## AS Level Independent Questions

# Exponentials and Logarithms

1. Express $10^3 = 1000$ in the form $\log_a b = c$ **(1)**

2. Express $\log_a 4 + \log_a 7$ in the form $\log_a n$ **(1)**

3. Express $4\log_a \frac{1}{\sqrt{x}}$ in the form $p\log_q x$ **(1)**

4. Given that $y = \log_q 8$, exress $\log_q 64$ in terms of $y$ **(1)**

5. Solve $\log_4(0.5x + 1) = 3.2$, giving your answer to 3 significant figures. **(2)**

6. Solve $\log_3 x + \log_3 5 = \log_3(2x + 3)$ **(2)**

7. Solve the simultaneous equations

$$\log_2 x = 3 - 2\log_2 y$$
$$\log_y 32 = -\frac{5}{2}$$

**(5)**

8. Solve $10^x = 14$ to 2 decimal places. **(1)**

9. Solve $16 - 3^{4+x} = 0$ **(3)**

10. Solve $3(16^x) - 4^{x+2} = 0$ **(3)**

11. Solve $3^{2x+1} + 15 = 2(3^{x+2})$ **(4)**

12. Sketch the following two cruves $y = 2^x$ and $y = 2^{x+3}$ on the same diagrams **(3)**

13. Given that $a = \log_{10} 2$ and $b = \log_{10} 3$, find an expression in terms of $a$ and $b$ for $\log_{10} 1.5$ **(2)**

14. Given that $2\log_2(x + 15) - \log_2 x = 6$
a. Show that $x^2 - 34x + 225 = 0$ **(4)**
b. Hence, or otherwise, solve the equation, $2\log_2(x + 15) - \log_2 x = 6$ **(2)**

15a. Find, to 3 significant figures, the values of x for which $5^x = 7$. **(2)**
b. Solve the equation $5^{2x} - 12(5^x) + 35 = 0$ **(4)**

16. The mass, m grams, of a leaf t dats after it has been picked from a tree is given by

$$m = pe^{-kt}$$

Where $k$ and $p$ are postive constants.
When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.
a. Write down the value of $p$ **(1)**
b. Show that $k = ¼ \ln 3$ **(4)**

17. A heated metal ball is dropped into a liquid. As the ball colls, its temperature $T^oC$, $t$ minutes after it enters the liquid is given by,
$T = 400e^{-0.05t} + 25$, $t \geq 0$
a. Find the temperature of the ball as it enters the liquid. **(1)**
b. Find the value of $t$ for which $T = 300$, giving your answer to 3 significant figures. **(3)**

18. The temperature of a mug of coffee at time $t$ can be modelled by the equation

$T(t) = T_R + (90 - T_R)e^{-\frac{1}{20}t}$ where $T(t)$ is the temperature, in °C, of the coffee at time $t$ minutes after the coffee was poured into the mug and $T_R$ is the room temperature in °C.

Using the equation for this model,

a. Explain why the initial temperature of the coffee is independent of the initial room temperature. **(2)**

b. Calculate the temperature of the coffee after 10 minutes if its room temperature is 20 °C. **(2)**

19. Given that $\log_3(3b + 1) - \log_3(a - 2) = -1$, express $b$ in terms of $a$ **(3)**

20. Joan brings a cup of hot tea into a room and places the cup on a table. At time t minutes after Joan places the cup on the table, the temperature, $\theta$ °C, of the tea is modelled by the equation

$$\theta = 20 + Ae^{-kt}$$

where A and k are positive constants. Given that the initial temperature of the tea was 90°C.

a. Find the value of A **(2)**

The tea takes 5 minutes to decrease in temperature from 90°C to 55°C.

b. Show that $k = \frac{1}{5}\ln 2$ **(3)**

21. The area, $A$ mm$^2$, of a bacterial culture growing in milk, $t$ hours after midday, is given by

$A = 20e^{1.5t}$, $t \geq 0$

a. Write down the area of the culture at midday. **(1)**

b. Find the time at which the area of the culture is twice its area at midday. Give your answer to the nearest minute. **(4)**

