A-Level Unit Test: Coordinate Geometry

Equation of a Line

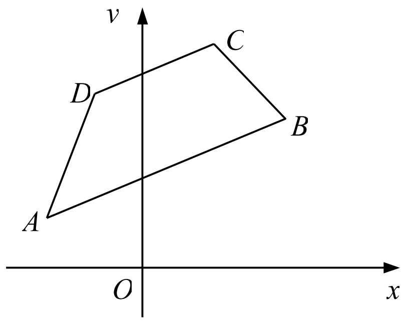
Jethwa Maths

1. The line l has the equation $5x - 18y - 30 = 0$.
a. Find the coordinates of the points A and B where the line l crosses the coordinate axes. **(2)**
b. Find the area of triangle OAB where O is the origin. **(1)**

2. The straight line l_1 passes through the points P (−2, 1) and Q (4, −1).
a. Find the equation of l_1 in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**
The straight line l_2 passes through the point R (2, 4) and through the mid-point of PQ.
b. Find the equation of l_2 in the form $y = mx + c$. **(3)**

3. The straight line l passes through the points A (−5, 5) and B (1, 7).
a. Find an equation of the line l. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**
The point M is the mid-point of AB.
b. Prove that the line OM, where O is the origin, is perpendicular to line l. **(3)**

4. The diagram shows trapezium $ABCD$ in which sides AB and DC are parallel. The point A has coordinates (−4, 2) and the point B has coordinates (6, 6).



a. Find the equation of the straight line passing through A and B, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**
Given that the gradient of BC is −1,
b. Find an equation of the straight line passing through B and C. **(1)**
Given also that the point D has coordinates (−2, 7),
c. Find the coordinates of the point C **(4)**
d. Show that $\angle ACB = 90°$. **(3)**

5. The straight line l has gradient $\frac{1}{2}$ and passes through the point with coordinates (2, 4).
a. Find the equation of l in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**
The straight line m has the equation $y = 2x - 6$.
b. Find the coordinates of the point where line m intersects line l. **(3)**
c. Show that the quadrilateral enclosed by line l, line m and the positive coordinate axes is a kite. **(4)**

6. The straight line l_1 passes through the point P (1, 3) and the point Q (13, 12).
a. Find the length of PQ. **(2)**
b. Find the equation of l_1 in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**
The straight line l_2 is perpendicular to $l1$ and passes through the point R (2, 10).
c. Find an equation of line l_2. **(2)**
d. Find the coordinates of the point where lines l_1 and l_2 intersect. **(3)**
e. Find the area of triangle PQR. **(2)**

Mark Scheme

1a

$x = 0 \rightarrow y = -\frac{5}{3}$	**M1**
$y = 0 \rightarrow x = 6$	**M1**
$(0\ , -\frac{5}{3})$ and $(6, 0)$	

1b.

Area = 0.5 x 6 x $\frac{5}{3}$ = 5	**M1**

2a.

Gradient of $l_1 = \frac{-1-1}{4+2} = -\frac{1}{3}$	**M1**
$y - 1 = -\frac{1}{3}(x + 2)$ $3y - 3 = = -x - 2$	**M1**
$x + 3y - 1 = 0$	**M1**

2b.

mid-point of $PQ = (\frac{-2+4}{2}, \frac{1-1}{2}) = (1, 0)$	**M1**
grad of $l_2 = \frac{0-4}{1-2} = 4$	**M1**
$y = 4(x - 1)$ $y = 4x - 4$	**M1**

3a.

Gradient = $\frac{7-5}{1+5} = \frac{1}{3}$	**M1**
$y - 5 = \frac{1}{3}(x + 5)$ $3y - 15 = x + 5$	**M1**
$x - 3y + 20 = 0$	**M1**

3b.

M = $(\frac{-5+1}{2}, \frac{5+7}{2}) = (-2, 6)$	**M1**
Gradient of OM = $\frac{6-0}{-2-0} = -3$	**M1**
Gradient l x gradient OM = $\frac{1}{3}$ x -3 -1 Therefore, OM is perpendicular to l	**M1**

4a.

