A-Level Unit Test: Algebra and Functions

Index Laws & Surds

Jethwa Maths

1. a) Write down the value of $64^{\frac{1}{6}}$ **(1)**

b) Simplify fully $(64x^6)^{-\frac{2}{6}}$ **(3)**

2. Express 16^{2x+4} in the form 4^y, stating y in terms of x **(2)**

3. Given that $y = 3^x$,

a) Express 9^x in terms of y **(1)**

b) Hence or otherwise, solve $(9^x) - 3^x + 1 = 0$ **(4)**

4. Solve, $64^{4x-5} = 16^{6x-10}$ **(3)**

5. a) Simplify $\sqrt{50} + \sqrt{32} - \sqrt{128}$, giving your answer in the form $a\sqrt{2}$ where a is an integer. **(2)**

b) Hence or otherwise, simplify, $\frac{12\sqrt{6}}{\sqrt{50}+\sqrt{32}-\sqrt{128}}$, giving your answer in the form $b\sqrt{c}$, where b and c are integers and $b \neq 1$

(3)

6. a) Write $\sqrt{80}$ in the form $c\sqrt{5}$, where c is a positive constant.

A rectangle R has a length of $(1 + \sqrt{5})$ cm and an area of $\sqrt{80}$ cm^2 **(1)**

b) Calculate the width of R in cm. Express your answer in the form $p + q\sqrt{5}$, where p and q are integers to be found.

(3)

7. Show that $\dfrac{2}{\sqrt{12} - \sqrt{8}}$ can be written in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers **(5)**

Total marks: 28

Mark Scheme

1a.

$64^{\frac{1}{6}} = \sqrt[6]{64} = 2$	**M1**

1b.

$(64x^6)^{-\frac{2}{6}} = (64)^{-\frac{2}{6}}(x^6)^{-\frac{2}{6}}$	**M1**
$= \frac{1}{4}x^{-2}$	**M1** **M1**

2.

$16^{2x+4} = 4^{2(2x+4)}$	**M1**
$= 4^{4x+4}$ Therefore, $y = 4x + 4$	**M1**

3a.

$9^x = 3^{2x} = 3^x \text{ x } 3^x$ $= y \text{ x } y$ $= y^2$	**M1**

3b.

$9y^2 - 240y - 81 = 0$ $(3y - 81)(3y + 1) = 0$	**M1**
$3y - 81 = 0, y = 27$ $3y + 1 = 0, y = -\frac{1}{3}$	**M1**
$3^x = 27, x = 3$	**M1**
$3^x = -\frac{1}{3}$, no solution	**M1**

4.

$64^{4x+2} = 16^{6x-10}$ $4^{4(4x+2)} = 4^{2(6x-10)}$	**M1**
$4^{16x+8} = 4^{12x-20}$	**M1**
$16x + 8 = 12x - 20$ $4x = -28$ $x = -7$	**M1**

5.

$\sqrt{50} + \sqrt{32} - \sqrt{128}$ $= \sqrt{25}\sqrt{2} + \sqrt{16}\sqrt{2} - \sqrt{64}\sqrt{2}$ $= 5\sqrt{2} + 4\sqrt{2} - 8\sqrt{2}$	**M1**
$= \sqrt{2}$	**M1**

5b.

$\frac{12\sqrt{6}}{\sqrt{50}+\sqrt{32}-\sqrt{128}} = \frac{12\sqrt{6}}{\sqrt{2}}$	**M1**
$\frac{12\sqrt{6}}{\sqrt{2}} \text{ x } \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{12}}{2}$	**M1**
$= 6\sqrt{12}$ $= 6\sqrt{4}\sqrt{3}$ $= 12\sqrt{3}$	**M1**

6a.

$\sqrt{80} = \sqrt{16} \text{ x } \sqrt{5} = 4\sqrt{5}$ $c = 5$	**M1**

6b.

