Integration

Pt. 1: Basic Integration

AS Level
Pt. 1: Basic Integration
Pt. 2: Equations of Curves
Pt. 3: Definite Integration

Jethwa Maths

1. Find $\int 8x^3 + 4\ dx$, giving each term in its simplest form. **(2)**

2. Find $\int 5x^3 - 6x + 1\ dx$ **(2)**

3. Find $\int 24x^{-3}\ dx$ **(2)**

4. Find $\int 10x^4 - 4x - \frac{3}{\sqrt{x}}\ dx$ **(3)**

5. Given that $y = 2x^5 + 7 + \frac{1}{x^3}$, $x \neq 0$, find, in their simplest form,
a. $\frac{dy}{dx}$ **(2)**
b. $\int y\ dx$, simplifying each term **(2)**

6. Find $\int 12x^5 - 3x^2 + 4x^{\frac{1}{3}}\ dx$, giving each term in its simplest form. **(3)**

7. Find $\int 8x^3 + 6x^{\frac{1}{2}} - 5\ dx$, giving each term in its simplest form. **(2)**

8. Find $\int 12x^5 - 8x^3 + 3\ dx$, giving each term in its simplest form. **(2)**

Mark Scheme

1.

$\int 8x^3 + 4\ dx = \frac{8x^4}{4} + 4x + c$	**M1**
$= 2x^4 + 4x + c$	**M1**

2.

$\int 5x^3 - 6x + 1\ dx = \frac{5x^4}{4} - \frac{6x^2}{2} + x + c$	**M1**
$= \frac{5x^4}{4} - 3x^2 + x + c$	**M1**

3.

$\int 24x^{-3}\ dx = \frac{24}{-2}x^{-2} + c$	**M1**
$= -12x^{-2} + c$	**M1**

4.

$\int 10x^4 - 4x - \frac{3}{\sqrt{x}}\ dx = \int 10x^4 - 4x - 3x^{-\frac{1}{2}}\ dx$	**M1**
$\frac{10x^5}{5} - \frac{4x^2}{2} - \frac{3x^{\frac{1}{2}}}{\frac{1}{2}} + c$	**M1**
$= 2x^5 - 2x^2 - 6x^{\frac{1}{2}} + c$	**M1**

5a.

$y = 2x^5 + 7 + \frac{1}{x^3} = 2x^5 + 7 + x^{-3}$	**M1**
$\frac{dy}{dx} = 10x^4 - 3x^{-4}$	**M1**

5b.

$y = 2x^5 + 7 + \frac{1}{x^3} = 2x^5 + 7 + x^{-3}$	
$\int y\ dx = \frac{2x^6}{6} + 7x - \frac{x^{-2}}{-2} + c$	**M1 M1**

6.

$\int 12x^5 - 3x^2 + 4x^{\frac{1}{3}}\ dx = \int 12x^5 - 3x^2 + 4x^{-\frac{1}{3}}\ dx$	**M1**
$= \frac{12x^6}{6} - \frac{3x^3}{3} + \frac{4x^{\frac{2}{3}}}{\frac{2}{3}} + c$	**M1**
$= 2x^6 - x^3 + 6x^{\frac{2}{3}} + c$	**M1**

7.

$\int 8x^3 + 6x^{\frac{1}{2}} - 5\ dx = \frac{8x^4}{4} + \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} - 5x + c$	**M1**
$= 2x^4 + 4x^{\frac{3}{2}} - 5x + c$	**M1**

8.

