AS Level Independent Questions

Exponentials and Logarithms

1. Express $10^3 = 1000$ in the form $\log_a b = c$ **(1)**

2. Express $\log_a 4 + \log_a 7$ in the form $\log_a n$ **(1)**

3. Express $4\log_a \frac{1}{\sqrt{x}}$ in the form $p\log_q x$ **(1)**

4. Given that $y = \log_q 8$, exress $\log_q 64$ in terms of y **(1)**

5. Solve $\log_4(0.5x + 1) = 3.2$, giving your answer to 3 significant figures. **(2)**

6. Solve $\log_3 x + \log_3 5 = \log_3(2x + 3)$ **(2)**

7. Solve the simultaneous equations

$$\log_2 x = 3 - 2\log_2 y$$
$$\log_y 32 = -\frac{5}{2}$$

(5)

8. Solve $10^x = 14$ to 2 decimal places. **(1)**

9. Solve $16 - 3^{4+x} = 0$ **(3)**

10. Solve $3(16^x) - 4^{x+2} = 0$ **(3)**

11. Solve $3^{2x+1} + 15 = 2(3^{x+2})$ **(4)**

12. Sketch the following two cruves $y = 2^x$ and $y = 2^{x+3}$ on the same diagrams **(3)**

13. Given that $a = \log_{10}2$ and $b = \log_{10}3$, find an expression in terms of a and b for $\log_{10}1.5$ **(2)**

14. Given that $2\log_2(x + 15) - \log_2 x = 6$
a. Show that $x^2 - 34x + 225 = 0$ **(4)**
b. Hence, or otherwise, solve the equation, $2\log_2(x + 15) - \log_2 x = 6$ **(2)**

15a. Find, to 3 significant figures, the values of x for which $5^x = 7$. **(2)**
b. Solve the equation $5^{2x} - 12(5^x) + 35 = 0$ **(4)**

16. The mass, m grams, of a leaf t dats after it has been picked from a tree is given by

$$m = pe^{-kt}$$

Where k and p are postive constants.
When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.
a. Write down the value of p **(1)**
b. Show that $k = \frac{1}{4}\ln 3$ **(4)**

17. A heated metal ball is dropped into a liquid. As the ball colls, its temperature T°C, t minutes after it enters the liquid is given by,
$T = 400e^{-0.05t} + 25, t \geq 0$
a. Find the temperature of the ball as it enters the liquid. **(1)**
b. Find the value of t for which $T = 300$, giving your answer to 3 significant figures. **(3)**

18. The temperature of a mug of coffee at time t can be modelled by the equation

$T(t) = T_R + (90 - T_R)e^{-\frac{1}{20}t}$ where $T(t)$ is the temperature, in °C, of the coffee at time t minutes after the coffee was poured into the mug and T_R is the room temperature in °C.

Using the equation for this model,

a. Explain why the initial temperature of the coffee is independent of the initial room temperature. **(2)**

b. Calculate the temperature of the coffee after 10 minutes if its room temperature is 20 °C. **(2)**

19. Given that $\log_3(3b + 1) - \log_3(a - 2) = -1$, express b in terms of a **(3)**

20. Joan brings a cup of hot tea into a room and places the cup on a table. At time t minutes after Joan places the cup on the table, the temperature, θ °C, of the tea is modelled by the equation

$$\theta = 20 + Ae^{-kt}$$

where A and k are positive constants. Given that the initial temperature of the tea was 90°C.

a. Find the value of A **(2)**

The tea takes 5 minutes to decrease in temperature from 90°C to 55°C.

b. Show that $k = \frac{1}{5}\ln 2$ **(3)**

21. The area, A mm^2, of a bacterial culture growing in milk, t hours after midday, is given by

$A = 20e^{1.5t}$, $t \geq 0$

a. Write down the area of the culture at midday. **(1)**

b. Find the time at which the area of the culture is twice its area at midday. Give your answer to the nearest minute. **(4)**

22. The value of Bob's car can be calculated from the formula

$$V = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$$

Where V is the values of the car in pounds (£) and t is the age in yeats.

a. Find the value of the car when $t = 0$ **(1)**

b. Calculate the exact values of t when $V = 9500$ **(4)**

23. The population of a town is being studied. The population P, at time t years from the start of the study, is assumed to be,

$$P = \frac{8000}{1+7e^{-kt}},\ t \geq 0$$

Where k is a postive constant.

a. Use the given equation to find the population at the start of the study. **(2)**

b. Find a value for the expected upper limit of the population **(1)**

Given also that the population reaches 2500 at 3 years from the start of the study,

c. Calculate the value of k to 3 decimal places. **(5)**

Using this value for k,

d. Find the population at 10 years from the start of the study, giving your answer to 3 significant figures **(2)**

24. Find algebraically the exact solutions to the equations

a. $\ln(4 - 2x) + \ln(9 - 3c) = 2\ln(x + 1) \qquad -1 < x < 2$ **(5)**

b. $2^x e^{3x+1} = 10$

Give your answer to (b) in the form $\frac{a+\ln b}{c+\ln d}$ where $a, b, c,$ and d are integers **(5)**

Mark Scheme

1.

