A-Level Unit Test: Coordinate Geometry

Circles

1. Find the cartesian equation for the curve with parametric equations,

$$x = t^3 \qquad y = 2t^2$$

(2)

2. Find the cartesian equation for the curve with parametric equations,

$$x = 2 \sec \Theta \qquad y = 4 \tan \Theta$$

(3)

3. Write down the parametric equation for a circle with centre (a, b), radius r, where a, b, and r are constant and $r > 0$.

(2)

4. A curve is given by the parametric equations,

$$x = t - 3 \qquad y = 4 - t^2$$

a. Find the cartesian equation of the curve **(2)**
b. Sketch the curve showing any points where it meets the coordinate axes. **(3)**

5. The curve C and has parametric equations:

$$x = \sqrt{3} \sin 2t \qquad y = 4 \cos^2 t \qquad 0 \leq t \leq \pi$$

Find a cartesian equation of C **(4)**

6. The curve C and has parametric equations:

$$x = 2 \cos 2t \qquad y = 6 \sin t \qquad 0 \leq t \leq \frac{\pi}{2}$$

a. Find the cartesian equation of the curve in the form

$$y = f(x) \qquad -k \leq x \leq k$$

Stating the value of the constant k. **(4)**
b. Write down the range of $f(x)$ **(2)**

Mark Scheme

1.

$x^2 = (t^3)^2 = t^6$ $y^3 = (2t)^3 = 8t^6$	**M1**
$y^3 = 8x^2$	**M1**

2.

$\sec \Theta = \frac{x}{2}$ $\tan \Theta = \frac{y}{4}$	**M1**
$1 + \tan^2 \Theta = \sec^2 \Theta$ $1 + (\frac{y}{4})^2 = (\frac{x}{2})^2$	**M1**
$16 + y^2 = 4x^2$ $y^2 = 4x^2 - 16$	**M1**

3.

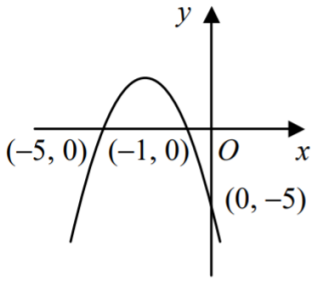
$x = a + r \cos \theta$	**M1**
$y = b + r \sin \theta$	**M1**

4a.

$t = x + 3$	**M1**
$y = 4 - (x + 3)^2$	**M1**

4b.

Shape **M1**
(-5, 0) and (-1,0) **M1**
(0, -5) **M1**



5.

$y = 4 \cos^2 t \rightarrow \frac{y}{2} = 2 \cos^2 t$ $2 \cos^2 t = 1 + \cos 2t = \frac{y}{2} \rightarrow \cos 2t = \frac{y-2}{2}$	**M1**
$\sin 2t = \frac{x}{\sqrt{3}}$	**M1**
$\sin^2 2t + \cos^2 2t = 1$	**M1**
$\frac{x^2}{3} + \frac{(y-3)^2}{4} = 1$	**M1**

6.

$x = 2 \cos 2t = 2(1 - 2\sin^2 t) = 2[1 - 2(\frac{y}{6})^2]$	**M1**
$x = 2 - \frac{4y^2}{36}$ $9x = 18 - y^2$ $y^2 = 9(2-x)$	**M1**
$y = \sqrt{9(2-x)}$ $y = 3\sqrt{2-x}$	**M1**
When $t = \frac{\pi}{2}$	**M1**

$x(\frac{\pi}{2}) = 2\cos\pi = -2$ $-2 \leq x \leq 2$ $k = 2$	

6b.

$0 \leq f(x) \leq 6$	**M1**