

Exponentials and Logarithms



1. Solve the equation $2^{2x+5} - 7(2^x) = 0$ (4)

2a. $2\log(x + a) = \log(16a^6)$, where a is a positive constant. Find x in terms of a , giving your answer in its simplest form. (3)

b. $\log_3(9y + b) - \log_3(2y - b) = 2$, where b is a positive constant. Find y in terms of b , giving your answer in its simplest form. (4)

3. Given that $\log_3 x = a$, find in terms of a , giving each answer in its simplest form

a. $\log_3(9x)$ (2)

b. $\log_3 \frac{x^5}{81}$ (3)

c. Solve for x , $\log_3(9x) + \log_3 \frac{x^5}{81} = 3$ (4)

4a. Simplify fully: $\frac{2x^2+9x-5}{x^2+2x-15}$ (2)

b. Given that $\ln(2x^2 + 9x - 5) = 1 + \ln(x^2 + 2x - 15)$, find x in terms of e . (4)

5. On 1 January 1990, a sculpture was valued at £80. When the sculpture was sold on 1 January 1956, its value was £5000. The value of £ V , of the sculpture is modelled by the formula $V = Ak^t$, where t is the time in years since 1 January 1990 and A and K are constants.

a. Write down the value of A (1)

b. Show that $k \approx 1.07664$ (3)

c. Use this model to show that the value of the sculpture on 1 January 2006 will be greater than £200 000 (2)

d. Find the year in which the value of the sculpture will first exceed £800,000 (2)

6. A biologist is researching the growth of a certain species of hamster. She proposes that the length x cm of a hamster t days after its birth is given by,

$$x = 15 - 12e^{-\frac{t}{14}}$$

a. Use this model to find the length of a hamster when it is born (1)

b. Find the length of a hamster after 14 days, giving your answer to three significant figures. (2)

c. Show that the time for a hamster to grow to 10cm in length is given by $t = 14\ln\left(\frac{a}{b}\right)$, where a and b are integers (2)

d. Find this time to the nearest day (1)

7. A particular species of orchid is being studied. The population p at time t years after the study started is assumed to be

$$p = \frac{2800ae^{0.2t}}{1+ae^{0.2t}}, \text{ where } a \text{ is a constant}$$

Given that there were 300 orchids when the study started,

a. Show that $a = 0.12$ (3)

b. Use the equation with $a = 0.12$ to predict the number of years before the population of orchids reaches 1850 (4)

c. Show that $p = \frac{336}{0.12+e^{-0.2t}}$ (1)

d. Hence show that the population cannot exceed 2800. (2)

8. Find the exact solutions to the equations,

a. $\ln x + \ln 3 + \ln 6$ (2)

b. $e^x + 3e^{-x} = 4$ (4)

9. The amount of a certain type of drug in the bloodstream t hours after it has been taken is given by the formula, $x = De^{-\frac{1}{8}t}$,

Where x is the amount of the drug in the bloodstream in milligrams and D is the dose given in milligrams.

A dose of 10mg of the drug is given.

a. Find the amount of the drug in the bloodstream 5 hours after the dose is given. Give your answer in mg to 3 decimal places. (2)

A second dose of 10mg is given after 5 years.

b. Show that the amount of the drug in the bloodstream 1 hour after the second dose is 13.549 mg to 3 decimal places. (2)

No more doses of the drug are given. At time T hours after the second dose is given, the amount of the drug in the bloodstream is 3 mg.

c. Find the value of T (3)

10. The radioactive decay of a substance is given by

$$R = -1000e^{-ct}, t \geq 0.$$

where R is the number of atoms at time t years and c is a positive constant.

a. Find the number of atoms when the substance started to decay. (1)

It takes 5730 years for half of the substance to decay.

b. Find the value of c to 3 significant figures. (4)

c. Calculate the number of atoms that will be left when $t = 22\,920$. (2)

d. In the space provided on page 13, sketch the graph of R against t . (2)

11. Find the exact solutions to the equations

a. $\ln(3x - 7) = 5$ (3)

b. $3^x e^{7x+2} = 15$ (5)

The function f and g are defined by

$$f(x) = e^{2x} + 3$$

$$g(x) = \ln(x - 1)$$

c. Find f^{-1} and state its domain (4)

d. Find fg and state its range (3)

Total marks: 87

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Mark Scheme

1.