$\int 12x^5 - 8x^3 + 3\ dx = \frac{12x^6}{6} - \frac{8x^4}{4} + 3x + c$	**M1**
$= 2x^6 - 2x^4 + 3x + c$	**M1**

Integration

Pt. 2: Equation of Curves

AS Level

Pt. 1: Basic Integration

Pt. 2: Equations of Curves

Pt. 3: Definite Integration

Jethwa Maths

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1. A curve has equation $y = f(x)$. It is given that $f'(x) = \frac{1}{\sqrt{x+6}} + \frac{6}{x^2}$ and that f(3) = 1. Find f(x). **(5)**

2. $f'(x) = \frac{(3-x^2)^2}{x2}$, $x \neq 0$.

a. Show that $f'(x) = 9x^{-2} + A + Bx^2$, where A and B are constants to be found. **(3)**

b. Find $f''(x)$ **(2)**

Given that the point (-3, 10) lies on the curve with equation $y = f(x)$.

c. Find $f(x)$ **(5)**

3. $\frac{dy}{dx} = -x^3 + \frac{4x-5}{2x^3}$, $x \neq 0$

Given that y = 7 at x = 1, find y in terms of x, giving each term in its simplest form **(5)**

4a. Find $\int (x^2 - 2x + 5)\, dx$ **(2)**

b. Hence, find the equation of the curve for which $\frac{dy}{dx} = x^2 - 2x + 5$ and which passes through the point (3, 11) **(2)**

5. Given that $\frac{6x+3x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $6x^p + 3x^q$.

a. Write down the value of p and the value of q. **(2)**

Given that $\frac{dy}{dx} = \frac{6x+3x^{\frac{5}{2}}}{\sqrt{x}}$, and that $y = 90$ when $x = 4$,

b. Find y in terms of x, simplifying the coefficient of each term. **(5)**

6. The curve equation $f(x)$ passes through the point (-1, 0). Given that $f'(x) = 12x^2 - 8x + 1$, find $f(x)$ **(4)**

7. $\frac{dy}{dx} = 5x^{-\frac{1}{2}} + x\sqrt{x}$, $x > 0$

Given that y = 35 and x = 4, find y in terms of x, giving each term in its simplest form. **(5)**

8. A curve has equation y = f(x) and passes through the point (4, 22).

Given that, $f'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7$, use integration to find f(x), givin each term in ites simplest form **(5)**

Mark Scheme

1.

f'(x) = $\frac{1}{\sqrt{x+6}} + \frac{6}{x^2}$ = $(x+6)^{-0.5} + 6x^{-2}$	**M1**
f(x) = $\frac{(x+6)^{\frac{1}{2}}}{\frac{1}{2}} + \frac{6x^{-1}}{-1} + c$	**M1**
= $2(x+6)^{\frac{1}{2}} - \frac{6}{x} + c$	**M1**
When x = 3, f(3) = 1 1 = 6 – 2 + c c = -3	**M1**
f(x) = $2(x+6)^{\frac{1}{2}} - \frac{6}{x} - 3$	**M1**

2a.

f'(x) = $\frac{(3-x^2)^2}{x^2} = \frac{(3-x^2)(3-x^2)}{x^2} = \frac{9-6x^2+x^4}{x^2}$	**M1**
= $\frac{9}{x^2} - \frac{6x^2}{x^2} + \frac{x^4}{x^2}$	**M1**
f'(x) = $9x^{-2} - 6 + x^2$ Where A = -6, B = 1	**M1**

2b.

f''(x) = $-18x^{-3} + 2x$	**M1 M1**

2c.

y = f(x) = $\int 9x^{-2} - 6 + x^2\,dx$	**M1**
= $\frac{9x^{-1}}{-1} - 6x + \frac{x^3}{3} + c$	**M1**
= $-\frac{9}{x} - 6x + \frac{x^3}{3} + c$	**M1**
When x = -3, y = 10 10 = 3 + 18 – 9 + c	**M1**
c = -2	**M1**
f(x) = $\frac{9x^{-1}}{-1} - 6x + \frac{x^3}{3} - 2$	

3.

