Algebra and Functions

Part 4: Graph Functions and Transformations

1. $f(x) = (x + 4)(x + 2)(x - 2)$

a) Sketch the curve of $f(x)$, showing the points of intersection with the co-ordinate axis. **(4)**

b) Showing the co-ordinates of the points of intersection with the co-ordinate axis, sketch on separate diagrams the curves,

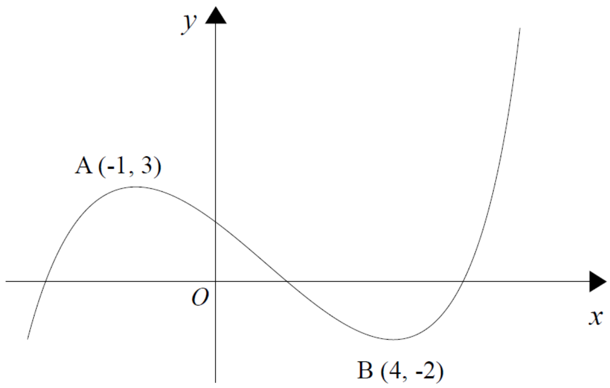
i. $y = f(x - 3)$ **(4)**

ii. $y = f(-x)$ **(5)**

2. The sketch shows the graph of $f(x)$. The curve has a minimum at
The points A (1, -3) and B (4, -2) are the turning points of the graph. On separate diagrams sketch the graphs of:

a) $y = f(x) + 2$ **(3)**

b) $y = -f(x)$ **(3)**



3. $f(x) = x^2 + 4x + 5$

a) Express $f(x)$ in the form $(x + a)^2 + b$, and state the coordinates of the minimum point of $y = f(x)$. **(3)**

b) Sketch the graph of $y = f(x)$ showing the co-ordinates of intersection with the coordinate axis **(3)**

c) Find the minimum points of these curves:

i. $y = 2f(x)$

ii. $y = f(2x)$

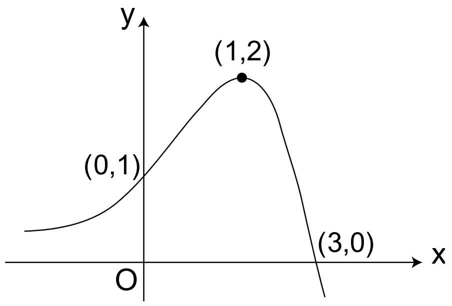
4a. Draw a graph of $f(x) = \frac{1}{x}$, $x \neq 0$. **(2)**

b. On a separate diagram, sketch the curbe with equation $y = \frac{3}{x+2}$, $x \neq -2$. Show the coordinates of any point at which the curve crosses a coordinate axis.

(4)

c. Write down the equations of the asympotoes of the curve in part (a).

5. Figure 1 shows a sketch of the curve with equation $y = \text{f}(x)$.
The curve crosses the co-ordinate axes at the points (0,1) and (3,0). The maximum point on the curve is (1, 2).



Sketch on separate diagrams the curve with equation:
a. $y = \text{f}(x + 1)$ **(3)**
b. $y = \text{f}(3x)$ **(3)**

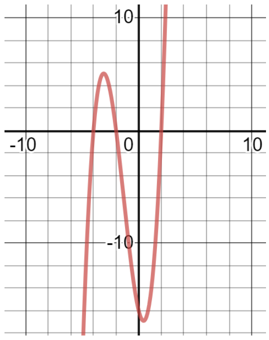
On each diagram, show clearly the co-ordinates of the maximum point, and each point at which the cruve crosses the coordinate axes.

6. Given that $\text{f}(x) = \ln x$, $x > 0$. Sketch on separate axes the graphs of,
a) $y = \text{f}(x)$ **(2)**
b) $y = -\text{f}(x - 4)$ **(3)**
Show on each diagram, the point where the graph meets or crosses the x-axis. In each case, state the equation of the aympotote.

Mark Scheme

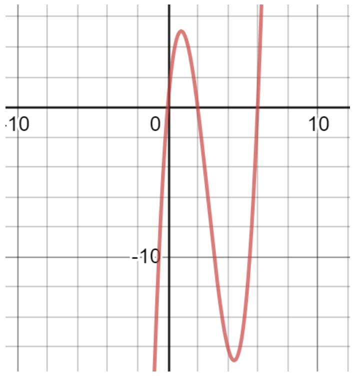
1a.

$f(x) = (x + 4)(x + 2)(x - 2)$
When $y = 0$, $0 = (x + 4)(x + 2)(x - 2) \rightarrow x = -4, -2$ and 2. **M1 M1**
When $x = 0$, $y = (4)(2)(-2) = -16$ **M1**
Shape **M1**



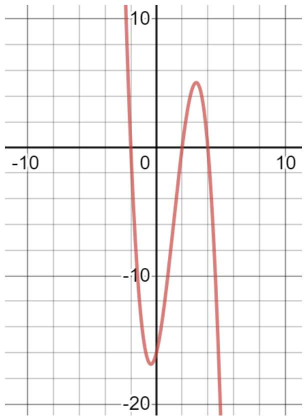
1bi.

$y = f(x - 4) \rightarrow$ Shift x co-ordinates 4 units to the right.
New roots: $x = 0$, $x = 2$, $x = 6$ **M1 M1**
Y-intercetpt: (0,0) **M1**
Shape **M1**



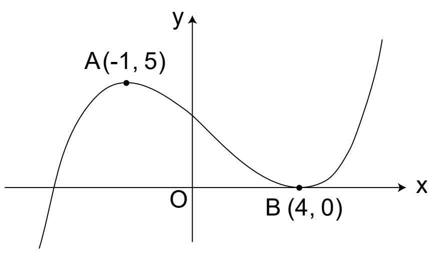
1bii.