$2^{2x+5} - 7(2^x) = 0$ $2^{2x+5} = 7(2^x)$	M1
$\log_2(2^{2x+5}) = \log_2(7(2^x))$	M1
$2x + 5 = \log_2 7 + \log_2 2^x$ $2x + 5 = \log_2 7 + x$	M1
$x = \log_2 7 - 5$ $x = -2.19$	M1

2a.

$\log(x+a) = \frac{1}{2} \log(16a^6)$ $\log(x+a) = \log \sqrt{16a^6}$	M1
$x+a = \sqrt{16a^6}$ $x+a = 4a^3$	M1
$x = 4a^3 - a$	M1

2b.

$\log_3(9y+b) - \log_3(2y-b) = 2$ $\log_3\left(\frac{9y+b}{2y-b}\right) = 2$	M1
$\frac{9y+b}{2y-b} = 3^2$	M1
$9y+b = 18y-9b$ $9y = 10b$	M1
$y = \frac{10}{9}b$	M1

3a.

$\log_3(9x) = \log_3(9) + \log_3(x)$	M1
$= 2 + \log_3 x$ $= 2 + a$	M1

3b.

$\log_3 \frac{x^5}{81} = \log_3(x^5) - \log_3(81)$	M1
$= 5\log_3(x) - \log_3(81)$ $= 5\log_3(x) - 4$	M1
$= 5a - 4$	M1

3c.

$2 + a + 5a - 4 = 3$ $6a = 5$	M1
$a = \frac{5}{6}$	M1
$\frac{5}{6} = \log_3(x)$ $3^{\frac{5}{6}} = x$	M1
$x = 2.498$	M1



4a.

$\frac{2x^2+9x-5}{x^2+2x-15} = \frac{(2x-1)(x+5)}{(x-3)(x+5)}$	M1
$= \frac{2x-1}{x-3}$	M1

4b.

$\ln \frac{2x^2+9x-5}{x^2+2x-15} = 1$	M1
$e = \frac{2x^2+9x-5}{x^2+2x-15} = \frac{2x-1}{x-3}$	M1
$\begin{aligned} ex - 3e &= 2x - 1 \\ ex - 2x &= 3e - 1 \\ x(e - 2) &= 3e - 1 \end{aligned}$	M1
$x = \frac{3e-1}{e-2}$	M1

5a.

$A = 80$	M1
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5b.

$\begin{aligned} 5000 &= 80 \times k^{56} \\ k &= \sqrt[56]{\frac{5000}{80}} = 1.07664 \end{aligned}$	M1
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5c.

$\begin{aligned} V &= 80 \times k^{106} \\ &= 200707 \end{aligned}$	M1
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5d.

$\begin{aligned} \ln 10000 &= \ln k^t \\ t &= \frac{\ln 10000}{\ln k} = 124.7 \\ &= 2024 \end{aligned}$	M1
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6a.

$t = 0, x = 3$	M1
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6b.

$\begin{aligned} t &= 14 \\ x &= 15 - 12e^{-1} \end{aligned}$	M1
$x = 10.6$	M1

6c.

$\begin{aligned} -5 &= -12e^{-\frac{t}{14}} \\ \ln\left(\frac{5}{12}\right) &= -\frac{t}{14} \end{aligned}$	M1
$t = 14 \ln\left(\frac{12}{5}\right)$	M1

6d.

$t = 12.256 \approx 12 \text{ days}$	M1
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7a.

