# Algebra and Functions

# Part 1: Equation of a Line

| AS Level | | A-Level |
|---|---|---|
| **Pt 1**: Equation of a Line | **Pt. 2**: Circles | **Pt 3:** Parametric and Cartesian Equations |

1. Find in the form $y = mx + c$, the equation of the straight line passing through the pair of co-ordinates $(-\frac{1}{2}, -2)$ and (2, 8). **(3)**

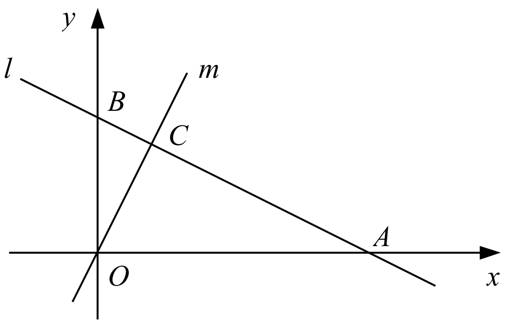
2. The straight line $l$ passes through the points A (-6, 8) and B (3, 2).
a. Find an equation of the line $l$ **(3)**
b. Show that the points C (9, -2) lies on $l$. **(2)**

3. The straight line $l_1$ passes through the points P(-2, 1) and Q(4, -1).
a. Find the equation of $l_1$ in thr form a$x$ + b$y$ + $c$ = 0, where $a$, $b$, and $c$ are integers. **(3)**
The straight line $l_2$ passes through the points R (2, 4) and through the mid-point PQ.
b. Find the equation of $l_2$ in the form $y = mx + c$. **(3)**

4. The straight line $p$ has the equation $3x - 4y + 8 = 0$.
The straight line $q$ is parallel to $p$ and passes through the point with coordinates (8, 5).
a. Find the equation of $q$ in the form $y = mx + c$. **(2)**
The straight line $r$ is perpendicular to $p$ and passes through the point with coordinates (−4, 6).
b. Find the equation of $r$ in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers. **(3)**
c. Find the coordinates of the point where lines $q$ and $r$ intersect. **(4)**

5. The vertices of a triangle are the points $A$ (5, 4), $B$ (−5, 8) and $C$ (1, 11).
a. Find the equation of the straight line passing through $A$ and $B$, giving your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers. **(2)**
b. Find the coordinates of the point $M$, the mid-point of $AC$. **(1)**
c. Show that $OM$ is perpendicular to $AB$, where $O$ is the origin. **(2)**

6. The diagram shows the straight line $l$ with equation $x + 2y - 20 = 0$ and the straight line $m$ which is perpendicular to $l$ and passes through the origin $O$.




a. Find the coordinates of the points $A$ and $B$ where $l$ meets the $x$-axis and $y$-axis respectively. **(2)**

Given that $l$ and $m$ intersect at the point $C$,
b. find the ratio of the area of triangle $OAC$ to the area of triangle $OBC$. **(5)**

7. The vertices of a triangle are the points $P$ (3, $c$), $Q$ (9, 2) and $R$ (3$c$, 11) where $c$ is a constant.
Given that $\angle PQR = 90°$
a. Find the value of $c$ **(5)**
b. Show that the length of $PQ$ is $k\sqrt{10}$, where $k$ is an integer to be found **(3)**
c. Find the area of triangle $PQR$. **(4)**

# Mark Scheme

1.

| | |
|---|---|
| Gradient = $\frac{8+2}{2+0.5} = 4$ | **M1** |
| $y - 8 = 4(x - 2)$ | **M1** |
| $y = 4x$ | **M1** |

2a.

| | |
|---|---|
| Gradient = $\frac{2-8}{3+6} = -\frac{2}{3}$ | **M1** |
| $y - 8 = -\frac{2}{3}(x + 6)$ | **M1** |
| $2x + 3y - 12 = 0$ | **M1** |

2b.

| | |
|---|---|
| $2(9) + 3(-2) - 12 = 0$ | **M1** |
| Therefore, C lies on C. | **M1** |

3a.

| | |
|---|---|
| Gradient = $\frac{-1-1}{4+2} = -\frac{1}{3}$ | **M1** |
| $y - 1 = -\frac{1}{3}(x + 2)$<br>$3y - 3 = -x - 2$ | **M1** |
| $x + 3y - 1 = 0$ | **M1** |

3b.