Gradient = $\frac{6-2}{6+4} = \frac{2}{5}$	**M1**
$y - 2 = \frac{2}{5}(x + 4)$	**M1**
$5y - 10 = 2x + 8$ $2x - 5y + 18 = 0$	**M1**

4b.

$y - 6 = -(x - 6)$ $y = 12 - x$	**M1**

4c.

grad DC = grad $AB = \frac{2}{5}$	**M1**
Equation DC = Gradient AB = $\frac{2}{5}$	**M1**
Therefore, equation of DC = $y - 7 = \frac{2}{5}(x + 2)$ $y = \frac{2}{5}x + 7\frac{4}{5}$	**M1**
At C: $12 - x = \frac{2}{5}x + 7\frac{4}{5}$ $60 - 5x = 2x + 39$ $x = 3$ Therefore C = (3, 9)	**M1**

4d.

Gradient of AC = $\frac{9-2}{3+4} = 1$	**M1**
Gradient AC x gradient BD = 1 x -1 = -1	**M1**
Therefore, AC is perpendicular to BC and the angle ACB is 90^{O}	**M1**

6a.

$PQ^2 = 12^2 + 9^2 = 225$	**M1**
PQ = 15	**M1**

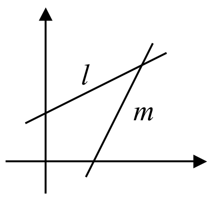
5a.

$y - 4 = \frac{1}{2}(x - 2)$	**M1**
$2y - 8 = x - 2$	**M1**
$x - 2y + 6 = 0$	**M1**

5b.

$x - 2(2x - 6) + 6 = 0$	**M1**
$18 - 3x = 0$	**M1**
$x = 6$, therefore coordinates are (6, 6)	**M1**

5c.

l meets y-axis at (0, 3) m meets x-axis at (3, 0)	**M1**
(0, 0) and (6, 6) are on the line $y = x$.	**M1**
(0, 3) and (3, 0) symmetrical about $y = x$	**M1**
Therefore, quadrilateral is a kite. 	**M1**

6b.

Gradient = $\frac{12-3}{13-1} = \frac{3}{4}$	**M1**
$y - 3 = \frac{3}{4}(x - 1)$	**M1**
$4y - 12 = 3x - 3$	**M1**
$3x - 4y + 9 = 0$	**M1**

6c.

Gradient of $l_2 = -\frac{4}{3}$	**M1**
$y - 10 = -\frac{4}{3}(x - 2)$	**M1**

6d.

L1 → $9x - 12y + 27 = 0$	**M1**
L2 → $16x + 12y - 152 = 0$	**M1**
Adding equations: $25x - 125 = 0$ $x = 5$ Therefore, coordinates are (5, 6)	**M1**

6e.

Distance R to (5, 6) = $\sqrt{3^2 + 4^2} = 5$	**M1**
Area = ½ x 15 x 5 = 37.5	**M1**

A-Level Unit Test: Coordinate Geometry

Circles

1. Find an equation of the circle which crosses the x-axis at the points (2, 0) and (8, 0) and touches the y-axis at the point (0, 4). **(3)**

2. Given that the circle with equation $x^2 + y^2 + 8x - 12y + k = 0$ does not touch or cross either of the coordinate axes, find the set of possible values of the constant k. **(5)**

3. Find an equation of the tangent to the circle with equation $x^2 + y^2 - 10x + 4y + 4 = 0$ at the point (2, 2). **(4)**

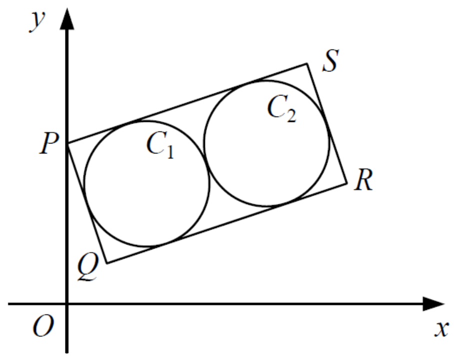
4. The circle C has equation, $x^2 + y^2 - 10x + 6y + 30 = 0$

a. Find the coordinates of the centre of C **(2)**

b. The radius of C **(2)**

c. The y coordinates of the points where the circle C crosses the line with equation $x = 4$, giving your answers as simplified surds. **(3)**