Width = $\frac{\sqrt{80}}{1+\sqrt{5}} \text{ x } \frac{1-\sqrt{5}}{1-\sqrt{5}}$	**M1**
$= \frac{\sqrt{80}-\sqrt{400}}{1-5} =$	**M1**
$= \frac{4\sqrt{5}-20}{-4} = 5 - \sqrt{5}$ $p = 5, q = -1$	**M1**

7.

$\left\{\frac{2}{\sqrt{12}-\sqrt{8}}\right\} = \frac{2}{\left(\sqrt{12}-\sqrt{8}\right)} \times \frac{\left(\sqrt{12}+\sqrt{8}\right)}{\left(\sqrt{12}+\sqrt{8}\right)}$	**M1**
$= \frac{\left\{2\left(\sqrt{12}+\sqrt{8}\right)\right\}}{12-8}$	**M1**
$= \frac{2\left(2\sqrt{3}+2\sqrt{2}\right)}{12-8}$	**M1 M1**
$= \sqrt{3}+\sqrt{2}$	**M1**

A-Level Unit Test: Algebra and Functions

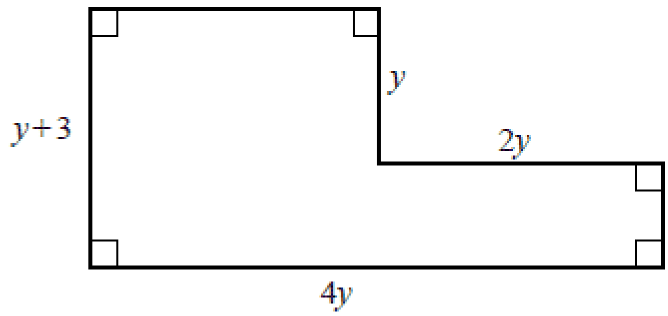
Index Laws & Surds

1. Find the set of value of x for which
a) $3x - 7 < 3 - x$ **(2)**
b) $x^2 - 9x \leq 36$ **(4)**
c) Both, $3x - 7 < 3 - x$ and $x^2 - 9x \leq 36$ **(1)**

2a) A rectangular tile has length $4x$ cm and width $(x + 3)$ cm. The area of the rectangle is less than 112 cm^2. By writing down and solving an inequality, determine the set of possible values of x.

(6)

b) A second rectangular tile of length $4y$ cm and width $(y + 3)$ has a rectangle of length $2y$ cm and width y cm removed from one corner as shown in the diagram.



Given that the perimeter of this tile is between 20 cm and 54 cm, determine the set of possible values of y. **(5)**

3. The equation $(k + 3)x^2 + 6x + k = 5$, where k is a constant, has two distinct real solutions for x.

a) Show that k satisfies, $k^2 - 2k - 24 < 0$ **(4)**
b) Hence find the set of possible values of k **(3)**

4. The equation $x^2 + kx + 8 = k$ has no real solutions for x. Find the set of possible values for k. **(6)**

5a. Express $2x^2 - 20x + 49$ in the form $p(x - q)^2 + r$ **(4)**
5b. State the co-ordinates of the vertex of the curve $y = 2x^2 - 20x + 49$ **(2)**

6. Given than $3x^2 + px + 16 = q(x - 2)^2 + r$ for all values of x, find the values of the constants p, q and r. **(4)**

7. $f(x) = x^2 + (k + 3)x + k$ where k is a real constant.
a) Find the discriminant of $f(x)$ in terms of k **(2)**
b) Show that the discriminant of $f(x)$ can be expressed in the form $(k + a)^2 + b$, where a and b are integers to be found.

(2)

c) Show that, for all values of k, the equation $f(x) = 0$ has real roots. **(2)**

8. The straight line with equation $y = 3x - 7$ does not cross or touch the curve with equation $y = 2px^2 - 6px + 4p$, where p is a constant.

a) Show that $4p^2 - 20p + 9 < 0$. **(4)**

b) Hence, find the set of possible values of p. **(4)**

Total marks: 53

Mark Scheme

1a.

$3x - 7 < 3 - x$	**M1**
$4x < 10$	**M1**
$x < \frac{5}{2}$	**M1**

1b.

