AS-Level Unit Test: Integration

1. Find $\int 3x^2 + 4x^5 - 7 \ dx$, giving each term in its simplest form. **(2)**

2. Given that $y = 2x^3 + \frac{3}{x^2}$, $x \neq 0$ find,

a. $\frac{dy}{dx}$ **(2)**

b. $\int y \ dx$, simplidying each term **(1)**

3. A curve has an equation which satisfies $\frac{dy}{dx}$ = kx(2x – 1) for all values of x. The point P (2, 7) lies on the curve and the gradient of the curve at P is 9.

a. Find the value of the constant k **(2)**

b. Find the equation of the curve **(5)**

4. The curve C has equation y = f(x), x > 0, and f'(x) = 4x - $6\sqrt{x} + \frac{8}{x^2}$. Given that the points P(4, 1) lies on C.

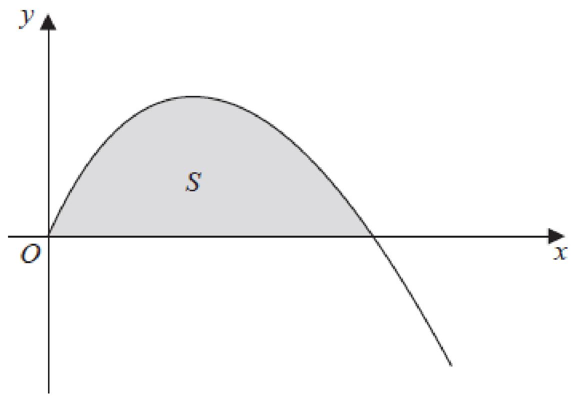
a. Find f(x) and simplify your answer **(4)**

b. Find an equation of the normal to C at the point P(4, 1) **(4)**

5. Find the exact value of $\int_2^1 \frac{2x^3-6x^3}{x^2} \ dx$ **(4)**

6. Evaluate $\int_1^8 \frac{1}{\sqrt{x}} \ dx$, giving your answer in the form a + b$\sqrt{2}$, where a and b are integers **(5)**

7. The Figure shows a sketch of part of the curve with equation, y = 3x - $x^{\frac{3}{2}}$, x ≥ 0.



The finite region S, bounded by the x-axis and the curve, is shown shaded in the figure.

a. Find $\int 3x - x^{\frac{3}{2}} dx$ **(3)**

b. Hence, find the area of S. **(5)**

Total marks: 37

Mark Scheme

1.

$\int 3x^2 + 4x^5 - 7 \ dx = \frac{3x^3}{3} + \frac{4x^5}{5} - 7x + c$	**M1**
$= x^3 + \frac{4}{5}x^5 - 7x + c$	**M1**

2a.

$y = 2x^3 + \frac{3}{x^2} = 2x^3 + 3x^{-2}$	**M1**
$\frac{dy}{dx} = 6x^2 - 6x^{-3}$	**M1**

2b.

$y = 2x^3 + \frac{3}{x^2} = 2x^3 + 3x^{-2}$	
$\int y \, dx = \frac{2}{4}x^3 + \frac{3x^{-1}}{-1} = \frac{1}{2}x^3 - 3x^{-1} + c$	**M1**

3a.

$\frac{dy}{dx} = kx(2x-1)$ When x = 2, $\frac{dy}{dx} = 9$	**M1**
9 = k(2)(3) $k = \frac{3}{2}$	**M1**

3b.

$y = \frac{3}{2}\int x(2x-1)dx$ $= \frac{3}{2}\int 2x^2 - 2 \, dx$	**M1**
$= \frac{3}{2}\left[\frac{2x^3}{3} - \frac{x^2}{2}\right] + c$	**M1**
$= x^3 - \frac{3}{4}x^2 + c$	**M1**
When x = 2, y = 7 7 = 5 + c	**M1**
c = 2	**M1**
$y = x^3 - \frac{3}{4}x^2 + 2$	

4a.

$y = f(x) = \int f'(x) \, dx$ $\int (4x - 6\sqrt{x} + \frac{8}{x^2}) \, dx = \frac{4x^2}{2} - \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{8x^{-1}}{-1} + c$	**M1**
$y = 2x^2 - 4x^{\frac{3}{2}} - \frac{8}{x} + c$	**M1**
When f(4) = 1, $2(4)^2 - 4(4)^{1.5} - 2 + c = 1$	**M1**
32 – 32 – 2 + c = 1 c = 3	**M1**
$y = 2x^2 - 4x^{\frac{3}{2}} - \frac{8}{x} + 3$	

4b.

f'(x) = $4x - 6\sqrt{x} + \frac{8}{x^2}$ x = 4, f'(x) = $\frac{9}{2}$	**M1**
Gradient of normal at P = $-\frac{2}{9}$	**M1**
Equation of normal at P is: $y - 1 = -\frac{2}{9}(x - 4)$	**M1**
$9y - 9 = -2(x - 4)$ $2x + 9y - 17 = 0$	**M1**

5.

$\frac{2x^3-6x^3}{x^2} = \frac{2x^3}{x^2} - \frac{6x^3}{x^2} = 2x - 6$	**M1**
$\int_2^1 \frac{2x^3-6x^3}{x^2}\,dx = \int_2^1 2x - 6\,dx$ $= [\frac{2x^2}{2} - 6x]_2^1$ $= [x^2 - 6x]_2^1$	**M1**
$((2)^2 - 6(2)) - ((1)^2 - 6(1))$	**M1**
$= -5 - (-8)$ $= 3$	**M1**

6.

$\int_1^8 \frac{1}{\sqrt{x}}\,dx = \int_1^8 x^{-\frac{1}{2}}\,dx$	**M1**
$= [\frac{x^{\frac{1}{2}}}{\frac{1}{2}}]_1^8 = [2x^{\frac{1}{2}}]_1^8$	**M1**
$= 2(8)^{\frac{1}{2}} - 2(1)^{\frac{1}{2}}$	**M1**
$2\sqrt{8} - 2\sqrt{1}$ $= 2\sqrt{8} - 2$ $= 2(2\sqrt{2}) - 2$ $= 4\sqrt{2} - 2$	**M1**
In the correct form: $-2 + 4\sqrt{2}$ a = -2 b = 4	**M1**

7a.

$\int 3x - x^{\frac{3}{2}}\,dx = \frac{3x^2}{2} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c$	**M1 M1 M1**

7b.

$3x - x^{\frac{3}{2}} = 0$ $x(3 - x^{\frac{1}{2}}) = 0$	**M1**
x = 0 $3 - x^{\frac{1}{2}} = 0 \rightarrow x^{\frac{1}{2}} = 3 \rightarrow x = 9$	**M1**
$[\frac{3x^2}{2} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}}]_0^9$	**M1**

$= \left(\frac{3(9)^2}{2} - \frac{(9)^{\frac{5}{2}}}{\frac{5}{2}}\right) - \left(\frac{3(0)^2}{2} - \frac{(0)^{\frac{5}{2}}}{\frac{5}{2}}\right)$	**M1**
$= \frac{243}{2} - \frac{486}{5}$ $= \frac{243}{10}$	**M1**