Vectors

Pt.1: 2D Vectors

AS-Level	A-Level
Pt. 1: 2D Vectors	Pt. 2: 3D Vectors

Jethwa Maths

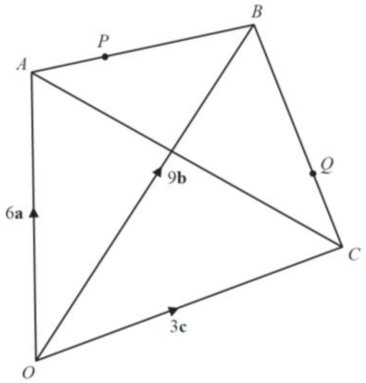
1. Given that $p = \mathbf{i} + 3\mathbf{j}$ and $q = 4\mathbf{i} - 2\mathbf{j}$,
a. Find the values of a and b such that $ap + bq = -5\mathbf{i} + 13\mathbf{j}$, **(3)**
b. Find the value of c such that $cp + q$ is parallel to the vector $\mathbf{j}$, **(2)**
c. Find the value of d such that $p + dq$ is parallel to the vector $3\mathbf{i} - \mathbf{j}$. **(3)**

2. Given that $p = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$, $q = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$ and $r = \begin{pmatrix} -2 \\ 5 \\ -3 \end{pmatrix}$, find as column vectors,
a. $p + 2q$ **(1)**
b. $2p - 3q + r$ **(1)**

3. A car is driving with a velocity of $(7\mathbf{i} - 5\mathbf{j})$ ms^{-1}.
a. Find the speed of the car **(2)**
b. Find the bearing the car is travelling on **(2)**

4. Given that the point A has position vector $4\mathbf{i} - 5\mathbf{j}$ and the point B has position vector $-5\mathbf{i} - 2\mathbf{j}$.
a. Find the vector $\overrightarrow{AB}$ **(2)**
b. Find $|\overrightarrow{AB}|$. Give your answer as a simplified surd. **(2)**

5. In the diagram, O, is the origin and $\overrightarrow{OA} = 6a$, $\overrightarrow{OB} = 9b$ and $\overrightarrow{OC} = 3c$.
The point P lies on AB such that $\overrightarrow{AP} = 3b - 2a$.
The point Q lies of BC such that $\overrightarrow{BQ} = 2c - 6b$.



a. Find, in terms of b and c, the position vector of Q. Give your answer in its simplest form. **(2)**
b. Find $\overrightarrow{AC}$, in terms of a and c, in its simplest form. **(2)**
c. Explain what your answers in part (b) tell you about PQ and AC. **(2)**

6. The quadrilateral $OABC$ has $\overrightarrow{OA} = 4\mathbf{i} + 2\mathbf{j}$, $\overrightarrow{OB} = 6\mathbf{i} - 3\mathbf{j}$ and $\overrightarrow{OC} = 8\mathbf{i} - 20\mathbf{j}$.
a. Find $\overrightarrow{AB}$ **(2)**
b. Show that quadrilateral $OABC$ is a trapezium. **(2)**

Mark Scheme

1a.

$a(\mathbf{i} + 3\mathbf{j}) + b(4\mathbf{i} - 2\mathbf{j}) = -5\mathbf{i} + 13\mathbf{j}$ $a + 4b = -5$ (1)	**M1**
$3a - 2b = 13$ (2) (1) + 2×(2) $\Rightarrow 7a = 21$	**M1**
$a = 3$, $b = -2$	**M1**

1b.

$c(\mathbf{i} + 3\mathbf{j}) + (4\mathbf{i} - 2\mathbf{j}) = k\mathbf{j}$	**M1**
$c + 4 = 0$ $c = -4$	**M1**

1c.

$(\mathbf{i} + 3\mathbf{j}) + d(4\mathbf{i} - 2\mathbf{j}) = k(3\mathbf{i} - \mathbf{j})$ $1 + 4d = 3k$	**M1**
$3 - 2d = -k$ (1) + 2×(2) $7 = k$	**M1**
$d = 5$	**M1**

2a.

$\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} + 2\begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ -1 \\ 1 \end{pmatrix}$	**M1**

2b.

$2\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} - 3\begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 5 \\ -3 \end{pmatrix} = \begin{pmatrix} -12 \\ 17 \\ -8 \end{pmatrix}$	**M1**

3a.

Speed = $\sqrt{7^2 + 5^2} = \sqrt{74}$ ms^{-1}	**M1**

3b.

$\tan x = \frac{5}{7}$ $x = \tan^{-1}(\frac{5}{7})$ $x = 35.5^{\circ}$	**M1**
Bearing = 90 + 35.5 = 126° (to the nearest degree)	**M1**

4a.

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$	**M1**
$= (-5\mathbf{i} - 2\mathbf{j}) - (2\mathbf{i} - 5\mathbf{j})$ $= -9\mathbf{i} + 3\mathbf{j}$	**M1**

4b.

$\lvert\overrightarrow{AB}\rvert = \sqrt{9^2 + 3^2}$ $= \sqrt{90}$	**M1**
$= 3\sqrt{10}$	**M1**

5a.

$\overrightarrow{OQ} = \overrightarrow{OB} + \overrightarrow{BQ}$ $9b + (2c - 6b)$	**M1**
$= 3b + 2c$	**M1**

5b.

$\overrightarrow{AC} = -\overrightarrow{OA} + \overrightarrow{OC}$	**M1**
$= 3c - 6a$	**M1**

5c.

$\overrightarrow{PQ} = \frac{3}{2}\overrightarrow{AC}$	**M1**
Therefore, *PQ* and *AC* are parallel	**M1**

6a.

$\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$ $=(-4 + 6)\mathbf{i} + (-2 - 3)\mathbf{j}$	**M1**
$= 2\mathbf{i} - 5\mathbf{j}$	**M1**

6b.

OC is parallel to *AB* because,	**M1**
$\overrightarrow{OC} = 4\overrightarrow{AB}$ They are not the same length, hence *OABC* is a trapezium.	**M1**