22. The value of Bob's car can be calculated from the formula

$$V = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$$

Where $V$ is the values of the car in pounds (£) and $t$ is the age in yeats.

a. Find the value of the car when $t = 0$ **(1)**

b. Calculate the exact values of t when $V = 9500$ **(4)**

23. The population of a town is being studied. The population P, at time $t$ years from the start of the study, is assumed to be,

$$P = \frac{8000}{1+7e^{-kt}}, \ t \geq 0$$

Where $k$ is a postive constant.

a. Use the given equation to find the population at the start of the study. **(2)**

b. Find a value for the expected upper limit of the population **(1)**

Given also that the population reaches 2500 at 3 years from the start of the study,

c. Calculate the value of $k$ to 3 decimal places. **(5)**

Using this value for $k$,

d. Find the population at 10 years from the start of the study, giving your answer to 3 significant figures **(2)**

24. Find algebraically the exact solutions to the equations

a. $\ln(4 - 2x) + \ln(9 - 3c) = 2\ln(x + 1)$ $\quad -1 < x < 2$ **(5)**

b. $2^x e^{3x+1} = 10$

Give your answer to (b) in the form $\frac{a+\ln b}{c+\ln d}$ where $a$, $b$, $c$, and $d$ are integers **(5)**

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# Mark Scheme

1.

| | |
|---|---|
| $\log_{10}1000 = 3$ | **M1** |

2.

| | |
|---|---|
| $\log_a(4 \times 7) = \log_a 28$ | **M1** |

3.

| | |
|---|---|
| $\log_q x^{-1/2}$<br>$= -2\log_q x$ | **M1** |

4.

| | |
|---|---|
| $= \log_q 8^2$<br>$= 2y$ | **M1** |

5.

| | |
|---|---|
| $\frac{1}{2}x + 1 = 4^{3.2}$ | **M1** |
| $x = 2(4^{3.2} - 1)$<br>$x = 167$ | **M1** |

6.

| | |
|---|---|
| $\log_3 5x = \log_3(2x + 3)$ | **M1** |
| $5x = 2x + 3$<br>$x = 1$ | **M1** |

7.

| | |
|---|---|
| $\log_y 32 = -\frac{5}{2}$<br>$y^{-2.5} = 32$ | **M1** |
| $y = 32^{-0.4} = \frac{1}{4}$ | **M1** |
| $\log_2 x = 3 - 2\log_2 \frac{1}{4}$ | **M1** |
| $\log_2 x = 3 - (-4) = 7$<br>$x = 2^7 = 128$ | **M1** |
| $x = 128$<br>$y = \frac{1}{4}$ | **M1** |

8.

| | |
|---|---|
| $x = \log 14 = 1.15$ | **M1** |

9.

| | |
|---|---|
| $(4 + x)\log 3 = \log 16$ | **M1** |
| $x = \frac{\log 16}{\log 3} - 4$ | **M1** |
| $x = -1.48$ | **M1** |

10.

| | |
|---|---|
| $3^{x+1} = 2^{4+x}$ | **M1** |
| $(x + 1)\log 3 = (4 + x)\log 2$ | **M1** |
| $x(\log 3 - \log 2) = 4\log 2 - \log 3$<br>$x = 4.13$ | **M1** |

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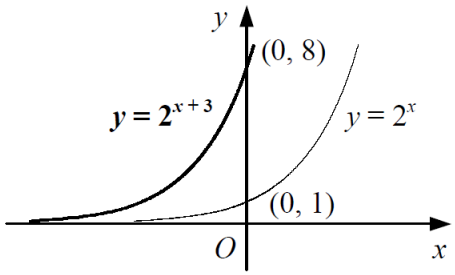
11.

| | |
|---|---|
| $3(3^{2x}) - 18(3^x) + 15 = 0$<br>$3(3^x - 1)(3^x - 5) = 0$ | **M1** |
| $3^x = 1$<br>$x = 0$ | **M1** |
| $3^x = 5$ | **M1** |
| $x = \frac{\log 5}{\log 3}$ | **M1** |

12.