$x^2 - 9x \leq 36$	**M1**
$x^2 - 9x - 36 \leq 0$	**M1**
$(x - 12)(x - 3)$ $x = 12, x = 3$	**M1**
$3 \leq x \leq 12$	**M1**

1c.

$3 \leq x < \frac{5}{2}$	**M1**

2a.

$4x(x + 3) < 112$	**M1 M1**
$4x^2 + 12x - 112 < 0$ $x^2 + 3x + 28 < 0$	**M1**
$(x + 7)(x - 4) < 0$	**M1**
$x = -7, x = -4$	**M1**
$-7 < x < -4$	**M1**

2b.

Perimeter $= y + 3 + 2y + y + 2y + 3 + 4y = 10y + 6$	**M1**
$20 < 10\,y + 6 < 54$	**M1**
$20 < 10\,y + 6$ $y < \frac{7}{5}$	**M1**
$10\,y + 6 < 54$ $y < \frac{24}{5}$	**M1**
$\frac{7}{5} < y < \frac{24}{5}$	**M1**

3a.

$(k + 3)x^2 + 6x + k = 5$ $(k + 3)x^2 + 6x + k - 5 = 0$ $(k + 3)x^2 + 6x + (k - 5) = 0$	**M1**
Equation has two distinct real solutions, $b^2 - 4ac > 0$ $a = k + 3, b = 6, c = k - 5$	**M1**
$6^2 - (4)(k+3)(k - 5) > 0$ $36 - 4(k^2 - 2k - 15) > 0$ $36 - 4k^2 + 8k + 60 > 0$	**M1**
$-4k^2 + 8k + 96 > 0$ $-k^2 + 2k + 24 > 0$ $k^2 - 2k - 24 < 0$	**M1**

3b.

$k^2 - 2k - 24 < 0$	**M1**
$(k-6)(k+4) = 0$ $k = 6, k = -4$	**M1**
$-4 < k < 6$	**M1**

4.

$x^2 + kx + 8 - k = 0$ $x^2 + kx + (8 - k) = 0$	**M1**
Equation has no real real solutions, $b^2 - 4ac < 0$ $a = 1, b = k, c = 8 - k$	**M1**
$k^2 - (4)(1)(8 - k) < 0$ $k^2 - 4k - 32 < 0$	**M1** **M1**
$(k-8)(k+4) = 0$ $k = 8, k = -4$	**M1**
$-4 < k < 8$	**M1**

5a.

$2x^2 - 20x + 49 = 2(x^2 - 10x) + 49$	**M1**
$= 2[(x-5)^2 - 25] + 49$	**M1**
$= 2(x-5)^2 - 50 + 49$	**M1**
$= 2(x-5)^2 - 1$	**M1**

5b.

From part a) $2(x-5)^2 - 1$ Vertex: (5, -1)	**M1 M1**

6.

$3x^2 + px + 16 = q(x-2)^2 + r$ $= q(x-2)(x-2) + r$ $= q(x^2 - 4x + 4) + r$	**M1**
$= qx^2 - 4qx + (4q + r)$	**M1**
Comparing coefficients: x^2: $q = 3$	**M1**
x: $p = -4q$, $p = -12$ constants: $q + r = 16$, $r = 13$	**M1**

7a.

$a = 1, b = k + 3, c = k$ $b^2 - 4ac = (k+3)^2 - (4)(1)(k)$	**M1**
$= k^2 + 6k + 9 - 4k$ $= k^2 + 2k + 9$	**M1**

7b.

$k^2 + 2k + 9 = (k+1)^2 - 1 + 9$	**M1**
$= (k+1)^2 + 8$ $a = 1, b = 8$	**M1**

7c.

$(k+1)^2$ will always give a positive value	**M1**
$(k+1)^2 + 8$ positive term plus a positive term will always be ≥ 0, therefore the function has two real roots.	**M1**

8a.