$y = f(-x) \rightarrow$ Swap signs of all x co-ordinates
New roots: $x = 4$, $x = 2$, $x = -2$ **M1 M1**
Y-intercetpt: (0,-16) **M1**
Shape **M1**



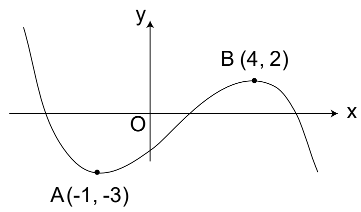
2a.

$y = f(x) + 2 \rightarrow$ move graph up by 2 units.
A (-1, 5) **M1**
B (4, 0) **M1**
Shape **M1**



2b.

y = -f(x) ⟶ swap signs of all y co-ordinates
A (-1, -3) **M1**
B (4, 2) **M1**
Shape **M1**

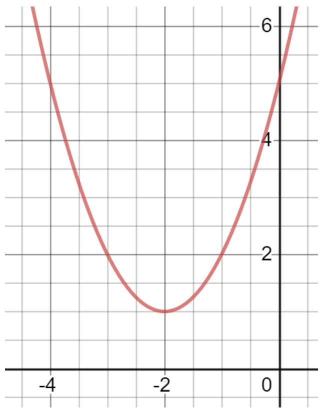


3a.

f(x) = x^2 + 4x + 5 f(x) = (x + 2)^2 – 4 + 5	**M1**
f(x) = (x + 2)^2 + 1	**M1**
Minimum point = (-2, 1)	**M1**

3b.

when y = 0, x = no solutions, does not cross x-axis. **M1**
When x = 0, y = 0^2 + 0 + 5 = 5 **M1**
Shape **M1**



3ci.

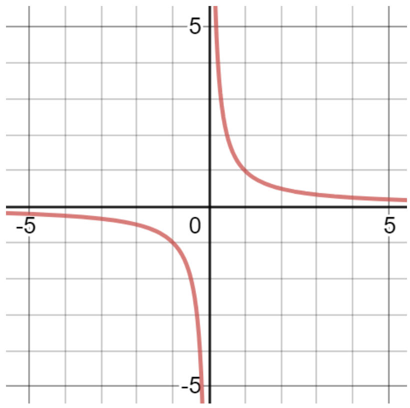
y = 2f(x) graph would be stretched by a factor of 2, y co-ordinates doubled. New minimum point: (-2, 2)	**M1 M1**

3cii.

y = f(2x) graph would be shrunk horizontally by a factor of 1/2, x co-ordinates halved. New minimum point: (-1, 1)	**M1 M1**

4a.

Asympotptes at x = 0, y = 0 **M1**
Shape **M1**

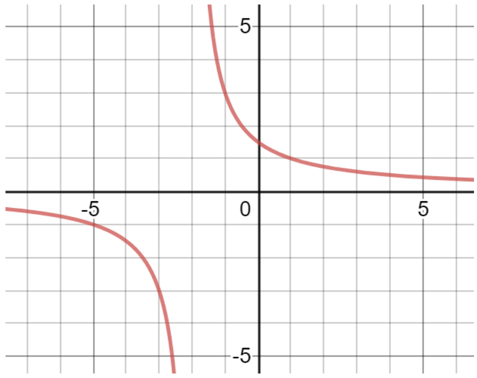


4b.

Graph does not cross the x-axis.

Graph crosses y axis at $(0, \frac{3}{2})$ **M1**

Shape **M1**



4c.

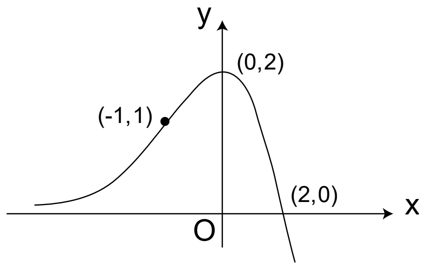
Assymptotes at $y = 0$, $x = -2$	**M1 M1**

5a.

$y = f(x + 1) \rightarrow$ graph moves one unit to the left.

Shape **M1**

Co-ordinates (0, 2) (-1, 1) (2, 0) **M1 M1**

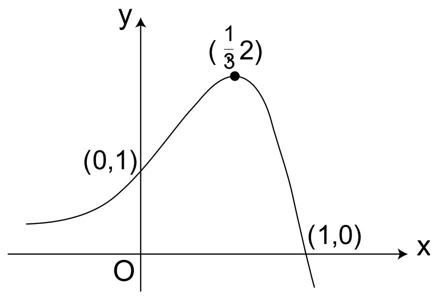


5b.

$y = f(3x) \rightarrow$ graph shrunk by a scale factor of $\frac{1}{3}$ (x co-ordinates affected only)

Shape **M1**

Co-ordinates (0, 1) $(\frac{1}{3}, 2)$ (1, 0) **M1 M1**

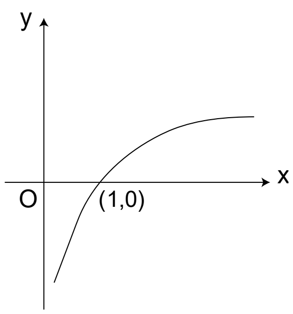


6a.

$y = \ln x$

Shape **M1**

Asympotote at y = 0 **M1**



6b.

$y = -f(x - 4) \rightarrow$ Two step transformation
Step 1: $y = f(x - 4) \rightarrow$ graph moves 4 units to the left
Step 2: $y = -f(x - 4) \rightarrow y$ values switch sign.

Shape **M1 M1**
Asympotote at $x = 4$ **M1**