

## Part 1a: Binomial Expansion



**AS Level**

Part 1a: Binomial Expansion I

**A-Level**

Part 1b: Binomial Expansion 2

Part 2: Arithmetic Sequences

Part 3: Geometric Sequences

1a. Find the first 3 terms, in ascending powers of  $x$ , of the binomial expansion of  $(1 + px)^9$ , where  $p$  is a constant. The first 3 terms are  $1, 36x$  and  $qx^2$ , where  $q$  is a constant. (2)

b. Find the value of  $p$  and the value of  $q$ . (2)

2a. Find the first 4 terms, in ascending powers of  $x$ , of the binomial expansion of  $(1 - 2x)^5$ . Give each term in its simplest form. (3)

b. If  $x$  is small, so that  $x^2$  and higher powers can be ignored, show that (3)  

$$(1 + x)(1 - 2x)^5 \approx 1 - 9x$$

3. Given that  $\binom{40}{4} = \frac{40!}{4!b!}$   
 a. Write down the value of  $b$  (2)

In the binomial expansion of  $(1 + x)^{40}$ , the coefficients of  $x^4$  and  $x^5$  are  $p$  and  $q$  respectively.

b. Find the value of  $\frac{q}{p}$  (2)

4a. Find the first 4 terms of the binomial expansion, in ascending powers of  $x$ , of  $(1 + \frac{x}{4})^8$  giving each term in its simplest form. (2)

b. Use your expansion to estimate the value of  $(1.025)^8$ , giving your answer to 4 decimal places. (2)

5a. Find the first 3 terms, in ascending powers of  $x$ , of the binomial expansion of  $(2 - 3x)^6$  giving each term in its simplest form. (3)

b. Hence, or otherwise, find the first 3 terms, in ascending powers of  $x$ , of the expansion of  $(1 + \frac{x}{2})(2 - 3x)^6$  (2)

6a. Find the first 3 terms, in ascending powers of  $x$ , of the binomial expansion of  $(2 - \frac{x}{16})^9$ , giving each term in its simplest form. (2)

$$f(x) = (a + bx)(2 - \frac{x}{16})^9, \text{ where } a \text{ and } b \text{ are constants}$$

b. Given that the first two terms, in ascending powers of  $x$ , in the series expansion of  $f(x)$  are  $128$  and  $36x$ , find the value of  $a$ . (2)

c. Find the value of  $b$  (2)

## Mark Scheme

1a.

$\binom{n}{0} (a)^n (bx)^0 + \binom{n}{1} (a)^{n-1} (bx)^1 + \dots + \binom{n}{n} (a)^0 (bx)^n$	<b>M1</b>
$\binom{9}{0} (1)^9 (px)^0 + \binom{9}{1} (1)^8 (px)^1 + \binom{9}{2} (1)^7 (px)^2$	<b>M1</b>
$= 1 + 9px + 36p^2x^2$	<b>M1</b>

1b.

Coefficients of $x$ : $9p = 36$ $p = 4$	<b>M1</b>
Coefficients of $x^2$ : $36p^2 = 3q$ $q = 576$	<b>M1</b>

2a.

$(x + a)^n = \sum_{k=0}^n \binom{n}{k} x^k a^{n-k} = \sum_{k=0}^5 \binom{5}{k} (-2)^k (1)^{5-k}$	<b>M1</b>
$(1 - 2x)^5 = \binom{5}{0} (1)^5 + \binom{5}{1} (1)^4 (-2x) + \binom{5}{2} (1)^4 (-2x)^2 + \binom{5}{3} (1)^3 (-2x)^3$	<b>M1</b>
$(1 - 2x)^5 = 1 - 10x + 40x^2 - 80x^3$	<b>M1</b>

2b.

$(1 - 2x)^5 = 1 - 10x + 40x^2 - 80x^3 \approx 1 - 10x$	<b>M1</b>
$(1 + x)(1 - 2x)^5 = (1 + x)(1 - 10x) = 1 + x - 10x - 10x^2$	<b>M1</b>
$(1 - 2x)^5 \approx 1 - 9x$	<b>M1</b>

3a.

$\binom{n}{k} = \frac{n!}{(n-k)!k!}$ $\binom{40}{4} = \frac{40!}{(40-4)!4!}$	<b>M1</b>
Therefore, $b = 36$	<b>M1</b>

3b.

$p = \binom{40}{4} = \frac{40!}{36!4!}$ $q = \binom{40}{5} = \frac{40!}{35!5!}$	<b>M1</b>
$\frac{q}{p} = \frac{40!}{35!5!} \times \frac{36!4!}{40!}$ $\frac{q}{p} = \frac{36}{5}$	<b>M1</b>

4a.

$(x + y)^n = \binom{n}{0} (x)^n (y)^0 + \binom{n}{1} (x)^{n-1} (y)^1 + \dots + \binom{n}{n} (x)^0 (y)^n$	<b>M1</b>
$\left(1 + \frac{x}{4}\right)^8 = \binom{8}{0} + \binom{8}{1} \left(\frac{x}{4}\right)^1 + 28 \left(\frac{x^2}{16}\right) + 56 \left(\frac{x^3}{64}\right) + \dots$	<b>M1</b>
$\left(1 + \frac{x}{4}\right)^8 = 1 + 2x + \frac{7}{4}x^2 + \frac{7}{8}x^3$	<b>M1</b>



4b.

If we let $x = 0.1$ then we can see that: $(1 + \frac{x}{4})^8 = (1 + \frac{0.1}{4})^8 = (1.025)^8 = 1.2184$	<b>M1</b>
If we let $x = 0.1$ in the expansion $1.025^8 = 1 + 2(0.1) + \frac{7}{4}(0.1)^2 + \frac{7}{8}(0.1)^3 = 1.2184$	<b>M1</b>

5a.

$(a + bx)^n = \sum_{k=0}^n \binom{n}{k} a^k (bx)^{n-k}$ $a = 2, b = -3, n = 6$	<b>M1</b>
$(2 - 3x)^6 = \binom{6}{0}2^6 + \binom{6}{1}2^5(-3x) + \binom{6}{2}2^4(-3x)^2 + \dots$	<b>M1</b>
$(2 - 3x)^6 = 64 + 6(32)(-3x) + (15)(16)(9x^2)$ $= 64 - 576x + 2160x^2$	<b>M1</b>

5b.

$(1 + \frac{x}{2})(2 - 3x)^6 = (1 + \frac{x}{2})(64 - 576x + 2160x^2) = 64 - 576x + 2160x^2 + (\frac{x}{2})(64 - 576x + 2160x^2)$	<b>M1</b>
.... (expanding brackets) .... $= 64 - 544x + 1872x^2 + \dots$	<b>M1</b>

6a.

$(2 - \frac{x}{16})^9 = 2^9 + 9(2)^8(-\frac{x}{16}) + 36(2)^7(-\frac{x}{16})^2$	<b>M1</b>
$= 512 - 144x + 18x^2$	<b>M1</b>

6b.

$(a + bx)(512 - 144x + 18x^2 + \dots) = 128 + 36x + \dots$	<b>M1</b>
Therefore, $512a = 128$ $a = \frac{1}{4}$	<b>M1</b>

6c.

$512b - 144a = 36$	<b>M1</b>
$512b - 36 = 36$ $b = \frac{9}{64}$	<b>M1</b>