$\log_{10}1000 = 3$	**M1**

2.

$\log_{\mathrm{a}}(4 \times 7) = \log_a 28$	**M1**

3.

$\log_q x^{-1/2}$ $= -2\mathrm{loq}_q x$	**M1**

4.

$= \log_q 8^2$ $= 2y$	**M1**

5.

$\frac{1}{2}x + 1 = 4^{3.2}$	**M1**
$x = 2(4^{3.2} - 1)$ $x = 167$	**M1**

6.

$\log_3 5x = \log_3(2x + 3)$	**M1**
$5x = 2x + 3$ $x = 1$	**M1**

7.

$\log_y 32 = -\frac{5}{2}$ $y^{-2.5} = 32$	**M1**
$y = 32^{-0.4} = \frac{1}{4}$	**M1**
$\log_2 x = 3 - 2\log_2\frac{1}{4}$	**M1**
$\log_2 x = 3 - (-4) = 7$ $x = 2^7 = 128$	**M1**
$x = 128$ $y = \frac{1}{4}$	**M1**

8.

$x = \log 14 = 1.15$	**M1**

9.

$(4 + x)\log 3 = \log 16$	**M1**
$x = \frac{\log 16}{\log 3} - 4$	**M1**
$x = -1.48$	**M1**

10.

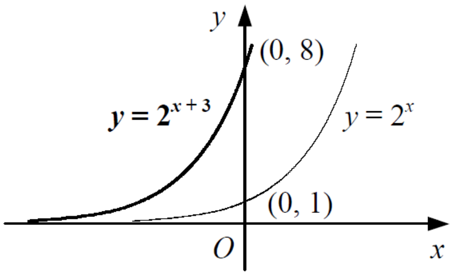
$3^{x+1} = 2^{4+x}$	**M1**
$(x + 1)\log 3 = (4 + x)\log 2$	**M1**
$x(\log 3 - \log 2) = 4\log 2 - \log 3$ $x = 4.13$	**M1**

11.

$3(3^{2x}) - 18(3^x) + 15 = 0$ $3(3^x - 1)(3^x - 5) = 0$	**M1**
$3^x = 1$ $x = 0$	**M1**
$3^x = 5$	**M1**
$x = \frac{\log 5}{\log 3}$	**M1**

12.

Shape of $y = 2^{x+3}$ **M1**
Shape of $y = 2^x$ **M1**
Point (0,8) and (0,1) labelled. **M1**



13.

$= \log_{10}\frac{3}{2}$	**M1**
$= \log_{10}3 - \log_{10}2$ $= b - a$	**M1**

14a.

$2\log_2(x + 15) - \log_2 x = 6$ $\log_2(x + 15)^2 - \log_2 x = 6$	**M1**
$\log_2\frac{(x+15)^2}{x} = 6$ $\frac{(x+15)^2}{x} = 2^6$	**M1**
$(x + 15)^2 = 64x$ $x^2 + 30x + 225 = 64x$	**M1**
$x^2 - 34x + 225 = 0$	**M1**

14b.

$x^2 - 34x + 225 = 0$ $(x - 25)(x - 9) = 0$	**M1**
$x = 25$ $x = 9$	**M1**

15a.

$\log 5^x = \log 7$ $x \log 5 = \log 7$	**M1**
$x = \frac{\log 7}{\log 5}$ $x = 1.21$ (to 3 s.f)	**M1**

15b.

$5^{2x} - 12(5^x) + 35 = 0$ $(5^x)^2 - 12(5^x) + 35 = 0$	**M1**
let $y = 5^x$ $y^2 - 12y + 35 = 0$ $(y - 5)(y - 7) = 0$	**M1**
$y = 5$ $5^x = 5$ $x = 1$	**M1**
$y = 7$ $5^x = 7$ $x = 1.21$ (to 3 s.f)	**M1**

16a.

When $t = 0$, $m = 7.5$ Therefore, $7.5 = pe^0$ $p = 7.5$	**M1**

16b.