$2px^2 - 6px + 4p = 3x - 7$ $2px^2 - 6px + 4p - 3x + 7 = 0$ $2px^2 - (6p - 3)x + (4p + 7) = 0$	**M1**
Curve and line do not cross, therefore, $b^2 - 4ac < 0$ $a = 2p$, $b = 6p - 3$, $c = 4p + 7$ $(6p - 3)^2 - 4(2p)(4p + 7) < 0$	**M1**
$(6p - 3)(6p - 3) - 8p(4p + 7) < 0$ $36p^2 - 36p + 9 - 32p^2 - 56p < 0$	**M1**
$4p^2 - 20p + 9 < 0$	**M1**

8b.

$4p^2 - 20p + 9 < 0$	**M1**
$(2p - 9)(2p - 1) = 0$	**M1**
$2p - 9 = 0$, $p = \frac{9}{2}$ $2p - 1 = 0$, $p = \frac{1}{2}$	**M1**
$\frac{1}{2} < p < \frac{9}{2}$	**M1**

A-Level Unit Test: Algebra and Functions

Simultaneous Equations

Jethwa Maths

1. Solve the simultaneous equations

$$x + y = 2$$
$$4y^2 - x^2 = 11$$ **(7)**

2. Solve the simultaneous equations:

$$y + 4x + 1 = 0$$
$$y^2 + 5x^2 + 2x = 0$$ **(7)**

3. Find the co-ordinates of the points where the circle C with equation $x^2 + y^2 - 2x = 19$ meets the line L with equation $y = 3x - 1$.

(5)

4. Find the pair of values (x, y) which satisfy the simultaneous equations

$$x^2 + 2xy + y^2 = 9$$
$$x - 3y = 1$$ **(7)**

5a. Sketch the curve $y = 12 - x - x^2$ giving the co-ordinates of all intercepts with the axes. **(5)**

b. Solve the inequality $12 - x - x^2 > 0$ **(2)**

c. Find the co-ordinates of the points of intersection of the curve $y = 12 - x - x^2$ and the line $3x + y = 4$. **(5)**

Total marks: 38

Mark Scheme

1.

$x + y = 2 \rightarrow y = 2 - x$	**M1**
$4(2 - x)^2 - x^2 = 11$ $4(2 - x)(2 - x) - x^2 = 11$ $4(4 + x^2 - 4x) - x^2 = 11$	**M1**
$16 + 4x^2 - 16x - x^2 - 11 = 0$ $3x^2 - 16x + 5 = 0$	**M1**
$(3x - 1)(x - 5) = 0$ $3x - 1 = 0$ $x - 5 = 0$	**M1**
$3x - 1 = 0, x = \frac{1}{3}$ $x - 5 = 0, x = 5$	**M1**
$x = \frac{1}{3}, y = 2 - \frac{1}{3} = \frac{5}{3}$ $x = 5, y = 2 - 5 = -3$ Solutions: $x = \frac{1}{3}, y = \frac{5}{3}$ $x = 5, y = -3$	**M1 M1**

2.

$y + 4x + 1 = 0 \rightarrow y = -4x - 1$	**M1**
$(-4x - 1)^2 + 5x^2 + 2x = 0$ $(-4x - 1)(-4x - 1) + 5x^2 + 2x = 0$	**M1**
$16x^2 + 1 + 8x + 5x^2 + 2x = 0$ $21x^2 + 10x + 1 = 0$	**M1**
$(7x + 1)(3x + 1) = 0$	**M1**
$7x + 1 = 0, x = -\frac{1}{7}$ $3x + 1 = 0, x = -\frac{1}{3}$	**M1**
$x = -\frac{1}{7}, y = -4(-\frac{1}{7}) - 1 = -\frac{3}{7}$ $x = -\frac{1}{3}, y = -4(-\frac{1}{3}) - 1 = \frac{1}{3}$ Solutions: $x = -\frac{1}{7}, y = -\frac{3}{7}$ $x = -\frac{1}{3}, y = \frac{1}{3}$	**M1 M1**

3.

