Algebra and Functions

Part 6: Modulus Functions

Jethwa Maths

AS Level		A-Level
Pt. 1: Index Laws & Surds	**Pt. 2**: Quadratic Functions	**Pt. 5**: Composite Functions
Pt. 3: Simultaneous Equations	**Pt. 4**: Graph Functions & Transformations	**Pt. 6**: Modulus Functions
		Pt. 7: Partial Fractions

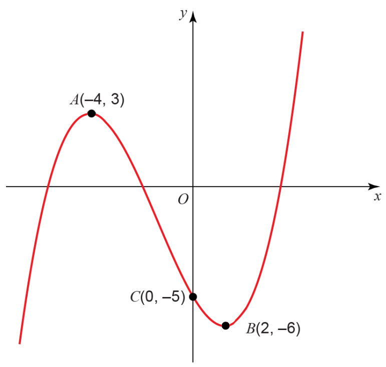
1. f(x) = $|2x + 3| - 4, x \in \mathbb{R}$

a. Sketch the graph of y = f(x), labelling its vertex and any points of intersection with the coordinate axes. **(5)**

b. Find the coordinates of the points of intersection of

$$y = |2x + 3| - 4 \text{ and } y = -\frac{1}{4}x + 2$$ **(5)**

2. The diagram shows the graph of h(x).



The points A(−4, 3) and B(2, −6) are turning points on the graph and C(0, −5) is the y-intercept. Sketch on separate diagrams, the graphs of

a. $y = |f(x)|$ **(3)**
b. $y = f(|x|)$ **(3)**
c. $y = 2f(x + 3)$ **(3)**

Where possible, label clearly the transformations of the points A, B and C on your new diagrams and give their coordinates.

3. For the constant k, where $k > 1$, the functions f and g are defined by,

$$f : x \to \ln(x + k) \qquad x > -k$$
$$g : x \to |2x - k| \qquad x \in \mathbb{R}$$

a. On separate axes, sketch the graph of f and the graph of g(x). On each sketch state, in terms of k, the coordinates of points where the graph meets to coordinate axes.

(6)

b. Write down the range of f(x). **(1)**

c. The curve C has equation y = f(x). The tangent to C at the point with x-coordinate 3 is parallel to the line with equation $9y = 2x + 1$.

(2)

d. Find the value of k. **(4)**

4. The functions f and g are defined by

$$f : x \to |x - a| + a, \qquad x \in \mathbb{R}$$
$$g : x \to 4x + x, \qquad x \in \mathbb{R}$$

Where a is a positive constant.

a. One the same diagram, sketch the graphs of f and g, showing clearly the coordinates of any points at which your graphs meet the axes.

(5)

b. Use algebra to find, in terms of a, the co-ordinates of the point at which the graphs f and g intersect. **(3)**

c. Find an expression for fg(x) **(2)**

d. Solve for x in terms of a, the equation **(3)**

$$fg(x) = 3a$$

5. Solve the equation $|x| = |2x - 3|$ **(2)**

6. A function is defined as f(x) = $|2x + 5|,\ x \in \mathbb{R}$.

a. Sketch the graph of y = f(x), showing the co-ordinates of any points where the graph meets the co-ordinate axes. **(4)**

b. Evaluate ff(-4). **(2)**

$$g(x) = f(x + k),\ x \in \mathbb{R}.$$

c. State the value of the constant k for which g(x) is symmetrical about the y-axis. **(1)**

7. Solve the equation $|3x - 4| = |2x + 3|$ **(5)**

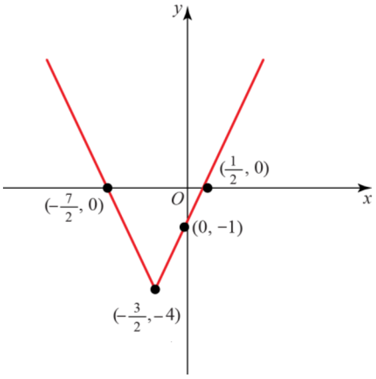
8. For each of the following, sketch y = $|f(x)|$ and y = f($|x|$) on separate axes showing the coorindates of any points of intersection with the coordinate axes.

a. f : $x \to \tan x,\ x \in \mathbb{R}, \frac{-\pi}{2} < x < \frac{\pi}{2}$ **(2)**

b. f : $x \to (1 + x)(5 - x),\ x \in \mathbb{R}$ **(4)**

Mark Scheme

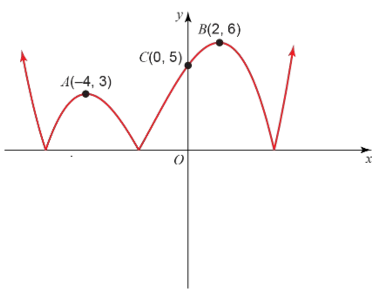
1a.