When $t = 4$, $m = 2.5$ $2.5 = 7.5e^{-4k}$	**M1**
$e^{-4k} = \frac{7.5}{2.5} = \frac{1}{3}$	**M1**
$\ln(\frac{1}{3}) = \ln e^{-4k}$ $\ln(\frac{1}{3}) = -4k \ln e$ $\ln 1 - \ln 3 = -4k$ $-\ln 3 = -4k$	**M1**
$k = \frac{1}{4} \ln 3$	**M1**

17a.

425°C	**M1**

17b.

$300 = 400e^{-0.05t} + 25$	**M1**
400	**M1**

18a.

$t = 0$ $T(0) = T_R + (90 - T_R)e^{-\frac{1}{20}\times 0}$ $= T_R + (90 - T_R)e^0$ $= T_R + 90 - T_R$ $= 90$	**M1**
The room temperature T_R will always cancel itself out in $T(0)$, so the initial temperature is independent of T_R.	**M1**

18b.

Substitute $T_R = 20$ and t = 10 into $T(t)$ $T(t = 10, T_R = 20) = 20 + (90\text{-}20)\ e^{-\frac{1}{20}\times 10}$	**M1**
$= 20 + 70e^{-\frac{1}{2}}$ $= 60.457$°C	**M1**

19.

$\log_3(3b-1) - \log_3(a-2) = -1$ $\log_3\frac{3b-1}{a-2} = -1$	**M1**
$\frac{3b-1}{a-2} = 3^{-1}$ $3b - 1 = \frac{1}{3}(a-2)$	**M1**
$b = \frac{1}{9}a - \frac{5}{9}$	**M1**

20a.

$\theta = 20 + Ae^{-kt}$ $t = 0, \theta = 90$ $90 = 20 + Ae^{-kt}$	**M1**
$90 = 20 + A$ $A = 70$	**M1**

20b.

$\theta = 20 + 70e^{-kt}$ $t = 5, \theta = 55$ $\frac{35}{70} = e^{-kt}$	**M1**
$\ln(\frac{35}{70}) = -5k$	**M1**
$-5k = \ln\frac{1}{2}$ $-5k = \ln 1 - \ln 2$ $-5k = -\ln 2$	**M1**
$k = \frac{1}{2}\ln 2$	**M1**

21a.

20 mm^2	**M1**

21b.

$40 = 20\, e^{1.5t}$ $e^{1.5t} = c$ $e^{1.5t} = \frac{40}{20}$	**M1**
$1.5t = \ln 2$ $t = \frac{\ln c}{1.5}$	**M1**
$t = \frac{\ln 2}{1.5} = 0.486$	**M1**
therefore $t = 12.28$ or 28 minutes.	**M1**

22a.

£19500	**M1**

22b.

$9500 = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$ $17e^{-0.25t} + 2e^{-0.5t} = 9$ $17e^{0.25t} + 2 = 9e^{0.5t}$	**M1**
$0 = 9e^{0.5t} – 17e^{0.25t} - 2$	**M1**
$0 = (9e^{0.25t} + 1)(e^{0.25t} – 2)$	**M1**
$e^{0.25t} = 2$ $t = 4 \ln (2)$	**M1**

23a.

$t = 0$	**M1**
$P = \frac{8000}{1+7} = 1000$	

23b.

$t \longrightarrow \infty$ $P = \frac{8000}{1} = 8000$	**M1**

23c.

$t \longrightarrow 3$ $P = 2500$	**M1**
$2500 = \frac{8000}{1+7e^{-3k}}$	**M1**
$e^{-3k} = \frac{2.2}{7}$	**M1**
$k = -\frac{1}{3} \ln (\frac{2.2}{7})$	**M1**
$k = 0.386$	**M1**

23d.

$t = 10$ $P = \frac{8000}{(1+7e^{-0.386t})}$	**M1**
$P = 6970$	**M1**

24a.

$\ln(4 – 2x)(9 - 3x) = \ln(x + 1)^2$	**M1 M1**
$36 – 30x + 6x^2 = x^2 + 2x + 1$ $5x^2 – 32x + 35 = 0$	**M1**
$x = 5 \longrightarrow$ solution not valid.	**M1**
$x = \frac{7}{5}$	**M1**

24b.

$\ln 2^x + \ln e^{3x + 1} = \ln 10$	**M1**
$x\ln 2 + (3x + 1)\ln e = \ln 10$	**M1**
$x(\ln 2 + 3\ln e) = \ln 10 – \ln e$	**M1**
$\ln e = 1$	**M1**
$x = \frac{-1+\ln 10}{3+\ln 2}$	**M1**