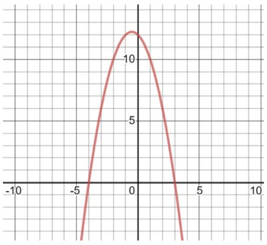
$x^2 + (3x - 1)^2 - 2x = 19$ $x^2 + (3x - 1)(3x - 1) - 2x - 19 = 0$	**M1**
$x^2 + 9x^2 - 6x + 1 - 2x - 19 = 0$ $10x^2 - 8x - 18 = 0$ $5x^2 - 4x - 9 = 0$	**M1**
$(5x - 9)(x + 1) = 0$ $5x - 9 = 0, x = \frac{9}{5}$ $x + 1 = 0, x = -1$	**M1**
$x = \frac{9}{5}, y = 3(\frac{9}{5}) - 1 = \frac{22}{5}$ $x = -1, y = 3(-1) - 1 = -4$ Solution: $x = \frac{9}{5}, y = \frac{22}{5}$ $x = -1, y = -4$	**M1 M1**

4.

$x - 3y = 1 \rightarrow x = 1 + 3y$ $(1 + 3y)^2 + 2y(1+3y) + y^2 - 9 = 0$	**M1**
$(1 + 3y)(1 + 3y) + 2y + 6y^2 + y^2 - 9 = 0$ $1 + 9y^2 + 6y + 2y + 6y^2 + y^2 - 9 = 0$	**M1**
$16y^2 + 8y - 8 = 0$ $2y^2 + y - 1 = 0$	**M1**
$(2y - 1)(y + 1) = 0$	**M1**
$2y - 1 = 0, y = \frac{1}{2}$ $y + 1 = 0, y = -1$	**M1**
$y = \frac{1}{2}, x = 1 + 3(\frac{1}{2}) = \frac{5}{2}$ $y = -1, x = 1 + 3(-1) = -2$ Solutions: $(\frac{5}{2}, \frac{1}{2})$ and $(-2, -1)$	**M1 M1**

5a.

$y = 12 - x - x^2$ when $x = 0, y = 12$ when $y = 0, 12 - x - x^2 = 0 \rightarrow x^2 + x - 12 = 0$	**M1**
$(x + 4)(x - 3) = 0$ $x = -4$ $x = 3$	**M1 M1**



M1 Shape
M1 Crossing at $x = -4$ and 3
$y = 12$

5b.

From the graph, $12 - x - x^2 > 0$ $-4 < x < 3$	**M1** **M1**

5c.

$3x + y = 4 \rightarrow y = 4 - 3x$ $y = 12 - x - x^2 = 4 - 3x$ $12 - x - x^2 - 4 + 3x = 0$	**M1**
$x^2 + x - 3x - 12 + 4 = 0$ $x^2 - 2x - 8 = 0$	**M1**
$(x - 4)(x + 2) = 0$ $x = 4$ $x = -2$	**M1**
$x = 4, y = 4 - 3(4) = -8$ $x = -2, y = 4 - 3(-2) = 10$ Solutions: $(4, -8)$ and $(-2, 10)$	**M1 M1**

A-Level Unit Test: Algebra and Functions

Graph Functions and Transformations

Jethwa Maths

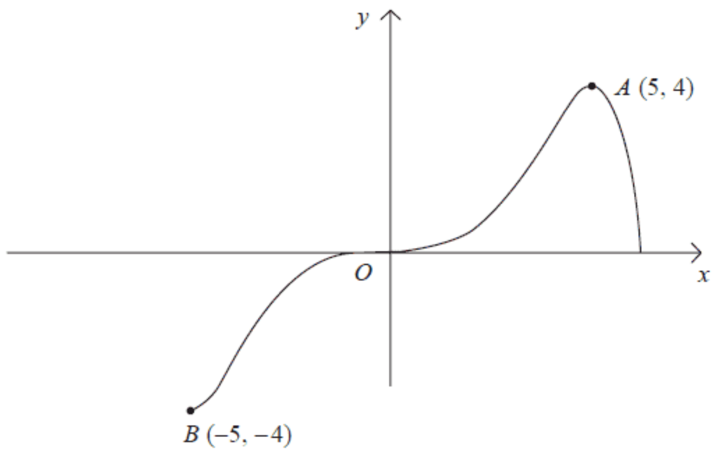
1. Figure 1 shows a sketch of the curve with equation $y = f(x)$. The curve passes through the origin O and the points (5, 4) and B (-5, -4). In separate diagrams, sketch the graph with equation.