Figure 1 	Graph has a distinct V-shape.	**M1**
	Labels vertex $\left(-\frac{3}{2}, -4\right)$	**A1**
	Finds intercept with the y-axis.	**M1**
	Makes attempt to find x-intercept, for example states that $\lvert 2x+3\rvert - 4 = 0$	**M1**
	Successfully finds both x-intercepts.	**A1**

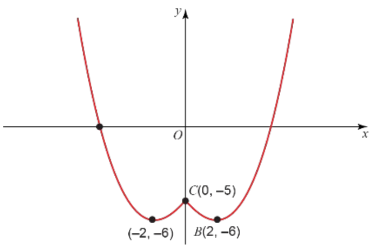
1b.

Recognises that there are two solutions. For example, writing $2x+3=-\frac{1}{4}x+2$ and $-(2x+3)=-\frac{1}{4}x+2$	**M1**
Makes an attempt to solve both questions for x, by manipulating the algebra.	**M1**
Correctly states $x = -\frac{4}{9}$ or $x = -\frac{20}{7}$. Must state both answers.	**A1**
Makes an attempt to substitute to find y.	**M1**
Correctly finds y and states both sets of coordinates correctly $\left(-\frac{4}{9}, -\frac{17}{9}\right)$ and $\left(-\frac{20}{7}, -\frac{9}{7}\right)$	**A1**

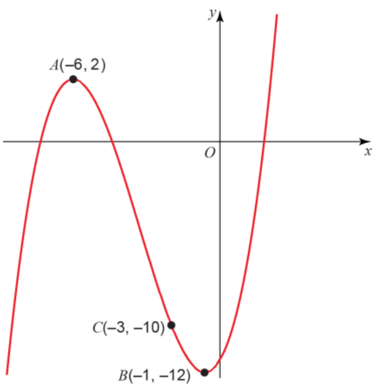
2.

Figure 2 	Clear attempt to reflect the negative part of the original graph in the x-axis.	**M1**
	Labels all three points correctly.	**A1**
	Fully correct graph.	**A1**

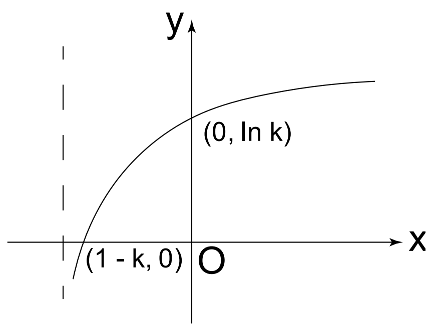
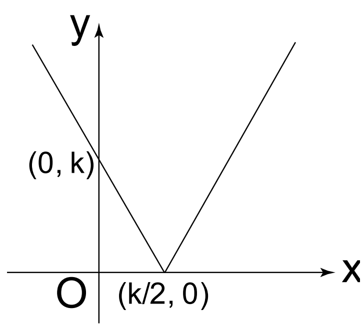
2b.

Figure 3 	Clear attempt to reflect the positive x part of the original graph in the y-axis.	**M1**
	Labels all three points correctly.	**A1**
	Fully correct graph.	**A1**

2c.

Figure 4 	Clear attempt to move the graph to the left 3 spaces.	**M1**
	Clear attempt to stretch the graph vertically by a factor of 2.	**M1**
	Fully correct graph.	**A1**

3a.

$f(x) = \ln(x + k)$	Shape **M1** Interaction with negative x-axis **M1** $(0, \ln k)$, $(1 - k, 0)$ **M1**	
$g(x) = \lvert 2x - k \rvert$	Modulus graph with V shape **M1** Vertex on positive x-axis **M1** $(0, k)$ and $(\frac{k}{2}, 0)$ **M1**	

3b.

