Algebra and Functions

Part 2: Quadratic Functions

Jethwa Maths

AS Level	
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A-Level
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1. Given that, $4x^2 + 8x + 3 = a(x + b)^2 + c$, find the values of the constants a, b and c. **(3)**

2. Given that, $5x^2 + px - 8 = q(x - q)^2 + r$, for all values of x, find the values of the constants p, q and r. **(4)**

3. The curve C has the equation,

$$x^2 + ax + b = 0$$

Where a and b are constants. Given that the minimum points of C has co-ordinates (5, -2), find the values of a and b.

4. By completing the square, find in terms of the consntant k, roots of the equation **(4)**

$$x^2 + kx - 9 = 0$$

b) Hence find the exact roots of the equation, $x^2 + 5x – 9 = 0$ **(2)**

5. State the co-ordinates of the minimum point of the curve, $y = x^2 + 12x + 3$ **(4)**

6. The equation $x^2 + kx + 5 = 0$, where k is a constant and has no real roots. Find the possible set of values for k. **(3)**

7. The equation $kx^2 + 6kx + 2 = 0$, where k is a constant and has two distinct roots. Find the possible set of values for k. **(4)**

8. The equation $x^2 + (2k - 3)x + (k + 3) = 0$, where k is a constant and has no real roots. Find the set of posibles values for k. **(4)**

9. The curve with equation y $= px^2 – 8px – 7p$, where p is a constant does not intersect the line with equation $y = 4x – 10$.

(a) Show that $9p^2 – 8p +1 < 0$ **(4)**

(b) Find the set of possible values for p. **(2)**

10. The line $y = mx + 2$ is a tangent to the circle $(x – 5)^2 + (y + 1)^2 = 15$. Find the two possible values of m, giving your answers in exact form. **(7)**

11. Find the set of value of x for which

a) $2(3x + 4) > 1- x$ **(2)**

b) $3x^2 + 8x – 3 < 0$ **(2)**

12. a) Solve the inequality $x^2 + 8x > 20$ **(3)**

b) Find the set of values of x for which satisfy both of the inequalities: **(2)**

$$x^2 + 8x > 20$$
$$18 + 3x < 23 + x$$

13. Solve the inequality $x(x + 1) \leq 12$ (3) **(3)**

14. Find the set of values for x which satisfy both of the inequalities **(5)**

$$x^2 + 3x - 10 < 0$$
$$9 + 3x \leq 12 + x$$

Mark Scheme

1.

$4x^2 + 8x + 3 = 4(x^2 + 2x) + 3$	**M1**
$= [4(x^2 + 1) - 1] + 3$ $= 4(x^2 + 1) - 4 + 3$	**M1**
$= 4(x^2 + 1) - 1$ $a = 4$, $b = 1$, $c = -1$	**M1**

2.

$5x^2 + px - 8 = q(x - 1)^2 + r$	**M1**
$= q(x - 1)(x - 1) + r$ $= q(x^2 - 2x + 1) + r$ $= qx^2 - 2qx + q + r$	 **M1** **M1**
Compare co-efficients: x^2: $q = 5$ x: $p = 2q$, $p = -10$ constants: $q + r = -8$, $r = -13$	**M1**

3.

Using co-ordinates: $(x - 5)^2 - 2$ $(x - 5)(x - 5) - 2$	**M1**
$x^2 - 10x + 23$ $a = -10$ $b = 23$	 **M1** **M1**

4a.

$x^2 + kx - 9 = 0$ $(x + \frac{k}{2})^2 - \left(\frac{k^2}{4}\right) - 9 = 0$	**M1**
$(x + \frac{k}{2})^2 = \frac{k^2}{4} + 9$	**M1**
$x + \frac{k}{2} = \pm\sqrt{\frac{k^2}{4} + 9}$	**M1**
$x = -\frac{k}{2} \pm\sqrt{\frac{k^2}{4} + 9}$	**M1**

4b.

$k = 5$ $x = -\frac{5}{2} \pm\sqrt{\frac{5^2}{4} + 9}$	**M1**
$x = \frac{-5 + \sqrt{61}}{2}$ or $x = -\frac{5 + \sqrt{61}}{2}$	**M1**

5.

$y = x^2 + 12x + 3 = (x + 6)^2 - 36 + 3$ $= (x + 6)^2 - 33$	**M1 M1**
Minimum point = $(-6, -33)$	**M1 M1**

6.

