A-Level Unit Test: Coordinate Geometry

Circles

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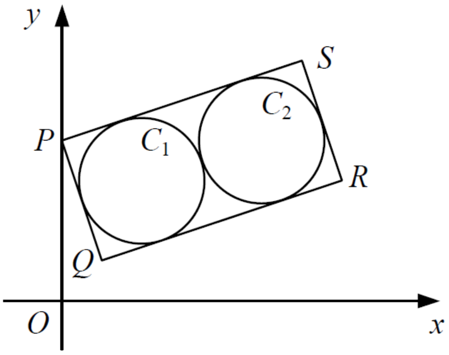
1. Find an equation of the circle which crosses the x-axis at the points (2, 0) and (8, 0) and touches the y-axis at the point (0, 4). **(3)**

2. Given that the circle with equation $x^2 + y^2 + 8x - 12y + k = 0$ does not touch or cross either of the coordinate axes, find the set of possible values of the constant k. **(5)**

3. Find an equation of the tangent to the circle with equation $x^2 + y^2 - 10x + 4y + 4 = 0$ at the point (2, 2). **(4)**

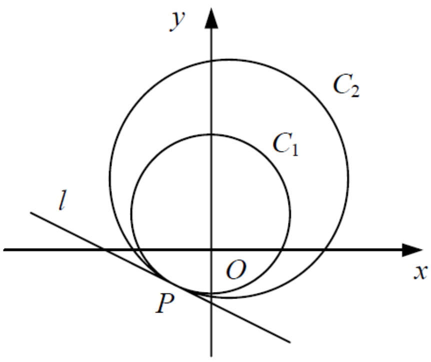
4. The circle C has equation, $x^2 + y^2 - 10x + 6y + 30 = 0$
a. Find the coordinates of the centre of C **(2)**
b. The radius of C **(2)**
c. The y coordinates of the points where the circle C crosses the line with equation $x = 4$, giving your answers as simplified surds. **(3)**

5. The diagram shows rectangle $PQRS$ and circles C_1 and C_2.



Each circle touches the other circle and three sides of the rectangle. The coordinates of the corners of the rectangle are P (0, 4), Q (1, 1), R (7, 3) and S (6, 6).
a. Find the radius of C_1. **(2)**
b. Find the coordinates of the point where the two circles touch. **(2)**
c. Show that C_1 has equation $2x^2 + 2y^2 - 8x - 12y + 21 = 0$. **(4)**

6. The diagram shows circles C_1 and C_2, which both pass through the point P, and the common tangent to the circles at P, the line l. Circle C_1 has the equation $x^2 + y^2 - 4y - 16 = 0$.



a. Find the coordinates of the centre of C_1.
Circle C_2 has the equation $x^2 + y^2 - 2x - 8y - 60 = 0$. **(2)**

b. Find an equation of the straight line passing through the centre of C_1 and the centre of C_2. **(4)**

c. Find an equation of line l. **(5)**

7. The points P (−10, 2), Q (8, 14) and R (−2, −10) all lie on circle C.
a. Show that PR is perpendicular to PQ. **(3)**
b. Hence, show that C has the equation $x^2 + y^2 - 6x - 4y - 156 = 0$. **(3)**

8. The circle C has equation $x^2 + y^2 - 4x - 6 = 0$ and the line l has equation $y = 3x - 6$.
a. Show that l passes through the centre of C. **(2)**
b. Find an equation for each tangent to C that is parallel to l. **(6)**

Mark Scheme

1.

x – coordinate of centre = $\frac{2+8}{2} = 5$ y - coordinate of centre = 4 Centre (5, 4)	**M1**
Radius = distance from (0, 4) to (5, 4) = 5	**M1**
Therefore equation = $(x – 5)^2 + (y – 4)^2 = 25$	**M1**

2.

$(x + 4)^2 – 16 + (y – 6)^2 – 36 + k = 0$ $(x + 4)^4 + (y – 6)^2 = 52 – k$	**M1**
Centre: (-4, 6) $r^2 = 52$ - k	**M1**
$r > 0$, therefore $k < 52$	**M1**
$r < 4$, therefore $52 – k < 16$, $k > 36$	**M1**
$36 < k < 52$	**M1**

3.

$(x – 5)^2 – 25 + (y + 2)^2 – 4 + 4 = 0$ Therefore centre: (5, -2)	**M1**
Gradient of normal = $\frac{-2-2}{5-2} = -\frac{4}{3}$	**M1**
Therefore, gradient of tangent = $\frac{4}{3}$	**M1**
Therefore, $y – 2 = \frac{3}{4}(x - 2)$ $3x – 4y + 2 = 0$	**M1**

4a.