| | |
|---|---|
| Mid-point of PQ = $(\frac{-2+4}{2}, \frac{1-1}{2}) = (1, 0)$ | **M1** |
| Gradient of $l_2$ = $\frac{0-4}{1-2} = 4$ | **M1** |
| $y = 4(x - 1)$<br>$y = 4x - 4$ | **M1** |

4a.

| | |
|---|---|
| p → $y = \frac{3}{4}x = 2$<br>gradient = $\frac{3}{4}$ | **M1** |
| $y - 5 = \frac{3}{4}(x - 8)$<br>$y = \frac{3}{4}x - 1$ | **M1** |

4b.

| | |
|---|---|
| Perpendicular gradient = $-\frac{4}{3}$ | **M1** |
| $y - 6 = -\frac{4}{3}(x + 4)$<br>$3y - 18 = -4x - 16$ | **M1** |
| $4x + 3y - 2 = 0$ | **M1** |

4c.

| | |
|---|---|
| q → $3x - 4y - 4 = 0$ → $9x - 12y - 12 = 0$ | **M1** |
| r → $16x + 12y - 8 = 0$ | **M1** |
| Adding, $25x - 20 = 0$<br>$x = \frac{4}{5}$ | **M1** |
| $y = \frac{3(0.8)-4}{4} = -\frac{2}{5}$<br>Co-oridnates = $(\frac{4}{5}, -\frac{2}{5})$ | **M1** |

5a.

| | |
|---|---|
| Gradient = $\frac{8-4}{-5-5} = -\frac{2}{5}$ | **M1** |
| $y - 4 = -\frac{2}{5}(x - 5)$<br>$5y - 20 = -2x + 10$<br>$2x + 5y - 30 = 0$ | **M1** |

5b.

| | |
|---|---|
| Midpoint = $(\frac{5+1}{2}, \frac{4+11}{2})$ = (3, 3.5) | **M1** |

5c.

| | |
|---|---|
| Gradient of OM = 3.5 ÷ 3 = $\frac{5}{2}$ | **M1** |
| Gradient OM x Gradient AB = $\frac{5}{2}$ x $-\frac{2}{5}$ = -1<br>Therefore, OM is perpendicular to AB. | **M1** |

6a.

| | |
|---|---|
| At A, $y = 0$, $x = 20 \rightarrow$ A(20, 0) | **M1** |
| At B, $x = 0$, $y = 10 \rightarrow$ B(0, 10) | **M1** |

6b.

| | |
|---|---|
| l $\rightarrow y = 10 - 0.5x$<br>Gradient of l = -0.5 | **M1** |
| Gradient of m = 2<br>Equation of line m: $y = 2x$ | **M1** |
| At C, $10 - 0.5x = 2x$<br>$x = 4$<br>Therefore, C = (4,8) | **M1** |
| Area of ΔOAC: area of ΔOBC<br>0.5 x 20 x 8 : 0.5 x 10 x 4 | **M1** |
| 4 : 1 | **M1** |

7a.

| | |
|---|---|
| Gradient of PQ = $\frac{2-c}{9-3} = \frac{2-c}{6}$ | **M1** |
| Gradient of QR = $\frac{11-2}{3c-9} = \frac{3}{c-3}$ | **M1** |
| $\angle PQR = 90^0$, therefore PQ is perpendicular to QR<br>$\frac{2-c}{6} \times \frac{3}{c-3}$ = -1<br>3(2 – c) = -6 (c – 3)<br>3c = 12<br>c = 4 | |

7b.

| | |
|---|---|
| $PQ^2 = 6^2 + 2^2 = 40$ | **M1** |
| PQ = $\sqrt{40} = 2\sqrt{10}$<br>k = 2 | **M1** |

7c.

| | |
|---|---|
| QR = $\sqrt{3^2 + 9^2} = \sqrt{90} = 3\sqrt{10}$ | **M1** |
| Area = ½ x PQ x QR = 30 | **M1** |

# Algebra and Functions

# Part 2: Circles

| AS Level | | A-Level |
| --- | --- | --- |
| **Pt 1**: Equation of a Line | **Pt. 2**: Circles | **Pt 3:** Parametric and Cartesian Equations |