$p = 300$ at $t = 0$ $300 = \frac{2800a}{1+a}$	M1
$300 = 2500a$	M1
$a = 0.12$	M1

7b.

$1850 = \frac{2800(0.12)e^{0.2t}}{1+0.12e^{0.2t}}$	M1
$e^{0.2t} = 16.2 \dots$	M1
$0.2t = \ln k$	M1
$t = 14$	M1

7c.

$p(0.12) = \frac{2800(0.12)e^{0.2t}}{1+(0.12)e^{0.2t}}$ $p = \frac{336e^{0.2t}}{1+(0.12)e^{0.2t}}$	M1
Divide numerator and denominator by $e^{0.2t}$ $p = \frac{336}{e^{-0.2t}+0.12}$	M1

7d.

$t \rightarrow \infty$ $e^{-0.2t} \rightarrow 0$	M1
$p \rightarrow \frac{336}{0.12} = 2800$	M1

8a.

$3x = 6$	M1
$x = 2$	M1

8b.

$e^x + 3e^{-x} = 4$ $e^x + \frac{3}{e^x} = 4$	M1
$(e^x)^2 + 3 = 4e^x$ $(e^x)^2 - 4e^x + 3 = 0$ $(e^x - 3)(e^x - 1) = 0$	M1
$e^x - 3 = 0$ $e^x = 3$ $x = \ln 3$	M1
$e^x - 1 = 0$ $e^x = 1$ $x = \ln 1 = 0$	M1

9a.

$D = 10, t = 5$ $x = 10e^{-\frac{1}{8} \times 5}$	M1
$x = 5.353$	M1

9b.

$D = 10 + 10e^{-\frac{5}{8}}, t = 1$ $x = 15.3526 \dots \times e^{-\frac{1}{8}}$	M1
$x = 13.549$	M1

9c.

$15.3526.. e^{-\frac{1}{8}T} = 3$	M1
$e^{-\frac{1}{8}T} = \frac{3}{15.3526} = 0.1954$	M1
$-\frac{1}{8}T = \ln 0.1954$ $T = 13.06$	M1

10a.

1000	M1
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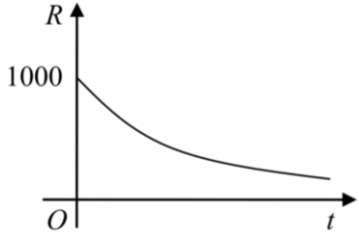
10b.

$1000e^{-5730c} = 500$ $e^{-5730c} = \frac{1}{2}$	M1
$-5730c = \ln \frac{1}{2}$ $c = 0.000121$	M1

10c.

$R = 1000 e^{-22920c} = 62.5$	M1
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10d.

Shape M1 1000 M1	
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11a.

$\ln(3x - 7) = 5$ $e^{\ln(3x - 7)} = e^5$	M1
$3x - 7 = e^5$	M1
$x = \frac{e^5 + 7}{3}$	M1

11b.

$3^x e^{7x+2} = 15$ $\ln(3^x e^{7x+2}) = \ln 15$	M1
$\ln 3^x + \ln e^{7x+2} = \ln 15$	M1
$\ln 3^x + 7x + 2 = \ln 15$ $x \ln 3 + 7x + 2 = \ln 15$	M1
$x(\ln 3 + 7) = -2 + \ln 15$	M1
$x = \frac{-2 + \ln 15}{7 + \ln 3}$	M1

11c.

$f(x) = e^{2x} + 3$ $y = e^{2x} + 3$	M1
$y - 3 = e^{2x}$ $\ln(y - 3) = 2x$	M1
$x = \frac{1}{2} \ln(y - 3)$ $f^{-1}(x) = \frac{1}{2} \ln(x - 3)$	M1
$f^{-1}(x)$ Domain: $x > 3$	M1

11d.

$g(x) = \ln(x - 1), x > 1$	M1
$fg(x) = e^{2\ln(x-1)} + 3$	M1
$fg(x)$: Range $y > 3$	M1