a) $y = f(x) + 3$ **(3)**

b) $y = f(x - 9)$ **(3)**

On each sketch, show the co-ordinates of the points corresponding to A and B.

c) The graph has a transformation of $y = 2f(x + 1)$. State the new coordinates of A and B. **(2)**



2. $f(x) = (x + 3)(x - 1)^2$

a) Sketch the curve $y = f(x)$, showing the points of intersection with the coordinate axis. **(3)**

b) Find the equation of $y = f(x + 2)$ in the form $y = (x + a)(x + b)^2$ **(2)**

3. Sketch the graph of $y = \frac{1}{x} + 2$, showing the points of intersection with the co-ordinate axis and stating the equations of any asymptotes.

(3)

4. $f(x) = x^3 + 4x^2 - 5x$

a) Sketch the curve $y = f(x)$, showing the points of intersection with the co-ordinate axis.

b) Showing the coordinates of the points of intersection with the coordinate axis, sketch on separate diagrams the curves:

i. $y = f(2x)$ **(2)**

ii. $y = 3f(x)$ **(2)**

5a. Sketch on the same diagram the curve of $y = x^2 + 5x$ and $y = -\frac{1}{x}$ **(4)**

b. State, giving a reason, the number of real solutions to the equation $x^2 + 5x + \frac{1}{x} = 0$ **(2)**

6. Figure 2 shows a sketch of the curve with equation $y = f(x)$ where,

$$f(x) = \frac{x}{x-2}, \; x \neq 2$$

The curve passes through the origin and has two asymptotes, with equations $y = 1$ and $x = 2$, as shown in Figure 1.

a) Sketch the curve with equation $y = f(x - 1)$ and state the equations of the asymptotes of this curve. **(4)**

b) Find the coordinates of the points where the curve with equation $y = f(x - 1)$ crosses the coordinate axes. **(4)**

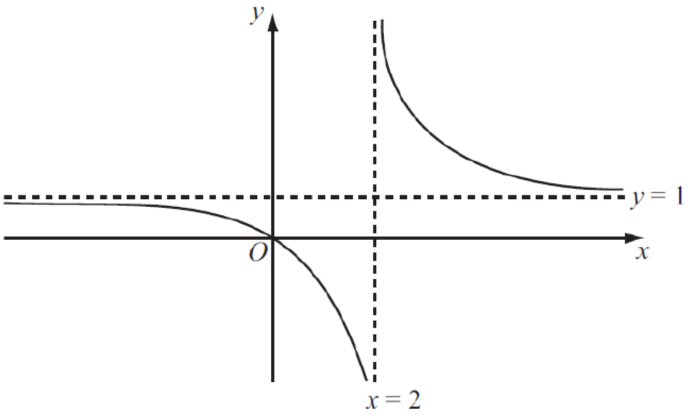


Figure 2

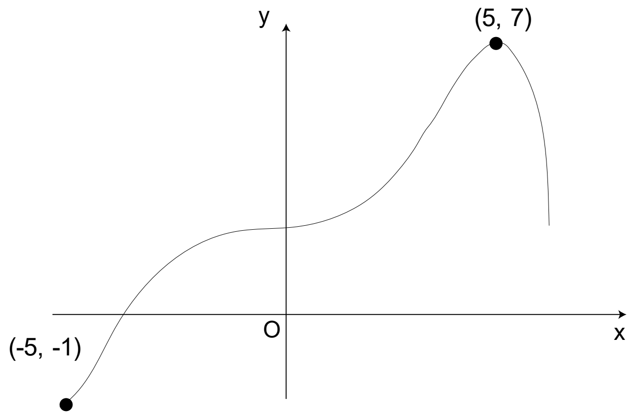
Total marks: 34

Mark Scheme

1a.