y = $\int(-x^3 + \frac{4x-5}{2x^3})\,dx = \int(-x^3 + \frac{4x}{2x^3} - \frac{5}{2x^3})\,dx$	**M1**
= $\int(-x^3 + \frac{2}{x^2} - \frac{5}{2x^3})\,dx$	**M1**
= $-\frac{x^4}{4} + \frac{2x^{-1}}{-1} - \frac{5x^{-2}}{2(-2)} + c$ = $-\frac{x^4}{4} - \frac{2}{x} - \frac{5}{4x^2} + c$	**M1**
When y = 7, x = 1 7 = $\frac{1}{4} - 2 + \frac{5}{4} + c$	**M1**
c = 8	**M1**
y = $-\frac{x^4}{4} - \frac{2}{x} - \frac{5}{4x^2} + 8$	

4a.

$\int(x^2 - 2x + 5)\,dx = \frac{x^3}{3} - \frac{2x^2}{2} + 5x + c$	**M1**
$= \frac{x^3}{3} - x^2 + 5x + c$	**M1**

4b.

$y = \frac{x^3}{3} - x^2 + 5x + c$ When x = 3, y = 11	**M1**
11 = 15 + c c = -4	**M1**
$y = \frac{x^3}{3} - x^2 + 5x - 4$	

5a.

$\frac{6x+3x^{\frac{5}{2}}}{\sqrt{x}} = \frac{6x+3x^{\frac{5}{2}}}{x^{\frac{1}{2}}}$	**M1**
$\frac{6x}{x^{\frac{1}{2}}} + \frac{3x^{\frac{5}{2}}}{x^{\frac{1}{2}}} = 6x^{0.5} + 3x^2$	**M1**
p = ½ q = 2	

5b.

$y = \int \frac{6x+3x^{\frac{5}{2}}}{\sqrt{x}}\,dx = \int(6x^{\frac{1}{2}} + 3x^2)\,dx$	**M1**
$y = \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{3x^3}{3} + c$ $y = 4x^{\frac{3}{2}} + x^3 + c$	**M1**
When x = 4, y = 90 90 = 32 + 64 + c	**M1**
c = -6	**M1**
$y = 4x^{\frac{3}{2}} + x^3 - 6$	

6.

$y = f(x) = \int f'(x)\,dx = \int 12x^2 - 8x + 1\,dx$	**M1**
$y = \frac{12x^3}{3} - \frac{8x^2}{2} + x + c$ $y = 4x^3 - 4x^2 + x + c$	**M1**
When x = -1, y = 0 0 = - 4 - 4 - 1 + c	**M1**
c = 9	**M1**
$y = 4x^3 - 4x^2 + x + 9$	

7.

$y = \int(5x^{\frac{1}{2}} + x^{\frac{3}{2}})\,dx$	**M1**
$= \frac{5x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c$	**M1**
$= 10x^{\frac{1}{2}} + \frac{2x^{\frac{5}{2}}}{5} + c$	**M1**
When x = 4, y = 35 $35 = 20 + \frac{64}{5} + c$	**M1**

c = $\frac{11}{5}$	**M1**
y = $10x^{\frac{1}{2}} + \frac{2x^{\frac{5}{2}}}{5} + \frac{11}{5}$	

8.

f'(x) = $3x^2 - 3x^{\frac{1}{2}} - 7 = 3x^2 - 3x^{0.5} - 7$	**M1**
f(x) = $\frac{3x^3}{3} - \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - 7x + c$	**M1**
f(x) = $x^3 - 2x^{\frac{3}{2}} - 7x + c$	**M1**
When x = 4, y = 22 22 = 64 – 16 – 28 + c	**M1**
c = 2	**M1**
f(x) = $x^3 - 2x^{\frac{3}{2}} - 7x + 2$	