Shape of $y = 2^{x+3}$ **M1**
Shape of $y = 2^x$ **M1**
Point (0,8) and (0,1) labelled. **M1**




13.

| | |
|---|---|
| $= \log_{10}\frac{3}{2}$ | **M1** |
| $= \log_{10}3 - \log_{10}2$<br>$= b - a$ | **M1** |

14a.

| | |
|---|---|
| $2\log_2(x + 15) - \log_2 x = 6$<br>$\log_2(x + 15)^2 - \log_2 x = 6$ | **M1** |
| $\log_2\frac{(x+15)^2}{x} = 6$<br>$\frac{(x+15)^2}{x} = 2^6$ | **M1** |
| $(x + 15)^2 = 64x$<br>$x^2 + 30x + 225 = 64x$ | **M1** |
| $x^2 - 34x + 225 = 0$ | **M1** |

14b.

| | |
|---|---|
| $x^2 - 34x + 225 = 0$<br>$(x - 25)(x - 9) = 0$ | **M1** |
| $x = 25$<br>$x = 9$ | **M1** |

15a.

| | |
|---|---|
| $\log 5^x = \log 7$<br>$x \log 5 = \log 7$ | **M1** |
| $x = \frac{\log 7}{\log 5}$<br>$x = 1.21$ (to 3 s.f) | **M1** |

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15b.

| | |
|---|---|
| $5^{2x} - 12(5^x) + 35 = 0$<br>$(5^x)^2 - 12(5^x) + 35 = 0$ | **M1** |
| let y = $5^x$<br>$y^2 - 12y + 35 = 0$<br>$(y - 5)(y - 7) = 0$ | **M1** |
| $y = 5$<br>$5^x = 5$<br>$x = 1$ | **M1** |
| $y = 7$<br>$5^x = 7$<br>$x = 1.21$ (to 3 s.f) | **M1** |

16a.

| | |
|---|---|
| When $t = 0$, $m = 7.5$<br>Therefore, $7.5 = pe^0$<br>$p = 7.5$ | **M1** |

16b.

| | |
|---|---|
| When $t = 4$, $m = 2.5$<br>$2.5 = 7.5e^{-4k}$ | **M1** |
| $e^{-4k} = \frac{7.5}{2.5} = \frac{1}{3}$ | **M1** |
| $\ln(\frac{1}{3}) = \ln e^{-4k}$<br>$\ln(\frac{1}{3}) = -4k \ln e$<br>$\ln 1 - \ln 3 = -4k$<br>$-\ln 3 = -4k$ | **M1** |
| $k = \frac{1}{4}\ln 3$ | **M1** |

17a.

| | |
|---|---|
| 425°C | **M1** |

17b.

| | |
|---|---|
| $300 = 400e^{-0.05t} + 25$ | **M1** |
| 400 | **M1** |

18a.

| | |
|---|---|
| $t = 0$<br>$T(0) = T_R + (90 - T_R)e^{-\frac{1}{20}\times 0}$<br>$= T_R + (90 - T_R)e^0$<br>$= T_R + 90 - T_R$<br>$= 90$ | **M1** |
| The room temperature $T_R$ will always cancel itself out in $T(0)$, so the initial temperature is independent of $T_R$. | **M1** |

18b.

| | |
|---|---|
| Substitute $T_R = 20$ and t = 10 into $T(t)$<br>$T(t = 10, T_R = 20) = 20 + (90\text{-}20)\, e^{-\frac{1}{20}\times 10}$ | **M1** |
| $= 20 + 70e^{-\frac{1}{2}}$<br>$= 60.457$°C | **M1** |

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19.

