A-Level Unit Test: Coordinate Geometry

Equation of a Line

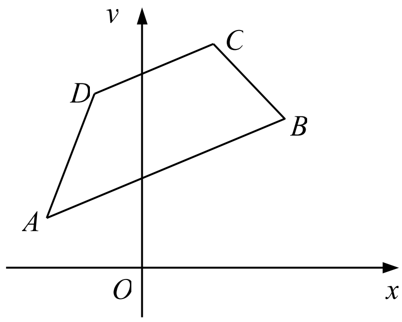
Jethwa Maths

1. The line l has the equation $5x - 18y - 30 = 0$.
a. Find the coordinates of the points A and B where the line l crosses the coordinate axes. **(2)**
b. Find the area of triangle OAB where O is the origin. **(1)**

2. The straight line l_1 passes through the points P $(-2, 1)$ and Q $(4, -1)$.
a. Find the equation of l_1 in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**
The straight line l_2 passes through the point R $(2, 4)$ and through the mid-point of PQ.
b. Find the equation of l_2 in the form $y = mx + c$. **(3)**

3. The straight line l passes through the points A $(-5, 5)$ and B $(1, 7)$.
a. Find an equation of the line l. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**
The point M is the mid-point of AB.
b. Prove that the line OM, where O is the origin, is perpendicular to line l. **(3)**

4. The diagram shows trapezium $ABCD$ in which sides AB and DC are parallel. The point A has coordinates $(-4, 2)$ and the point B has coordinates $(6, 6)$.



a. Find the equation of the straight line passing through A and B, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**
Given that the gradient of BC is -1,
b. Find an equation of the straight line passing through B and C. **(1)**
Given also that the point D has coordinates $(-2, 7)$,
c. Find the coordinates of the point C **(4)**
d. Show that $\angle ACB = 90°$. **(3)**

5. The straight line l has gradient $\frac{1}{2}$ and passes through the point with coordinates $(2, 4)$.
a. Find the equation of l in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**
The straight line m has the equation $y = 2x - 6$.
b. Find the coordinates of the point where line m intersects line l. **(3)**
c. Show that the quadrilateral enclosed by line l, line m and the positive coordinate axes is a kite. **(4)**

6. The straight line l_1 passes through the point P $(1, 3)$ and the point Q $(13, 12)$.
a. Find the length of PQ. **(2)**
b. Find the equation of l_1 in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**
The straight line l_2 is perpendicular to $l1$ and passes through the point R $(2, 10)$.
c. Find an equation of line l_2. **(2)**
d. Find the coordinates of the point where lines l_1 and l_2 intersect. **(3)**
e. Find the area of triangle PQR. **(2)**

Mark Scheme

1a

$x = 0 \rightarrow y = -\frac{5}{3}$	**M1**
$y = 0 \rightarrow x = 6$	**M1**
$(0, -\frac{5}{3})$ and $(6, 0)$	

1b.

Area = 0.5 x 6 x $\frac{5}{3}$ = 5	**M1**

2a.

Gradient of $l_1 = \frac{-1-1}{4+2} = -\frac{1}{3}$	**M1**
$y - 1 = -\frac{1}{3}(x + 2)$ $3y - 3 = = -x - 2$	**M1**
$x + 3y - 1 = 0$	**M1**

2b.

mid-point of $PQ = (\frac{-2+4}{2}, \frac{1-1}{2}) = (1, 0)$	**M1**
grad of $l_2 = \frac{0-4}{1-2} = 4$	**M1**
$y = 4(x - 1)$ $y = 4x - 4$	**M1**

3a.

Gradient $= \frac{7-5}{1+5} = \frac{1}{3}$	**M1**
$y - 5 = \frac{1}{3}(x + 5)$ $3y - 15 = x + 5$	**M1**
$x - 3y + 20 = 0$	**M1**

3b.

M $= (\frac{-5+1}{2}, \frac{5+7}{2}) = (-2, 6)$	**M1**
Gradient of OM $= \frac{6-0}{-2-0} = -3$	**M1**
Gradient l x gradient OM = $\frac{1}{3}$ x -3 -1 Therefore, OM is perpendicular to l	**M1**

4a.

Gradient $= \frac{6-2}{6+4} = \frac{2}{5}$	**M1**
$y - 2 = \frac{2}{5}(x + 4)$	**M1**
$5y - 10 = 2x + 8$ $2x - 5y + 18 = 0$	**M1**

4b.

$y - 6 = -(x - 6)$ $y = 12 - x$	**M1**

4c.

grad DC = grad AB = $\frac{2}{5}$	**M1**
Equation DC = Gradient AB = $\frac{2}{5}$	**M1**
Therefore, equation of DC = $y - 7 = \frac{2}{5}(x + 2)$ $y = \frac{2}{5}x + 7\frac{4}{5}$	**M1**
At C: $12 - x = \frac{2}{5}x + 7\frac{4}{5}$ $60 - 5x = 2x + 39$ $x = 3$ Therefore C = (3, 9)	**M1**

4d.

Gradient of AC = $\frac{9-2}{3+4} = 1$	**M1**
Gradient AC x gradient BD = 1 x -1 = -1	**M1**
Therefore, AC is perpendicular to BC and the angle ACB is 90^{O}	**M1**

6a.

$PQ^2 = 12^2 + 9^2 = 225$	**M1**
PQ = 15	**M1**

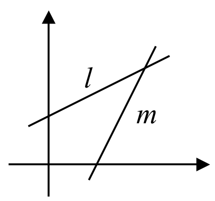
5a.

$y - 4 = \frac{1}{2}(x - 2)$	**M1**
$2y - 8 = x - 2$	**M1**
$x - 2y + 6 = 0$	**M1**

5b.

$x - 2(2x - 6) + 6 = 0$	**M1**
$18 - 3x = 0$	**M1**
$x = 6$, therefore coordinates are (6, 6)	**M1**

5c.

l meets y-axis at (0, 3) m meets x-axis at (3, 0)	**M1**
(0, 0) and (6, 6) are on the line $y = x$.	**M1**
(0, 3) and (3, 0) symmetrical about $y = x$	**M1**
Therefore, quadrilateral is a kite. 	**M1**

6b.

Gradient = $\frac{12-3}{13-1} = \frac{3}{4}$	**M1**
$y - 3 = ¾ (x - 1)$	**M1**
$4y - 12 = 3x - 3$	**M1**
$3x - 4y + 9 = 0$	**M1**

6c.

Gradient of $l_2 = -\frac{4}{3}$	**M1**
$y - 10 = -\frac{4}{3}(x - 2)$	**M1**

6d.

L1 → $9x - 12y + 27 = 0$	**M1**
L2 → $16x + 12y - 152 = 0$	**M1**
Adding equations: $25x - 125 = 0$ $x = 5$ Therefore, coordinates are (5, 6)	**M1**

6e.

Distance R to (5, 6) = $\sqrt{3^2 + 4^2} = 5$	**M1**
Area = ½ x 15 x 5 = 37.5	**M1**