Sequences and Series

Part 1a: Binomial Expansion

Jethwa Maths

AS Level	A-Level	
Part 1a: Binomial Expansion 1	Part 1b: Binomial Expansion 2	Part 2: Arithmetic Sequences Part 3: Geometric Sequences

1a. Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(1 + px)^9$, where p is a constant. The first 3 terms are 1, $36x$ and qx^2, where q is a constant.

(2)

b. Find the value of p and the value of q.

(2)

2a. Find the first 4 terms, in ascending powers of x, of the binomial expansion of $(1 – 2x)^5$. Give each term in its simplest form.

(3)

b. If x is small, so that x^2 and higher powers can be ignored, show that **(3)**

$$(1 + x)(1 – 2x)^5 \approx 1 – 9x$$

3. Given that $\binom{40}{4} = \frac{40!}{4!b!}$

a. Write down the value of b **(2)**

In the binomial expansion of $(1 + x)^{40}$, the coefficients of x^4 and x^5 are p and q respectively.

b. Find the value of $\frac{q}{p}$ **(2)**

4a. Find the first 4 terms of the binomial expansion, in ascending powers of x, of $(1 + \frac{x}{4})^8$ giving each term in its simplest form.

(2)

b. Use your expansion to estimate the value of $(1.025)^8$, giving your answer to 4 decimal places.

(2)

5a. Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(2 – 3x)^6$ giving each term in its simplest form.

(3)

b. Hence, or otherwise, find the first 3 terms, in ascending powers of x, of the expansion of

$$(1 + \frac{x}{2})(2 - 3x)^6$$

(2)

6a. Find the first 3 terms, in ascndeing powers of x, of the binomial expansion of, $(2 - \frac{x}{16})^9$, giving each term in its simplest form.

(2)

$$f(x) = (a + bx)(2 - \frac{x}{16})^9, \text{ where a and b are constants}$$

b. Given that the first two terms, in ascending powers of x, in the series expansion of f(x) are 128 and 63x, find the value of a.

(2)

c. Find the value of b

(2)

Mark Scheme

1a.

$\binom{n}{0}(a)^n(bx)^0 + \binom{n}{1}(a)^{n-1}(bx)^1 + \cdots + \binom{n}{n}(a)^0(bx)^n$ $\binom{9}{0}(1)^9(px)^0 + \binom{9}{1}(1)^8(px)^1 + \binom{9}{2}(1)^7(px)^2$	**M1**
$= 1 + 9px + 36p^2x^2$	**M1**

1b.

Coefficients of x: $9p = 36$ $p = 4$	**M1**
Coefficients of x^2: $36p^2 = 3q$ $q = 576$	**M1**

2a.

$(x + a)^n = \sum_{k=0}^{n}\binom{n}{k} x^k a^{n-k} = \sum_{k=0}^{5}\binom{5}{k}(-2)^k(1)^{5-k}$	**M1**
$(1-2x)^5 = \binom{5}{0}(1)^5 + \binom{5}{1}(1)^4(-2x) + \binom{5}{2}(1)^4(-2x)^2 + \binom{5}{3}(1)^3(-2x)^3$	**M1**
$(1-2x)^5 = 1 - 10x + 40x^2 - 80x^3$	**M1**

2b.

$(1-2x)^5 = 1 - 10x + 40x^2 - 80x^3 \approx 1 - 10x$	**M1**
$(1 + x)(1 - 2x)^5 = (1 + x)(1 - 10x) = 1 + x - 10x - 10x^2$	**M1**
$(1-2x)^5 \approx 1 - 9x$	**M1**

3a.

$\binom{n}{k} = \frac{n!}{(n-k)!k!}$ $\binom{40}{4} = \frac{40!}{(40-4)!4!}$	**M1**
Therefore, $b = 36$	**M1**

3b.

$p = \binom{40}{4} = \frac{40!}{36!4!}$ $q = \binom{40}{5} = \frac{40!}{35!5!}$ $\frac{q}{p} = \frac{40!}{35!5!} \times \frac{36!4!}{40!}$	**M1**
$\frac{q}{p} = \frac{36}{5}$	**M1**

4a.

$(x + y)^n = \binom{n}{0}(x)^n(y)^0 + \binom{n}{1}(x)^{n-1}(y)^1 + \cdots + \binom{n}{n}(x)^0(y)^n$ $(1 + \frac{x}{4})^8 = \binom{8}{0} + \binom{8}{1}(\frac{x}{4})^1 + 28\left(\frac{x^2}{16}\right) + 56\left(\frac{x^3}{64}\right) + \cdots$	**M1**
$(1 + \frac{x}{4})^8 = 1 + 2x + \frac{7}{4}x^2 + \frac{7}{8}x^3$	**M1**

4b.

If we let $x = 0.1$ then we can see that: $(1 + \frac{x}{4})^8 = (1 + \frac{0.1}{4})^8 = (1.025)^8 = 1.2184$	**M1**
If we let $x = 0.1$ in the expansion $1.025^8 = 1 + 2(0.1) + \frac{7}{4}(0.1)^2 + \frac{7}{8}(0.1)^3 = 1.2184$	**M1**

5a.

$(a + bx)^n = \sum_{k=0}^{n}\binom{n}{k} a^k (bx)^{n-k}$ $a = 2$, $b = -3$, $n = 6$	**M1**
$(2 - 3x)^6 = \binom{6}{0}2^6 + \binom{6}{1}2^5(-3x) + \binom{6}{2}2^4(-3x)^2 + \ldots..$	**M1**
$(2 - 3x)^6 = 64 + 6(32)(-3x) + (15)(16)(9x^2)$ $= 64 - 576x + 2160x^2$	**M1**

5b.

$(1 + \frac{x}{2})(2 - 3x)^6 = (1 + \frac{x}{2})(64 - 576x + 2160x^2) = 64 - 576x + 2160x^2 + (\frac{x}{2})(64 - 576x + 2160x^2)$	**M1**
…. (expanding brackets) ….. $= 64 - 544x + 1872x^2 + \ldots..$	**M1**

6a.

$(2 - \frac{x}{16})^9 = 2^9 + 9(2)^8(-\frac{x}{16}) + 36(2)^7(-\frac{x}{16})^2$	**M1**
$= 512 - 144x + 18x^2$	**M1**

6b.

$(a + bx)(512 - 144x + 18x^2 + \ldots) = 128 + 36x + \ldots$	**M1**
Therefore, $512a = 128$ $a = \frac{1}{4}$	**M1**

6c.

$512b - 144a = 36$	**M1**
$512b - 36 = 36$ $b = \frac{9}{64}$	**M1**