| | |
|---|---|
| $\log_3(3b-1) - \log_3(a-2) = -1$<br>$\log_3\frac{3b-1}{a-2} = -1$ | **M1** |
| $\frac{3b-1}{a-2} = 3^{-1}$<br>$3b - 1 = \frac{1}{3}(a-2)$ | **M1** |
| $b = \frac{1}{9}a - \frac{5}{9}$ | **M1** |

20a.

| | |
|---|---|
| $\theta = 20 + Ae^{-kt}$<br>$t = 0, \theta = 90$<br>$90 = 20 + Ae^{-kt}$ | **M1** |
| $90 = 20 + A$<br>$A = 70$ | **M1** |

20b.

| | |
|---|---|
| $\theta = 20 + 70e^{-kt}$<br>$t = 5, \theta = 55$<br>$\frac{35}{70} = e^{-kt}$ | **M1** |
| $\ln(\frac{35}{70}) = -5k$ | **M1** |
| $-5k = \ln\frac{1}{2}$<br>$-5k = \ln 1 - \ln 2$<br>$-5k = -\ln 2$ | **M1** |
| $k = \frac{1}{2}\ln 2$ | **M1** |

21a.

| | |
|---|---|
| $20\ mm^2$ | **M1** |

21b.

| | |
|---|---|
| $40 = 20\ e^{1.5t}$<br>$e^{1.5t} = c$<br>$e^{1.5t} = \frac{40}{20}$ | **M1** |
| $1.5t = \ln 2$<br>$t = \frac{\ln c}{1.5}$ | **M1** |
| $t = \frac{\ln 2}{1.5} = 0.486$ | **M1** |
| therefore $t = 12.28$ or 28 minutes. | **M1** |

22a.

| | |
|---|---|
| £19500 | **M1** |

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22b.

| | |
|---|---|
| $9500 = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$<br>$17e^{-0.25t} + 2e^{-0.5t} = 9$<br>$17e^{0.25t} + 2 = 9e^{0.5t}$ | **M1** |
| $0 = 9e^{0.5t} - 17e^{0.25t} - 2$ | **M1** |
| $0 = (9e^{0.25t} + 1)(e^{0.25t} - 2)$ | **M1** |
| $e^{0.25t} = 2$<br>$t = 4 \ln (2)$ | **M1** |

23a.

| | |
|---|---|
| $t = 0$ | **M1** |
| $P = \frac{8000}{1+7} = 1000$ | |

23b.

| | |
|---|---|
| $t \rightarrow \infty$<br>$P = \frac{8000}{1} = 8000$ | **M1** |

23c.

| | |
|---|---|
| $t \rightarrow 3$<br>$P = 2500$ | **M1** |
| $2500 = \frac{8000}{1+7e^{-3k}}$ | **M1** |
| $e^{-3k} = \frac{2.2}{7}$ | **M1** |
| $k = -\frac{1}{3} \ln (\frac{2.2}{7})$ | **M1** |
| $k = 0.386$ | **M1** |

23d.

| | |
|---|---|
| $t = 10$<br>$P = \frac{8000}{(1+7e^{-0.386t})}$ | **M1** |
| $P = 6970$ | **M1** |

24a.

| | |
|---|---|
| $\ln(4 - 2x)(9 - 3x) = \ln(x + 1)^2$ | **M1 M1** |
| $36 - 30x + 6x^2 = x^2 + 2x + 1$<br>$5x^2 - 32x + 35 = 0$ | **M1** |
| $x = 5 \rightarrow$ solution not valid. | **M1** |
| $x = \frac{7}{5}$ | **M1** |

24b.

| | |
|---|---|
| $\ln 2^x + \ln e^{3x+1} = \ln 10$ | **M1** |
| $x\ln 2 + (3x + 1)\ln e = \ln 10$ | **M1** |
| $x(\ln 2 + 3\ln e) = \ln 10 - \ln e$ | **M1** |
| $\ln e = 1$ | **M1** |
| $x = \frac{-1+\ln 10}{3+\ln 2}$ | **M1** |

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