$x^2 + y^2 – 10x + 6y + 30 = 0$ $(x – 5)^2 – 25 + (y + 3)^2 – 9 = 0$	**M1**
Centre: (5, -3)	**M1**

4b.

$r = \sqrt{25 + 9 - 30} = 2$	**M1**
$r = 2$	**M1**

4c.

When $x = 4$, $(4 – 5)^2 + (y + 3)^2 = 4$ $1 + y^2 + 6y + 9 = 4$	**M1**
$y^2 + 6y + 6 = 0$	**M1**
$y = -3 \pm \sqrt{3}$	**M1**

5a.

PQ = $\sqrt{1+9} = \sqrt{10}$	**M1**
Radius = $\frac{1}{2}$ PQ = $\frac{1}{2}\sqrt{10}$	**M1**

5b.

Midpoint of PR = $\left(\frac{0+7}{2}, \frac{4+3}{2}\right)$	**M1**
$= \left(\frac{7}{2}, \frac{7}{2}\right)$	**M1**

5c.

Midpoint of PQ = $\left(\frac{0+1}{2}, \frac{4+1}{2}\right) = \left(\frac{1}{2}, \frac{5}{2}\right)$	**M1**
Centre of C_1 = midpoint of $\left(\frac{1}{2}, \frac{5}{2}\right)$ and $\left(\frac{7}{2}, \frac{7}{2}\right)$ $= \frac{\frac{1}{2}+\frac{7}{2}}{2}, \frac{\frac{5}{2}+\frac{7}{2}}{2} = (2, 3)$	**M1**
Therefore, equation of $C_1 = (x-2)^2 + (y-3)^2 = \left(\frac{1}{2}\sqrt{10}\right)^2$ $x^2 - 4x + 4 + y^2 - 6y + 9 = \frac{5}{2}$	**M1**
$2x^2 - 8x + 8 + 2y^2 - 12y + 18 = 5$ $2x^2 + 2y^2 - 8x - 12y + 21 = 0$	**M1**

6a.

$x^2 + (y-2)^2 - 4 - 16 = 0$	**M1**
Centre = (0, 2)	**M1**

6b.

C2: $(x-1)^2 - 1 + (y-4)^2 - 16 - 60 = 0$	**M1**
Centre = (1, 4)	**M1**
Gradient = $\frac{4-2}{1-0} = 2$	**M1**
$y = 2x + 2$	**M1**

6c.

$x^2 + [(2x+2)-2]^2 - 20 = 0$	**M1**
$x^2 + (2x)^2 - 20 = 0$	**M1**
$x^2 = 4$ $x = \pm 2$	**M1**
From the diagram, $x = -2$, at P therefore P = (-2, -2) L is perpendicular to the line through the centres.	**M1**
Therefore, gradient = $-\frac{1}{2}$ $y + 2 = -\frac{1}{2}(x+2)$ $y = -\frac{1}{2}x - 3$	**M1**

7a.

Gradient PQ = $\frac{14-2}{8+10} = \frac{2}{3}$	**M1**
Gradient of PR = $\frac{-10-2}{-2+10} = -\frac{3}{2}$	**M1**
Gradient PR x Gradient PQ = $-\frac{3}{2} \times \frac{2}{3} = -1$ Therefore, PR is perpendicular to PQ	**M1**

7b.

Angle QPR = 90, therefore QR is a dimeter of the circle.	**M1**
Therefore, centre of circle is midpoint of QR = $\frac{8-2}{2}, \frac{14-10}{2} = (3, 2)$ Radius = $\sqrt{25+144} = 13$	**M1**
$(x-3)^2 + (y-2)^2 = 169$ $x^2 - 6x + 9 + y^2 - 4y + 4 - 169 = 0$ $x^2 + y^2 - 6x - 4y - 156 = 0$	**M1**

8a.

C: $(x-2)^2 - 4 + y^2 - 6 = 0$ Therefore, centre (2, 0)	**M1**
l: when $x = 2$, $y = 3(2) - 6 = 0$ Therefore, passes through centre of C.	**M1**

8b.

Equation of tangent: $y = 3x + k$ Substitute into equation of circle: $x^2 + (3x + k)^2 - 4x - 6 = 0$	**M1**
$10x^2 + (6k - 4)x + k^2 - 6 = 0$	**M1**
As the line is at a tangent, there is one repeated root, therefore $b^2 - 4ac = 0$	**M1**
$(6k - 4)^2 - 40(k^2 - 6) = 0$ $k^2 + 12k - 64 = 0$ $(k + 16)(k - 4) = 0$	**M1**
$k = -16 \rightarrow y = 3x - 16$ $k = 4 \rightarrow y = 3x + 4$	**M1 M1**