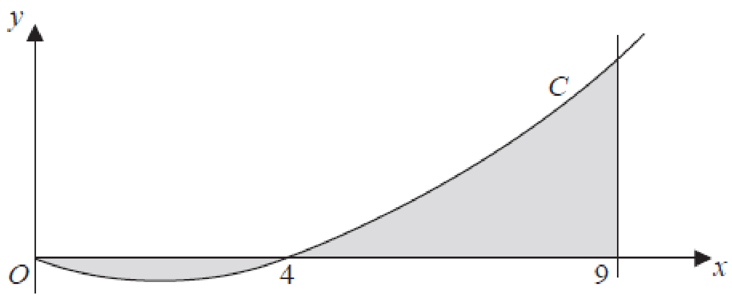
1. Given that A is constant and $\int_1^4 \left(3\sqrt{x} + A\right)dx = 2A^2$
Show that there are exactly two possible values for *A* **(6)**

2. Find the exact value of $\int_1^6 (12x^3 - 9x^2 + 2)dx$ **(4)**

3. Find the exact value of $\int_1^4 (\frac{8}{\sqrt{x}} - 12\sqrt{x^3})\ dx$ **(5)**

4. Find the exact value of $\int_1^2 \frac{\sqrt[3]{x^2}}{4} - \frac{1}{2x^3}\ dx$ **(4)**

5a. Find $\int 10x\left(x^{\frac{1}{2}} - 2\right)dx$, giving each term in its simplest form **(3)**



The figure shows a sketch of part of the curve *C* with equation, y = $10x(x^{\frac{1}{2}} - 2)$, $x \geq 0$.

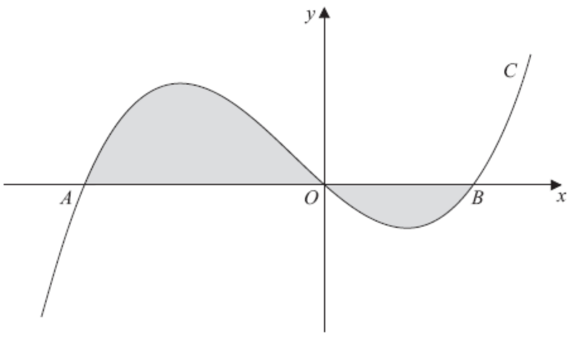
The curve *C* starts at the oringin and crosses the *x*-axis at the point (4, 0).

The area, shown shaded in the figure, consists of two finite regions and is bounded by the curve *C*, the *x*-axis and the line *x* = 9.

b. Use your answer from part *a*, to find the total area of the shaded regions **(5)**

6. The figure shows a sketch of part of the curve *C* with equation

$y = x(x + 4)(x - 2)$



The curve *C* crosses the *x*-axis at the origin *O* and at the point *A* and *B*.

a. Write down the *x*-coordinates of the points *A* and *B* **(1)**

The finite region, shown shaded in the figure, is bounded by the curve *C* and the *x*-axis.

b. Use integration to find the total area of the finite region shown shaded in Figure 3. **(6)**

Mark Scheme

1.

$\int_1^4 (3\sqrt{x} + A)dx = \int_1^4 \left(3x^{\frac{1}{2}} + A\right) dx$	**M1**
$= [2\left(\frac{3x^{\frac{3}{2}}}{3}\right) + Ax]_1^4$	**M1**
$= 16 + 4A - (2 + A)$ $= 14 + 3A$	**M1**
$2A^2 = 14 + 3A$ $2A^2 - 4A - 14 = 0$	**M1**
$(2A - 7)(A + 2) = 0$	**M1**
$A = \frac{7}{2}$ $A = -2$	**M1**

2.

$\int_1^6 (12x^3 - 9x^2 + 2)dx = [\frac{12x^4}{4} - \frac{9x^3}{3} + 2x]_1^6$	**M1**
$= [3x^4 - 3x^3 + 2x]_1^6$	**M1**
$= [3(6)^4 - 3(6)^3 + 2(6)] - [3(1)^4 - 3(1)^3 + 2(1)]$	**M1**
$= 3252 - 2 = 3250$	**M1**

3.

