Algebra and Functions

Part 3: Simultaneous Equations

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1. Solve the simultaneous equations,

$$x + y = 2$$
$$x^2 + 2y = 12$$

(6)

2. Solve the simultaneous equations,

$$y - 3x + 2 = 0$$
$$y^2 - x - 6x^2 = 0$$

(7)

3a) By eliminating y from the equations,

$$y = x - 4$$
$$2x^2 - xy = 8$$

Show that

$$x^2 + 4x - 8 = 0$$

(2)

b) Hence, or otherwise, solve the simultaneous equations,

$$y = x - 4$$
$$2x^2 - xy = 8$$

(4)

Giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers.

4. Solve the simultaneous equations,

$$y - 2x - 4 = 0$$
$$4x^2 + y^2 + 20x = 0$$

(6)

5. The curve C has the equation $y = x^2 - 2x + 7$. The line L has the equation $x + y = 7$. Find the coordinates of the points where L and C intersect.

(7)

6a. Solve the inequality, $x^2 + 3x - 10 < 0$ **(3)**

b. Find the set of values for x which satisfy both of the inequalities

$x^2 + 3x - 10 < 0$

$9 + 3x \leq 12 + x$ **(3)**

Mark Scheme

1.

$x + y = 2 \rightarrow y = 2 - x$	**M1**
Subing into second equation: $x^2 + 2(2- x) - 12 = 0$ $x^2 + 4 - 2x - 12 = 0$	**M1**
$x^2 - 2x - 8 = 0$ $(x + 2)(x - 4) = 0$	**M1**
$x = - 2$ $x = - 4$	**M1**
$x = - 2 \rightarrow y = 2 - (-2) = 4$ $x = - 4 \rightarrow y = 2 - (-4) = 6$ Solutions: $x = -2, y = 4$ $x = -4, y = 6$	**M1 M1**

2.

$y - 3x + 2 \rightarrow y = 3x - 2$	**M1**
Subing into second equation: $(3x - 2)^2 - x - 6x^2 = 0$ $9x^2 + 4 - 12x - x - 6x^2 = 0$	**M1**
$3x^2 - 13x + 4 = 0$ $(3x - 1)(x - 4) = 0$	**M1**
$x = \frac{1}{3}$ $x = 4$	**M1 M1**
$x = \frac{1}{3} \rightarrow y = 3(\frac{1}{3}) - 2 = -1$ $x = 4 \rightarrow y = 3(1) - 2 = 1$ Solutions: $x = \frac{1}{3}, y = -1$ $x = 4, y = 1$	**M1 M1**

3a.

$y = x - 4$ $2x^2 - x(x - 4) = 8$	**M1**
$2x^2 - x^2 + 4x - 8 = 0$ $x^2 + 4x - 8 = 0$	**M1**

3b.

$x^2 + 4x - 8 = 0$ $x = -2 \pm 2\sqrt{3}$	**M1 M1**
$y = -6 \pm 2\sqrt{3}$	**M1 M1**

4.

$y - 2x - 4 = 0 \rightarrow y = 2x + 4$	**M1**
$4x^2 + (2x + 4)^2 + 20x = 0$ $4x^2 + 4x^2 + 16x + 16 + 20x = 0$ $8x^2 + 36x + 16 = 0 \rightarrow 2x^2 + 9x + 4 = 0$	**M1**
$(2x + 1)(x + 4) = 0$ $x = -\frac{1}{2}$ $x = -4$	**M1**
$x = -\frac{1}{2} \rightarrow y = 2(-\frac{1}{2}) + 4 = 3$ $x = - 4 \rightarrow y = 2(-4) + 4 = -4$	**M1**
Solutions: $x = -\frac{1}{2}, y = 3$ $x = - 4, y = -4$	**M1 M1**

5.

$x + y = 7 \rightarrow x = 7 - y$	**M1**
$y = (7 - y)^2 - 2(7 - y) + 7$	**M1**
$y = 49 - 14y + y^2 - 14 + 2y + 7$ $y^2 - 13y + 42 = 0$	**M1**
$(y + 6)(y - 7) = 0$ $y = -6$ $y = 7$	**M1 M1**
$y = -6 \rightarrow x = 7 - (-6) = 13$ $y = 7 \rightarrow x = 7 - 7 = 0$ Solutions: $x = 13, y = -6$ $x = 0, y = 7$	**M1 M1**

6a.

$x^2 + 3x - 10 < 0$ $(x + 5)(x - 2) = 0$	**M1**
$x = -5$ $x = 2$	**M1**
$-5 < x < 2$	**M1**

6b.

$9 + 3x \leq 12 + x$ $9 + 2x \leq 12$	**M1**
$2x \leq 3$ $x \leq \frac{3}{2}$	**M1**
$-5 < x \leq \frac{3}{2}$	**M1**