No real roots, therefore, $b^2 - 4ac < 0$ $a = 1, b = k, c = 5$	**M1**
$k^2 - (4)(1)(5) < 0$ $k^2 - 20 < 0$ $k = \pm\sqrt{20}$	**M1**
$-\sqrt{20} < k < \sqrt{20}$	**M1**

7.

Equal roots, therefore, $b^2 - 4ac > 0$ $a = \text{k}, b = 6\text{k}, c = 2$	**M1**
$(6k)^2 - (4)(k)(2) > 0$ $36k^2 - 8k > 0$ $2k(16k - 4) > 0$ $2k > 0, k > 0$	**M1**
$16k - 4 > 0, k > \frac{1}{4}$	**M1**
$0 < \text{k} < \frac{1}{4}$	**M1**

8.

Two distinct roots, therefore, $b^2 - 4ac > 0$ $a = 1, b = 2k - 3, c = k + 3$	**M1**
$(2k - 3)^2 - (4)(1)(k + 3) < 0$ $4k^2 - 12k + 9 - 4k - 12 < 0$ $4k^2 - 16k - 3 < 0$	**M1**
Using calculator: $k = \frac{4+\sqrt{19}}{2}$ or $k = \frac{4-\sqrt{19}}{2}$	**M1**
$\frac{4-\sqrt{19}}{2} < k < \frac{4+\sqrt{19}}{2}$	**M1**

9a.

$\text{y} = px^2 - 8px - 7p = 4x - 10$ $px^2 - 8px - 7p - 4x + 10 = 0$	**M1**
$px^2 - (8p - 4)x + (10 - 7p) = 0$	**M1**
No intersection, therefore, no solutions, $b^2 - 4ac < 0$ $a = p, b = 8p - 4, c = 10 - 7p$	**M1**
$(8p - 4)^2 - (4)(p)(10 - 7p) < 0$ $64p^2 - 64p + 16 - 40p + 28p^2 < 0$ $92p^2 - 104p + 16 < 0$	**M1**

9b.

$92p^2 - 104p + 16 = 0$ $p = 11.39$ *(to 2 d.p) or* $p = 0.16$ *(to 2 d.p)*	**M1**
$0.16 < p < 0.16$	**M1**

10.

$(x-5)^2 + (mx+2+1)^2 = 15$	**M1**
$(x-5)^2 + (mx+3)(mx+3) = 15$ $x^2 - 5x - 5x + 25 + m^2x^2 + 3mx + 3mx + 9 = 15$	**M1**
$x^2 + m^2x^2 - 10x + 6mx + 19 = 0$ $(1+m^2)x^2 + (6m-10)x + 19 = 0$	**M1**
As the line is at a tnagenet, there is one solution, therefore $b^2 - 4ac = 0$ $a = 1 + m^2$ $b = 6m - 10$ $c = 19$	**M1**
$(6m-10)^2 - (4)(1+m^2)(19) = 0$ $(6m-10)(6m-10) - 76(1+m^2) = 0$	**M1**
$36m^2 - 60m - 60m + 100 - 76 - 76m^2 = 0$ $-40m^2 - 120m + 24 = 0$ $5m^2 + 15m - 3 = 0$	**M1**
$m = \frac{-15+\sqrt{285}}{10}$ or, $m = \frac{-15-\sqrt{285}}{10}$	**M1**

11a.

$2(3x+4) > 1 - x$ $6x + 8 > 1 - x$	**M1**
$7x > -7$ $x > -1$	**M1**

11b.

$(3x-1)(x+3) = 0$ $x = \frac{1}{3}$ or $x = -3$	**M1**
$-3 < x < \frac{1}{3}$	**M1**

12a.

$x^2 + 8x - 20 > 0$ $x = 2$ $x = -10$	**M1**
$x > 2,\ x < -10$	**M1 M1**

12b.

(From part *a*) $x^2 + 8x - 20 > 0$, $x > 2,\ x < -10$ $18 + 3x < 23 + x$ $2x < 5$ $x < 2.5$	**M1**
$2 < x < 2.5$	**M1**

13.

$x(x+1) \leq 12$ $x^2 + x - 12 \leq 0$	**M1**
$(x+4)(x-3) \leq 0$ $x \leq -4,\ x \leq 3$	**M1**
$-4 \leq x \leq 3$	**M1**

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14a.

$x^2 + 3x - 10 < 0$ $(x + 5)(x - 2) < 0$	**M1**
$x = -5$, $x = 2$	**M1**
$-5 < x < 2$	**M1**
$9 + 3x \leq 12 + x$ $2x \leq 3$ $x \leq 1.5$	**M1**
Both satisfied by: $-5 < x \leq 1.5$	**M1**