$f(x) \in \mathbb{R}$ $-\infty < f(x) < \infty$	**M1**

3c.

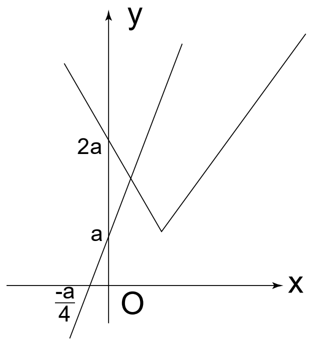
fg $(\frac{k}{4})$ = ln {k + $\left\|\frac{24}{4} - k\right\|$	**M1**
= ln$(\frac{3k}{2})$	**M1**

3d.

$\frac{dy}{dx} = \frac{1}{x+k}$	**M1**
when x = 3, $\frac{1}{3+k}$	**M1**
$= \frac{2}{9}$	**M1**
$k = 1\frac{1}{2}$	**M1**

4a.

V shape in correct orientation **M1**
Vertex in first quadrant **M1**
(0, 2a), (0, a) and $(\frac{-a}{4}, 0)$ **M1 M1 M1**



4b.

$4x + a = (a - x) + a$	**M1**
$5x = a, x = \frac{a}{5}$	**M1**
$y = \frac{9a}{5}$	**M1**

4c.

fg(x) = $\|4x + a - a\| + a$	**M1**
$\|4x\| + a$	**M1**

4d.

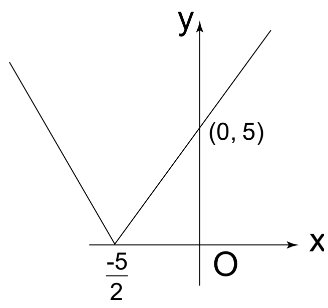
$\|4x\| + a = 3a$	**M1**
$\|4x\| = 2a$	**M1**
$x = \frac{a}{2}, -\frac{a}{2}$	**M1**

5.

$x = 2x - 3$, x = 3	**M1**
$x = (-2x - 3)$, $x = 1$	**M1**

6a.

V shape in correct orientation **M1**
Vertex in second quadrant **M1**
(0, 5) and ($\frac{-5}{2}$, 0) **M1 M1**



6b.

f(-4) = $\lvert -3 \rvert$ = 3	**M1**
ff(-4) = $\lvert -3 \rvert$ = $\lvert 11 \rvert$ = 11	**M1**

6c.

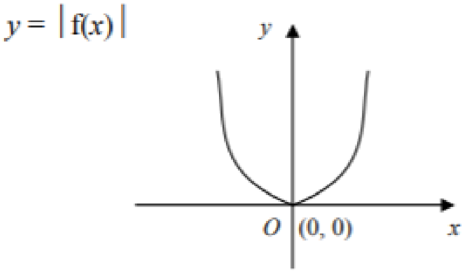
$\frac{-5}{2}$	**M1**

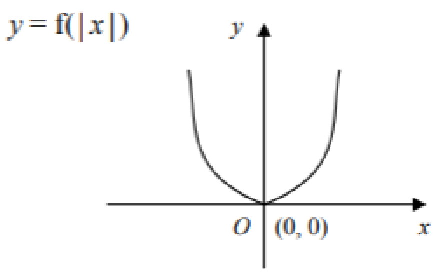
7.

$(3x-4)^2 = (2x+3)^2$	**M1**
$9x^2 - 24x + 16 = 4x^2 + 12x + 9$	**M1**
$5x^2 - 36x + 7 = 0$	**M1**
$(5x-1)(x-7) = 0$ $5x - 1 = 0, x = \frac{1}{2}$ $x - 7 = 9, x = 7$	**M1 M1**

8a.

Correct $y = \lvert f(x) \rvert$ **M1**
Correct $y = f(\lvert x \rvert)$ **M1**





8b.

Correct shape $y = \lvert f(x) \rvert$ **M1**
Coordinates: (0, 5), (-1, 0) and (5, 0) **M1**
Correct shape $y = f(\lvert x \rvert)$ **M1**
Coordinates: (-5, 0), (0, 5) and (5, 0) **M1**