



AS Level

Pt. 1: Index Laws & Surds
Pt. 3: Simultaneous Equations

Pt. 2: Quadratic Functions
Pt. 4: Graph Functions & Transformations

A-Level

Pt. 5: Composite Functions
Pt. 6: Modulus Functions
Pt. 7: Partial Fractions

1. Find the values of constants A and B in each identify:

$$6x + 7 \equiv A(2x - 1) + B(x + 2) \quad (2)$$

2. Find the values of the constants A and B in each identify:

$$\frac{x-9}{x^2-4x+3} \equiv \frac{A}{x-1} + \frac{B}{x-3} \quad (3)$$

3. Express in partial fractions $\frac{1-3x}{(3x+4)(2x+1)}$ (3)

4. Express in partial fractions $\frac{3x+2}{x^2-2x-24}$ (3)

5. Express in partial fractions $\frac{5x+7}{x^2+x}$ (3)

6. Express in partial fractions $\frac{4x+6}{x^2-9}$ (3)

7. Find the values of the constants A, B and C in each identity:

$$\frac{8x+14}{(x-2)(x+1)(x-6)} \equiv \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{x-6} \quad (4)$$

8. Find the values of the constants A, B and C in each identity:

$$\frac{3x^2-7x-4}{(x-3)(x-2)^2} \equiv \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2} \quad (4)$$

9. Express in partial fractions $\frac{2-9x}{(x-3)(2x-1)^2}$ (4)

10. Express in partial fractions $\frac{9x^2-2x-12}{x^3+x^2-6x}$ (5)

11. Find the values of the constants A, B and C in each identity

$$\frac{x^2+2x+9}{x^2+4x-5} \equiv A + \frac{B}{x-1} + \frac{C}{x+5} \quad (4)$$

12. Express in partial fractions $\frac{2x^2+7x}{x^2+6x+8}$ (4)

13. Express in partial fractions $\frac{x^2+3}{(x-3)(x+1)}$ (4)

Mark Scheme

1.

$x = \frac{1}{2}$ $10 = A(0) + B(2.5)$ $B = 4$	M1
$x = -2$ $-5 = -5A + B(0)$ $A = 1$	M1

2.

$x - 9 = A(x - 3) + B(x - 1)$	M1
$x = 3$ $-6 = A(0) + B(2)$ $B = -3$	M1
$x = 1$ $-8 = -2A + B(0)$ $A = 4$	M1
$\frac{x - 9}{x^2 - 4x + 3} \equiv \frac{4}{x - 1} - \frac{3}{x - 3}$	

3.

$\frac{1 - 3x}{(3x + 4)(2x + 1)} \equiv \frac{A}{(3x + 4)} + \frac{B}{(2x + 1)}$ $1 - 3x = A(2x + 1) + B(3x + 4)$	M1
$x = -1/2$ $2.5 = A(0) + B(2.5)$ $B = 1$	M1
$x = \frac{4}{3}$ $5 = A(\frac{5}{3}) + B(0)$ $A = -3$	M1
$\frac{1 - 3x}{(3x + 4)(2x + 1)} \equiv \frac{-3}{(3x + 4)} + \frac{1}{(2x + 1)}$	

4.

$\frac{3x + 2}{x^2 - 2x - 24} \equiv \frac{3x + 2}{(x - 6)(x + 4)} \equiv \frac{A}{(x - 6)} + \frac{B}{(x + 4)}$ $3x + 2 = A(x + 4) + B(x - 6)$	M1
$x = -4$ $-10 = A(0) + B(-10)$ $B = 1$	M1
$x = 6$ $20 = A(10) + B(0)$ $A = 2$	M1
$\frac{3x + 2}{x^2 - 2x - 24} \equiv \frac{2}{(x - 6)} + \frac{1}{(x + 4)}$	

5.

$\frac{5x+7}{x^2+x} \equiv \frac{5x+7}{x(x+1)} \equiv \frac{A}{x} + \frac{B}{x+1}$ $5x + 7 = A(x+1) + B(x)$	M1
$x = 0$ $7 = A(1) + B(0)$ $A = 7$	M1
$x = -1$ $2 = A(0) + B(-1)$ $B = 2$	M1
$\frac{5x+7}{x^2+x} \equiv \frac{7}{x} + \frac{2}{x+1}$	

6.

$\frac{4x+6}{x^2-9} \equiv \frac{4x+6}{(x+3)(x-3)} \equiv \frac{A}{x+3} + \frac{B}{x-3}$ $4x + 6 = A(x-3) + B(x+3)$	M1
$x = 3$ $18 = A(0) + B(6)$ $B = 3$	M1
$x = -3$ $-6 = A(-6) + B(0)$ $A = 1$	M1
$\frac{4x+6}{x^2-9} \equiv \frac{1}{x+3} + \frac{3}{x-3}$	

7.