5. The diagram shows rectangle $PQRS$ and circles C_1 and C_2.



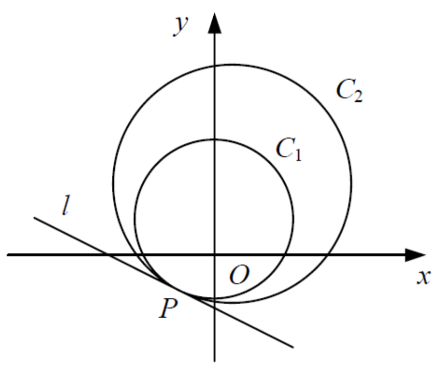
Each circle touches the other circle and three sides of the rectangle. The coordinates of the corners of the rectangle are P (0, 4), Q (1, 1), R (7, 3) and S (6, 6).

a. Find the radius of C_1. **(2)**

b. Find the coordinates of the point where the two circles touch. **(2)**

c. Show that C_1 has equation $2x^2 + 2y^2 - 8x - 12y + 21 = 0$. **(4)**

6. The diagram shows circles C_1 and C_2, which both pass through the point P, and the common tangent to the circles at P, the line l. Circle C_1 has the equation $x^2 + y^2 - 4y - 16 = 0$.



a. Find the coordinates of the centre of C_1.

Circle C_2 has the equation $x^2 + y^2 - 2x - 8y - 60 = 0$. **(2)**

b. Find an equation of the straight line passing through the centre of C_1 and the centre of C_2. **(4)**

c. Find an equation of line l. **(5)**

7. The points P (−10, 2), Q (8, 14) and R (−2, −10) all lie on circle C.
a. Show that PR is perpendicular to PQ. **(3)**
b. Hence, show that C has the equation $x^2 + y^2 - 6x - 4y - 156 = 0$. **(3)**

8. The circle C has equation $x^2 + y^2 - 4x - 6 = 0$ and the line l has equation $y = 3x - 6$.
a. Show that l passes through the centre of C. **(2)**
b. Find an equation for each tangent to C that is parallel to l. **(6)**

Mark Scheme

1.

x – coordinate of centre = $\frac{2+8}{2} = 5$ y - coordinate of centre = 4 Centre (5, 4)	**M1**
Radius = distance from (0, 4) to (5, 4) = 5	**M1**
Therefore equation = $(x - 5)^2 + (y - 4)^2 = 25$	**M1**

2.

$(x + 4)^2 - 16 + (y - 6)^2 - 36 + k = 0$ $(x + 4)^4 + (y - 6)^2 = 52 - k$	**M1**
Centre: (-4, 6) $r^2 = 52 - k$	**M1**
$r > 0$, therefore $k < 52$	**M1**
$r < 4$, therefore $52 - k < 16$, $k > 36$	**M1**
$36 < k < 52$	**M1**

3.

$(x - 5)^2 - 25 + (y + 2)^2 - 4 + 4 = 0$ Therefore centre: (5, -2)	**M1**
Gradient of normal = $\frac{-2-2}{5-2} = -\frac{4}{3}$	**M1**
Therefore, gradient of tangent = $\frac{4}{3}$	**M1**
Therefore, $y - 2 = \frac{3}{4}(x - 2)$ $3x - 4y + 2 = 0$	**M1**

4a.

$x^2 + y^2 - 10x + 6y + 30 = 0$ $(x - 5)^2 - 25 + (y + 3)^2 - 9 = 0$	**M1**
Centre: (5, -3)	**M1**

4b.

$r = \sqrt{25 + 9 - 30} = 2$	**M1**
$r = 2$	**M1**

4c.

When $x = 4$, $(4 - 5)^2 + (y + 3)^2 = 4$ $1 + y^2 + 6y + 9 = 4$	**M1**
$y^2 + 6y + 6 = 0$	**M1**
$y = -3 \pm \sqrt{3}$	**M1**

5a.

