n = 4U_{n-1} - k, n > 1, U_1 = k$, find expression for U_2 and U_3 in terms of the constant *k*. **(2)**

2. For the sequence $U_{n+1} = \frac{U^n}{k}, n > 1, U_1 = 4$, find expression for U_2 and U_3 in terms of the constant *k*. **(2)**

3. For the sequence $U_{n+1} = \sqrt[3]{61k^3 + {U_n}^3}, n > 0, U_1 = k\sqrt[3]{3}$, find expression for U_2 and U_3 in terms of the constant *k*. **(2)**

4. The terms of a sequence $U_1, U_2, U_3 \ldots$ are given by $U_n = 3(U_{n-1} - k)$, n > 1 where k is constant. Given that $U_1 = -4$

a. Find expressions for U_2 and U_3 in terms of *k*. **(2)**

Given also that $U_3 = 7U_2 + 3$, find

b. The value of *k*, **(2)**

c. The value of U_4. **(2)**

5. The *n*th term of a sequence, U_n , is given by

$$U_n = k^n - 3.$$

Given that $U_1 + U_2 = 0$,

a. Find the two possible values of the constant *k*. **(3)**

b. For each value of *k* found in part a, find the corresponding value of U_5. **(2)**

6. A sequence $U_1, U_2, U_3, \ldots$ is defined by

$$U_{n+1} = \frac{1}{U_n} \qquad U_1 = \frac{2}{3}$$

Find the exact value of $\sum_{r=1}^{100} U_r$ **(4)**

Total marks: 21

Mark Scheme

1.

$U_1 = k$ $U_2 = 4k - k = 3k$	**M1**
$U_3 = 4(3k) - k = 16 + 15k$	**M1**

2.

$U_1 = 4$ $U_2 = \frac{4}{k}$	**M1**
$U_3 = \frac{4}{k} \div k = \frac{4}{k^2}$	**M1**

3.

$U_1 = k\sqrt[3]{3}$ $U_2 = \sqrt[3]{61k^3 + (k\sqrt[3]{3})^3} = \sqrt[3]{61k^3 + 3k^3} = \sqrt[3]{64k^3} = 4k$	**M1**
$U_3 = \sqrt[3]{61k^3 + (4k)^3} = \sqrt[3]{61k^3 + 64k^3} = \sqrt[3]{125k^3} = 5k$	**M1**

4a.

$U_1 = -4$ $U_2 = 3(-4 - k) = -12 - 3k$	**M1**
$U_3 = 3[(-12 - 3k) - k] = -36 - 12k$	**M1**

4b.

$-36 - 12k = 7(-12 - 3k) + 3$ $9k = -45$	**M1**
$k = -5$	**M1**

4c.

$U_3 = -36 - 12(-5) = 24$	**M1**
Therefore, $U_4 = 3(24 + 5) = 87$	**M1**

5a.

$U_1 = k - 3$ $U_2 = k^2 - 3$	**M1**
$U_1 + U_2 = 0$ $k - 3 + (k^2 - 3) = 0$ $k^2 + k - 6 = 0$	**M1**
$(k + 3)(k - 2) = 0$ $k = -3 \; or \; 2$	**M1**

5b.

$k = -3$ $U_5 = (-3)^5 - 3 = -242 - 3 = -246$	**M1**
$k = 2$ $U_5 = (2)^5 - 3 = 32 - 3 = 29$	**M1**

6.

$U_1 = \frac{2}{3}$ $U_2 = \frac{3}{2}$ $U_3 = \frac{2}{3}$	**M1**

Jethwa Maths

$U_4 = \frac{3}{2}$	
Sequence is periodic with order 2. Therefore, $\sum_{r=1}^{100} U_r$ = (50 x U_1) + (50 x U_2)	**M1**
$\sum_{r=1}^{100} U_r = \left(50 \times \frac{2}{3}\right) + \left(50 \times \frac{3}{2}\right)$	**M1**
$= \frac{325}{3}$	**M1**