$\frac{8x+14}{(x-2)(x+1)(x-6)} \equiv \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{x-6}$ $8x + 14 = A(x+1)(x-6) + B(x-2)(x-6) + C(x-2)(x+1)$	M1
$x = 2$ $30 = A(-12)$ $A = -\frac{2}{5}$	M1
$x = -1$ $6 = A(0) + B(21) + C(0)$ $B = \frac{7}{2}$	M1
$x = 6$ $62 = A(0) + B(0) + C(28)$ $C = \frac{31}{14}$	M1
$\frac{8x + 4}{(x-2)(x+1)(x-6)} \equiv -\frac{2}{5(x-2)} + \frac{7}{2(x+1)} + \frac{31}{14(x-6)}$	

8.

$\frac{3x^2 - 7x - 4}{(x-3)(x-2)^2} \equiv \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$ $3x^2 - 7x - 4 = A(x-2)^2 + B(x-3)(x-2) + C(x-3)$	M1
$x = 3$ $2 = 1A + B(0) + C(0)$ $A = 2$	M1
$x = 2$ $-6 = A(0) + B(0) + C(-1)$ $C = 6$	M1
$x = 0, A = 2, C = 6$ $-4 = 2 + B(6)$ $-6 = B(6)$ $B = 1$	M1
$\frac{3x^2 - 7x - 4}{(x-3)(x-2)^2} \equiv \frac{2}{x-3} + \frac{1}{x-2} + \frac{6}{(x-2)^2}$	



9.

$\frac{2-9x}{(x-3)(2x-1)^2} \equiv \frac{A}{x-3} + \frac{B}{(2x-1)} + \frac{C}{(2x-1)^2}$ $2-9x = A(2x-1)^2 + B(x-3)(2x-1) + C(x-3)$	M1
$x = \frac{1}{2}$ $-2.5 = A(0) + B(0) + C(-2.5)$ $C = 1$	M1
$x = 3$ $-25 = 25A + B(0) + C(0)$ $A = -1$	M1
$x = 0, A = -1, C = 1$ $2 = -4 + 3B + -3$ $6 = 3B$ $B = 2$	M1
$\frac{2-9x}{(x-3)(2x-1)^2} \equiv \frac{-1}{x-3} + \frac{2}{(2x-1)} + \frac{1}{(2x-1)^2}$	

10.

$\frac{9x^2-2x-12}{x^3+x^2-6x} \equiv \frac{9x^2-2x-12}{x(x+3)(x-2)} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-2}$	M1
$9x^2 - 2x - 12 = A(x+3)(x-2) + B(x)(x-2) + C(x)(x+3)$	M1
$x = 0$ $-12 = A(-6) + B(0) + C(0)$ $A = 2$	M1
$x = -3$ $75 = A(0) + B(15) + C(0)$ $B = 5$	M1
$x = 2$ $20 = A(0) + B(0) + C(10)$ $C = 2$	M1
$\frac{9x^2-2x-12}{x^3+x^2-6x} \equiv \frac{2}{x} + \frac{5}{x+3} + \frac{2}{x-2}$	

11.

$\frac{x^2 + 2x + 9}{x^2 + 4x - 5} \equiv A + \frac{B}{x-1} + \frac{C}{x+5}$ $x^2 + 2x + 9 = A(x-1)(x+5) + B(x+5) + C(x-1)$	M1
Co-efficients of x^2 : $1 = A1, A = 1$	M1
$x = 1$ $12 = A(0) + B(6) + C(0)$ $B = 2$	M1
$x = -5$ $24 = A(0) + B(0) + C(-6)$ $C = 4$	M1
$\frac{x^2+2x+9}{x^2+4x-5} \equiv 1 + \frac{2}{x-1} + \frac{4}{x+5}$	



12.

$\frac{2x^2 + 7x}{x^2 + 6x + 8} \equiv \frac{2x^2 + 7x}{(x+4)(x+2)} \equiv A + \frac{B}{x+4} + \frac{C}{x+2}$ $2x^2 + 7x = A(x+4)(x+2) + B(x-2) + C(x+4)$	M1
Co-efficients x^2 : $2 = A(1)$ $A = 2$	M1
$x = -4$ $4 = A(0) + B(-2) + C(0)$ $B = -2$	M1
$x = -2$ $-6 = A(0) + B(0) + C(2)$ $C = -3$	M1
$\frac{2x^2 + 7x}{x^2 + 6x + 8} \equiv 2 - \frac{2}{x+4} - \frac{3}{x+2}$	

13.

$\frac{x^2+3}{(x-3)(x+1)} = A + \frac{B}{x-3} + \frac{C}{x+1}$ $x^2 + 3 = A(x-3)(x+1) + B(x+1) + C(x-3)$	M1
Coefficients of x^2 : $1 = A(1)$ $A = 1$	M1
$x = 3$ $12 = A(0) + B(4) + C(0)$ $B = 3$	M1
$x = -1$ $4 = A(0) + B(0) + C(-4)$ $C = 1$	M1
$\frac{x^2 + 3}{(x-3)(x+1)} = 1 + \frac{3}{x-3} + \frac{1}{x+1}$	