$y = f(x) + 3 \rightarrow$ graphs moves up by 3 units

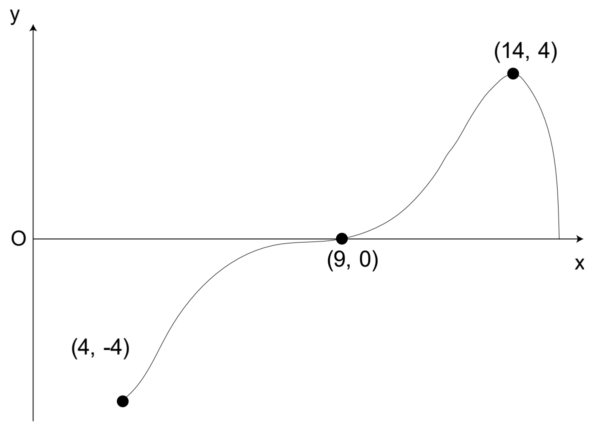
Shape **M1**
A: (5, 7) **M1**
B: (-5, -1) **M1**



1b.

$y = f(x - 9) \rightarrow$ graphs 9 units to the right

Shape **M1**
A: (14, 4) **M1**
B: (4, -4) **M1**

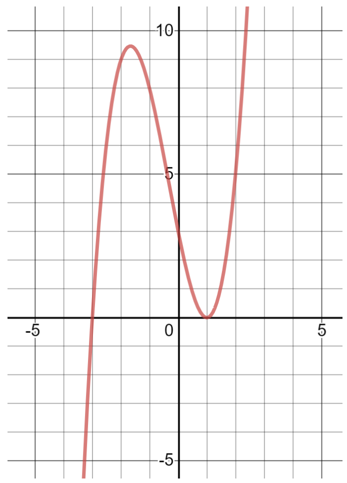


1c.

$y = 2f(x + 1) \rightarrow x$ coordinates 1 unit left, y coordinates doubled. A: (4, 8) B: (-6, -8)	**M1 M1**

2a.

$f(x) = (x + 3)(x - 1)^2$
Points of intersection on x axis: (-3, 0) and (1, 0) **M1**
Point of intersection on y axis: (0, 3) **M1**
Shape: **M1**



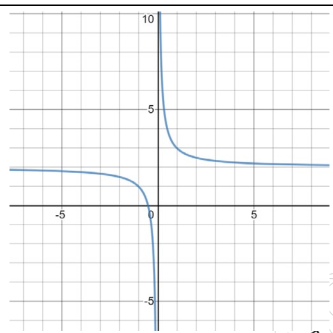
2b.

$y = f(x + 2) \rightarrow$ graph moves 2 units to the left New equation: $f(x) = (x + 5)(x + 1)^2$	**M1 M1**

3.

$y = \frac{1}{x} + 2 \rightarrow y = \frac{1}{x}$ graph shifted upwards by two units.

Asymptotes: $y = 2$ **M1**
Axis cuts: $(-\frac{1}{2}, 0)$ **M1**
Shape **M1**



4a.