$\int_1^4 (\frac{8}{\sqrt{x}} - 12\sqrt{x^3})\ dx = \int_1^4 (8x^{-\frac{1}{2}} - 12(x^{\frac{1}{2}})^3\ dx$	**M1**
$\int_1^4 (8x^{-\frac{1}{2}} - 12x^{\frac{3}{2}})\ dx$	**M1**
$= [\frac{8x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{12x^{\frac{5}{2}}}{\frac{5}{2}}]_1^4$ $= [16x^{\frac{1}{2}} - \frac{24}{5}x^{\frac{5}{2}}]_1^4$	**M1**
$= (16(4)^{\frac{1}{2}} - \frac{24}{5}(4)^{\frac{5}{2}}) - (16(1)^{\frac{1}{2}} - \frac{24}{5}(1)^{\frac{5}{2}})$ $= -\frac{608}{5} - \frac{56}{5}$	**M1**
$= -\frac{664}{5}$	**M1**

4.

$\int_1^2 \frac{\sqrt[3]{x^2}}{4} - \frac{1}{2x^3}\ dx = \int_1^2 (\frac{1}{4}(x^2)^{\frac{1}{3}} - \frac{1}{2}x^{-3})\ dx = \int_1^2 (\frac{1}{4}x^{\frac{2}{3}} - \frac{1}{2}x^{-3})\ dx$	**M1**
$=[\frac{1x^{\frac{5}{3}}}{4\times\frac{5}{3}} - \frac{1x^{-2}}{2(-2)}]_1^2$ $=[\frac{3x^{\frac{5}{3}}}{20} + \frac{x^{-2}}{4)}]_1^2$	**M1**
$= (\frac{3(2)^{\frac{5}{3}}}{20} + \frac{(2)^{-2}}{4}) - (\frac{3(1)^{\frac{5}{3}}}{20} + \frac{(1)^{-2}}{4})$	**M1**
$\frac{3(2)^{\frac{5}{3}}}{20} + \frac{1}{16} - \frac{3(1)^{\frac{5}{3}}}{20} - \frac{1}{4}$ $= \frac{3(2)^{\frac{5}{3}}}{20} - \frac{3(1)^{\frac{5}{3}}}{20} - \frac{3}{16}$	**M1**

5a.

$\int 10x\left(x^{\frac{1}{2}} - 2\right)dx = \int 10x^{\frac{3}{2}} - 20x\ dx$	**M1**

$= \frac{10}{\frac{5}{2}} x^{\frac{5}{2}} + \frac{-20x^2}{2} + c$	**M1**
$= 4x^{\frac{5}{2}} - 10x^2 + c$	**M1**

5b.

$\int_0^4 y\,dx$ $= (4(4)^{\frac{5}{2}} - 10(4)^2) - (4(0)^{\frac{5}{2}} - 10(0)^2)$	**M1**
= 128 – 160 = -32	**M1**
$\int_4^9 y\,dx$ $= (4(9)^{\frac{5}{2}} - 10(9)^2) - (4(4)^{\frac{5}{2}} - 10(4)^2)$	**M1**
= 972 – 810 – (128 – 160) = 194	**M1**
$\int_0^4 y\,dx + \int_4^9 y\,dx$ = 32 + 194 = 226	**M1**

6a.

$y = x(x + 4)(x - 2) = 0$ $x = 0$, $x = -4$ and $x = -2$	**M1**

6b.

$y = x(x + 4)(x - 2) = x^3 + 2x^2 - 8x$	**M1**
$\int x^3 + 2x^2 - 8x\,dx = \frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2} + c$	**M1**
Area = $\int_{-4}^0 y + \int_0^2 y$	**M1**
$[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2}]_{-4}^{0} = (0) - (64 - \frac{128}{3} - 64) = \frac{128}{3}$	**M1**
$[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2}]_0^2 = (4 + \frac{16}{3} - 16) - 0 = \frac{20}{3}$	**M1**
$= \frac{128}{3} + \frac{20}{3} = \frac{148}{3}$	**M1**