

A-Level Unit Test: Coordinate Geometry

Circles



1. Find the cartesian equation for the curve with parametric equations,

$$x = t^3 \quad y = 2t^2$$

(2)

2. Find the cartesian equation for the curve with parametric equations,

$$x = 2 \sec \Theta \quad y = 4 \tan \Theta$$

(3)

3. Write down the parametric equation for a circle with centre (a, b) , radius r , where a , b , and r are constant and $r > 0$.

(2)

4. A curve is given by the parametric equations,

$$x = t - 3 \quad y = 4 - t^2$$

- a. Find the cartesian equation of the curve

(2)

- b. Sketch the curve showing any points where it meets the coordinate axes.

(3)

5. The curve C and has parametric equations:

$$x = \sqrt{3} \sin 2t \quad y = 4 \cos^2 t \quad 0 \leq t \leq \pi$$

Find a cartesian equation of C

(4)

6. The curve C and has parametric equations:

$$x = 2 \cos 2t \quad y = 6 \sin t \quad 0 \leq t \leq \frac{\pi}{2}$$

- a. Find the cartesian equation of the curve in the form

$$y = f(x) \quad -k \leq x \leq k$$

Stating the value of the constant k .

(4)

- b. Write down the range of $f(x)$

(2)

Mark Scheme

1.

$x^2 = (t^3)^2 = t^6$	M1
$y^3 = (2t)^3 = 8t^6$	
$y^3 = 8x^2$	M1

2.

$\sec \Theta = \frac{x}{2}$	
$\tan \Theta = \frac{y}{4}$	M1
$1 + \tan^2 \Theta = \sec^2 \Theta$	
$1 + \left(\frac{y}{4}\right)^2 = \left(\frac{x}{2}\right)^2$	M1
$16 + y^2 = 4x^2$	
$y^2 = 4x^2 - 16$	M1

3.

$x = a + r \cos \theta$	M1
$y = b + r \sin \theta$	M1

4a.

$t = x + 3$	M1
$y = 4 - (x + 3)^2$	M1

4b.

Shape M1 (-5, 0) and (-1, 0) M1 (0, -5) M1	
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5.

$y = 4 \cos^2 t \rightarrow \frac{y}{2} = 2 \cos^2 t$	
$2 \cos^2 t = 1 + \cos 2t = \frac{y}{2} \rightarrow \cos 2t = \frac{y-2}{2}$	M1
$\sin 2t = \frac{x}{\sqrt{3}}$	M1
$\sin^2 2t + \cos^2 2t = 1$	M1
$\frac{x^2}{3} + \frac{(y-2)^2}{4} = 1$	M1

6.

$x = 2 \cos 2t = 2(1 - 2\sin^2 t) = 2\left[1 - 2\left(\frac{y}{6}\right)^2\right]$	M1
$x = 2 - \frac{4y^2}{36}$	
$9x = 18 - y^2$	M1
$y^2 = 9(2-x)$	
$y = \sqrt{9(2-x)}$	
$y = 3\sqrt{2-x}$	M1
When $t = \frac{\pi}{2}$	M1

$x\left(\frac{\pi}{2}\right) = 2 \cos \pi = -2$ $-2 \leq x \leq 2$ $k = 2$	
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6b.

$0 \leq f(x) \leq 6$	M1
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