1. Find the coordinates of the cente and the radius of the circles $9x^2 + 9y^2 + 6x - 24y + 8 = 0$ **(3)**

2. Find whether the (7, -3) lies inside or outside the circle $x^2 + y^2 + 10x - 4y = 140$ **(3)**

3. Find the equation of the normal to the circle with equation $x^2 + y^2 + 4x = 13$ at the point (-1, 4). **(3)**

4. The line with equaton $y = 1 - x$ intersects the circle with equation $x^2 + y^2 + 6x + 2y = 27$ at the points A and B.
Find the length of the chord AB, giving your answer in the form $k\sqrt{2}$
**(3)**

5. The circle $C$ has centre (3, −2) and radius 5.
a. Write down an equation of $C$ in cartesian form. **(1)**
The line $y = 2x - 3$ intersects $C$ at the points $A$ and $B$.
b. Show that $AB = 4\sqrt{5}$. **(5)**

6. The circle $C$ touches the $y$-axis at the point $A$ (0, 3) and passes through the point $B$ (2, 7).
a. Find an equation of the perpendicular bisector of $AB$. **(4)**
b. Find an equation for $C$. **(3)**
c. Show that the tangent to $C$ at $B$ has equation $3x - 4y + 22 = 0$. **(4)**

7. The circle $C$ has equation $x^2 + y^2 - 8x + 4y + 12 = 0$.
a. Find the coordinates of the centre of $C$ and the radius of $C$. **(2)**
The point $P$ has coordinates (3, 5) and the point $Q$ lies on $C$.
b. Find the largest and smallest values of the length $PQ$, giving your answers in the form $k\sqrt{2}$. **(3)**
c. Find the length of $PQ$ correct to 3 significant figures when the line $PQ$ is a tangent to $C$. **(2)**

8. Circle $C_1$ has the equation $x^2 + y^2 - 2ay = 0$, where $a$ is a positive constant.
a. Find the coordinates of the centre and the radius of $C_1$. **(2)**
Circle $C_2$ has the equation $x^2 + y^2 - 2bx = 0$, where $b$ is a constant and $b > a$.
b. Sketch $C_1$ and $C_2$ on the same diagram. **(3)**

9. The circle $C$ has equation $x^2 + y^2 - 8x - 16y + 72 = 0$.
a. Find the coordinates of the centre and the radius of $C$. **(2)**
b. Find the distance of the centre of $C$ from the origin in the form $k\sqrt{5}$ . **(2)**
The point $A$ lies on $C$ and the tangent to $C$ at $A$ passes through the origin $O$.
c. Show that $OA = 6\sqrt{2}$. **(2)**

# <u>Mark Scheme</u>

1.

| | |
|---|---|
| $x^2 + y^2 + \frac{2}{3}x - \frac{8}{3}y + \frac{8}{9} = 0$ | **M1** |
| $(x + \frac{1}{3})^2 - \frac{1}{9} + \left(y - \frac{4}{3}\right)^2 - \frac{16}{9} + \frac{8}{9} = 0$<br>$(x + \frac{1}{3})^2 + \left(y - \frac{4}{3}\right)^2 = 1$ | **M1** |
| Centre $(-\frac{1}{3}, 0)$<br>Radius 1 | **M1** |

2.

| | |
|---|---|
| $(x + 5)^2 - 25 + (y - 2)^2 - 4 = 140$<br>$(x + 5)^2 + (y - 2)^2 = 169$ | **M1** |
| Centre (-5, 2)<br>Radius 13 | **M1** |
| Distance to centre = $\sqrt{144 + 25} = 13$<br>Therefore point is on circle. | **M1** |

3.

| | |
|---|---|
| $(x + 2)^2 - 4 + y^2 = 13$<br>Therefore, centre (-2, 0) | **M1** |
| Gradient $= \frac{0-4}{-2+1} = 4$ | **M1** |
| Therefore, $y - 4 = 4(x + 1)$<br>$y = 4x + 8$ | **M1** |

4.

| | |
|---|---|
| $x^2 + (1 - x)^2 + 6x + 2(1 - x) = 27$ | **M1** |
| $x^2 + x - 12 = 0$<br>$(x + 4)(x - 3) = 0$<br>$x = -4, y = 1 - -4 = 5$<br>$x = 3, y = 1 - 3 = -2$ | **M1** |
| Therefore, AB = $\sqrt{49 + 49} = 7\sqrt{2}$ | **M1** |

5a.

| | |
|---|---|
| $(x - 3)^2 + (y + 2)^2 = 25$ | **M1** |

5b.