$f(x) = x^3 + 4x^2 - 5x$

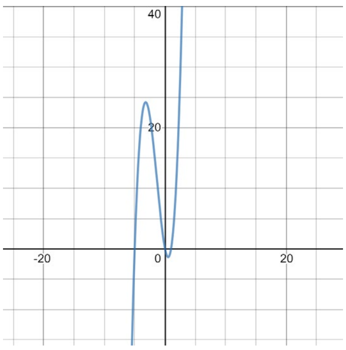
$f(x) = 0$, $x^3 + 4x^2 - 5x = 0$

$x(x + 5)(x - 1)$ **M1**

Roots = 0, -5 and 1 **M1**

When $x = 0$, $y = 0$ **M1**

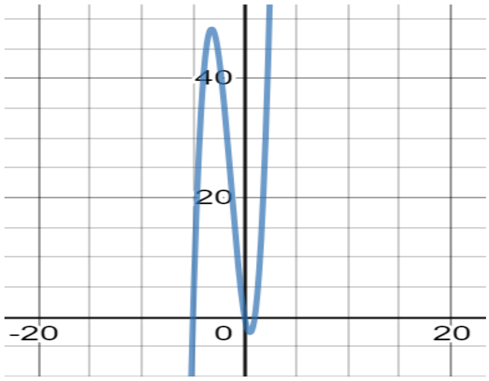
Shape **M1**



4bi.

$y = f(2x)$ graph is stretched by a scale factor of 2.

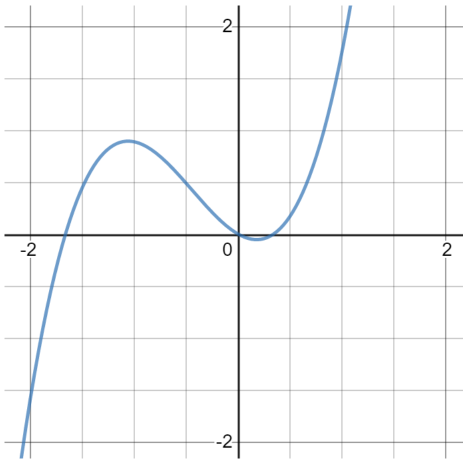
Roots remain the same: 0, -5, 1



4bii.

$y = 3f(x) \rightarrow$ graph is shrunk by a scale factor of $\frac{1}{3}$. x – coordinates are multiplied by a third.

Roots: $(-\frac{5}{3}, 0)$ $(0, 0)$ $(\frac{1}{3}, 0)$



5a.

$y = x^2 + 5x = x(x + 5)$

Roots at (0, 0) and (-5, 0) **M1**

Y intercept at (0, 0) **M1**

Shape **M1**

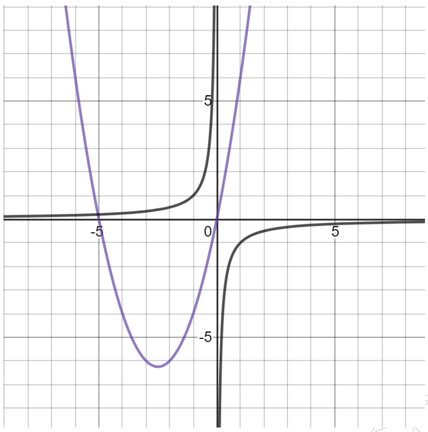
$y = -\frac{1}{x}$

No roots **M1**

No intercept **M1**

(Asymptotes at $x = 0$, $y = 0$)

Shape **M1**

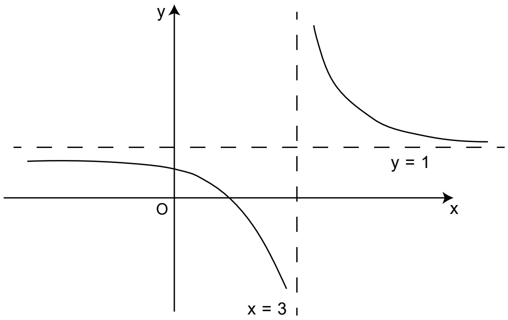


5b.

There is one real solution	**M1**
As the graphs cross once	**M1**

6a.

$y = \mathrm{f}(x - 1) \rightarrow$ graph shifts 1 unit to the right **M1**
Asymptotes: $x = 3$, $y = 1$ **M1 M1**
Shape **M1**



6b.

$\mathrm{f}(x - 1) = \frac{x-1}{x-1-2} = \frac{x-1}{x-3}$	**M1** **M1**
When $x = 0$, $y = \frac{1}{3}$ When $y = 0$, $\frac{x-1}{x-3} = 0$, $x = 1$ Co-ordinates cutting the axes: $(0, \frac{1}{3})$ and $(1, 0)$	**M1 M1**