A-Level Unit Test: Sequences and Series

Binomial Expansion 1a

Jethwa Maths

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1. Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(3 - \frac{1}{3}x\right)^5$$

giving each term in its simplest form **(3)**

2.
Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(1 + \frac{3x}{2}\right)^8$$

giving each term in its simplest form. **(3)**

3a. Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(3 + bx)^5$$

where b is a non-zero constant. Give each term in its simplest form. **(4)**

Given that, in this expansion, the coefficient of x^2 is twice the coefficient of x,

b. Find the value of b. **(2)**

4a. Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(2 - 9x)^4$, giving each term in its simplest form.

(4)

$$f(x) = (1 + kx)(2 - 9x)^4, \quad \text{where } k \text{ is a constant.}$$

The expansion, in ascending powers of x, of f(x) up to and including the term in x^2 is

$$A - 232x + Bx^2,$$

where A and B are constants.

b. Write down the value of A. **(1)**

c. Find the value of k. **(2)**

d. Hence find the value of B. **(2)**

Total marks: 21

Mark Scheme

1.

$3^5 + {}^5C_1 3^4(-\frac{1}{3}x) + {}^5C_2 3^3(-\frac{1}{3}x)^2 + {}^5C_3 3^2(-\frac{1}{3}x)^3$	**M1**
$243 - \frac{405}{3}x + \frac{270}{9}x^2 - \frac{90}{27}x^3$	**M1**
$243 - 135x + 30x^2 - \frac{10}{3}x^3$	**M1**

2.

$1 + 12x + \frac{8(7)}{2!}(\frac{3x}{2})^2 + \frac{8(7)(6)}{3!}(\frac{3x}{2})^3$	**M1**
$= 1 + 12x + 63x^2 + 189x^3$	**M1 M1**

3a.

$(3 + bx)^5 = 3^5 + {}^5C_1 3^4(bx) + {}^5C_2 3^3(bx)^2 + \dots$	**M1 M1**
$= 243 + 405bx + 270b^2x^2$	**M1 M1**

3b.

Coefficients of x^2: $2(405b) = 270b^2$	**M1**
$b = \frac{810}{270} = 3$	**M1**

4a.

$(2 - 9x)^4 = 2^4 + {}^4C_1 2^3(-9x) + {}^4C_2 2^2(-9x)^2 + \dots$	**M1 M1**
$f(x) = (1 + kx)(2 - 9x)^4 = A - 232x + Bx^2$ $= 16 - 288x + 1944x^2$	**M1 M1**

4b.

$A = 16$	**M1**

4c.

$(1 + kx)(2 - 9x)^4 = (1 + kx)(16 - 288x + 1944x^2)$	**M1**
Coefficient of x terms: $-288x + 16kx = -232x$ $16k = 56$ $k = \frac{7}{2}$	**M1**

4d.

Coefficient of x^2 terms: $1944x^2 - 288kx^2$	**M1**
Therefore $B = 1944 - 288(\frac{7}{2})$ $1944 - 1008 = 936$	**M1**