PQ = $\sqrt{1+9} = \sqrt{10}$	**M1**
Radius = $\frac{1}{2}$ PQ = $\frac{1}{2}\sqrt{10}$	**M1**

5b.

Midpoint of PR = $\left(\frac{0+7}{2}, \frac{4+3}{2}\right)$	**M1**
$= \left(\frac{7}{2}, \frac{7}{2}\right)$	**M1**

5c.

Midpoint of PQ = $\left(\frac{0+1}{2}, \frac{4+1}{2}\right) = \left(\frac{1}{2}, \frac{5}{2}\right)$	**M1**
Centre of C_1 = midpoint of $\left(\frac{1}{2}, \frac{5}{2}\right)$ and $\left(\frac{7}{2}, \frac{7}{2}\right)$ $= \frac{\frac{1}{2}+\frac{7}{2}}{2}, \frac{\frac{5}{2}+\frac{7}{2}}{2} = (2, 3)$	**M1**
Therefore, equation of $C_1 = (x-2)^2 + (y-3)^2 = \left(\frac{1}{2}\sqrt{10}\right)^2$ $x^2 - 4x + 4 + y^2 - 6y + 9 = \frac{5}{2}$	**M1**
$2x^2 - 8x + 8 + 2y^2 - 12y + 18 = 5$ $2x^2 + 2y^2 - 8x - 12y + 21 = 0$	**M1**

6a.

$x^2 + (y-2)^2 - 4 - 16 = 0$	**M1**
Centre = (0, 2)	**M1**

6b.

C2: $(x-1)^2 - 1 + (y-4)^2 - 16 - 60 = 0$	**M1**
Centre = (1, 4)	**M1**
Gradient = $\frac{4-2}{1-0} = 2$	**M1**
$y = 2x + 2$	**M1**

6c.

$x^2 + [(2x+2) - 2]^2 - 20 = 0$	**M1**
$x^2 + (2x)^2 - 20 = 0$	**M1**
$x^2 = 4$ $x = \pm 2$	**M1**
From the diagram, $x = -2$, at P therefore P = (-2, -2) L is perpendicular to the line through the centres.	**M1**
Therefore, gradient = $-\frac{1}{2}$ $y + 2 = -\frac{1}{2}(x+2)$ $y = -\frac{1}{2}x - 3$	**M1**

7a.

Gradient PQ = $\frac{14-2}{8+10} = \frac{2}{3}$	**M1**
Gradient of PR = $\frac{-10-2}{-2+10} = -\frac{3}{2}$	**M1**
Gradient PR x Gradient PQ = $-\frac{3}{2} \times \frac{2}{3} = -1$ Therefore, PR is perpendicular to PQ	**M1**

7b.

Angle QPR = 90, therefore QR is a dimeter of the circle.	**M1**
Therefore, centre of circle is midpoint of QR = $\frac{8-2}{2}, \frac{14-10}{2} = (3, 2)$ Radius = $\sqrt{25+144} = 13$	**M1**
$(x-3)^2 + (y-2)^2 = 169$ $x^2 - 6x + 9 + y^2 - 4y + 4 - 169 = 0$ $x^2 + y^2 - 6x - 4y - 156 = 0$	**M1**

8a.

C: $(x-2)^2 - 4 + y^2 - 6 = 0$ Therefore, centre (2, 0)	**M1**
l: when $x = 2$, $y = 3(2) - 6 = 0$ Therefore, passes through centre of C.	**M1**

8b.

Equation of tangent: $y = 3x + k$ Substitute into equation of circle: $x^2 + (3x + k)^2 - 4x - 6 = 0$	**M1**
$10x^2 + (6k - 4)x + k^2 - 6 = 0$	**M1**
As the line is at a tangent, there is one repeated root, therefore $b^2 - 4ac = 0$	**M1**
$(6k - 4)^2 - 40(k^2 - 6) = 0$ $k^2 + 12k - 64 = 0$ $(k + 16)(k - 4) = 0$	**M1**
$k = -16 \rightarrow y = 3x - 16$ $k = 4 \rightarrow y = 3x + 4$	**M1 M1**