| | |
|---|---|
| $(x - 3)^2 + [(2x - 3) + 2]^2 = 25$ | **M1** |
| $(x - 3)^2 + (2x - 1)^2 = 25$<br>$x^2 - 2x - 3 = 0$ | **M1** |
| $(x + 1)(x - 3) = 0$<br>$x = -1, y = 2(-1) - 3 = -5$<br>$x = 3, y = 2(3) - 3 = 3$ | **M1** |
| $AB^2 = 4^2 + 8^2 = 80$ | **M1** |
| AB = $\sqrt{80} = 4\sqrt{5}$ | **M1** |

6a.

| | |
|---|---|
| Midpoint AB = $(\frac{0+2}{2}, \frac{3+7}{2}) = (1, 5)$ | **M1** |
| Gradient AB = $\frac{7-3}{2-0} = 2$ | **M1** |
| Therefore perpendicular gradient = $-\frac{1}{2}$ | **M1** |
| $y - 5 = -\frac{1}{2}(x - 1)$ | **M1** |

| | |
|---|---|
| $y = \frac{11}{2} - \frac{1}{2}x$ | |

6b.

| | |
|---|---|
| Circle touches $y$-axis at (0, 3)<br>Therefore y-coordordinate of centre = 3 | **M1** |
| $3 = \frac{11}{2} - \frac{1}{2}x$<br>$x = 5$ | **M1** |
| Centre (5, 3) radius 5.<br>$(x - 5)^2 + (y - 3)^2 = 25$ | **M1** |

6c.

| | |
|---|---|
| Gradient of radius = $\frac{7-3}{2-5} = -\frac{4}{3}$ | **M1** |
| Therefore gradient of tangent = $\frac{3}{4}$ | **M1** |
| $y - 7 = \frac{3}{4}(x - 2)$<br>$4y - 28 = 3x - 6$ | **M1** |
| $3x - 4y + 22 = 0$ | **M1** |

7a.

| | |
|---|---|
| $(x - 4)^2 - 16 + (y + 2)^2 - 4 + 12 = 0$<br>$(x - 4)^2 + (y + 2)^2 = 8$ | **M1** |
| Centre: (4, −2)<br>Radius: $2\sqrt{2}$ | **M1** |

7b.

| | |
|---|---|
| Distance P to centre = $\sqrt{1 + 49} = \sqrt{50} = 5\sqrt{2}$ | **M1** |
| Therefore, max PQ = $5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$ | **M1** |
| Minimum PQ = $5\sqrt{2}$ - $2\sqrt{2} = 3\sqrt{2}$ | **M1** |

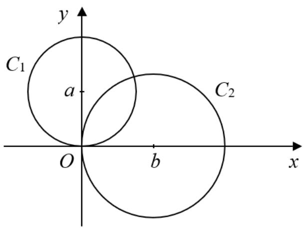
7c.

| | |
|---|---|
| Tangent perpendicular to radius:<br>$PQ^2 = (5\sqrt{2})^2 - (5\sqrt{2})^2 = 50 - 8 = 42$ | **M1** |
| $PQ = \sqrt{42} = 6.48$ | **M1** |

8a.

| | |
|---|---|
| $x^2 + (y - a)^2 - a^2 = 0$<br>$x^2 + (y - a)^2 = a^2$ | **M1** |
| Centre: (0, a)<br>Radius: a | **M1** |

8b.

| | |
|---|---|
| $C_2$: $(x - b)^2 - b^2 + y^2 = 0$<br>$(x - b)^2 + y^2 = b^2$ | **M1** |
| Centre: (b, 0)<br>Radius: b | **M1** |
|   | **M1** |

9a.

| | |
|---|---|
| $(x-4)^2 - 16 + (y-8)^2 - 64 + 72 = 0$<br>$(x-4)^2 + (y-8)^2 = 8$ | **M1** |
| Centre: (4, 8)<br>Radius: $2\sqrt{2}$ | **M1** |

9b.

| | |
|---|---|
| $\sqrt{16+64} = \sqrt{80}$ | **M1** |
| $= 4\sqrt{5}$ | **M1** |

9c.

| | |
|---|---|
| Tangent perpendicular to radius: | **M1** |
| $OA^2 = (\sqrt{80})^2 - (2\sqrt{2})^2 = 72$<br>$OA = \sqrt{72} = 6\sqrt{2}$ | **M1** |