

Integration

Pt. 1: Basic Integration

AS Level

Pt. 1: Basic Integration

Pt. 2: Equations of Curves

Pt. 3: Definite Integration



1. Find $\int 8x^3 + 4 dx$, giving each term in its simplest form. (2)
2. Find $\int 5x^3 - 6x + 1 dx$ (2)
3. Find $\int 24x^{-3} dx$ (2)
4. Find $\int 10x^4 - 4x - \frac{3}{\sqrt{x}} dx$ (3)
5. Given that $y = 2x^5 + 7 + \frac{1}{x^3}$, $x \neq 0$, find, in their simplest form,
 - a. $\frac{dy}{dx}$ (2)
 - b. $\int y dx$, simplifying each term (2)
6. Find $\int 12x^5 - 3x^2 + 4x^{\frac{1}{3}} dx$, giving each term in its simplest form. (3)
7. Find $\int 8x^3 + 6x^{\frac{1}{2}} - 5 dx$, giving each term in its simplest form. (2)
8. Find $\int 12x^5 - 8x^3 + 3 dx$, giving each term in its simplest form. (2)

Mark Scheme

1.

$\int 8x^3 + 4 dx = \frac{8x^4}{4} + 4x + c$	M1
$= 2x^4 + 4x + c$	M1

2.

$\int 5x^3 - 6x + 1 dx = \frac{5x^4}{4} - \frac{6x^2}{2} + x + c$	M1
$= \frac{5x^4}{4} - 3x^2 + x + c$	M1

3.

$\int 24x^{-3} dx = \frac{24}{-2} x^{-2} + c$	M1
$= -12x^{-2} + c$	M1

4.

$\int 10x^4 - 4x - \frac{3}{\sqrt{x}} dx = \int 10x^4 - 4x - 3x^{-\frac{1}{2}} dx$	M1
$\frac{10x^5}{5} - \frac{4x^2}{2} - \frac{3x^{\frac{1}{2}}}{\frac{1}{2}} + c$	M1
$= 2x^5 - 2x^2 - 6x^{\frac{1}{2}} + c$	M1

5a.

$y = 2x^5 + 7 + \frac{1}{x^3} = 2x^5 + 7 + x^{-3}$	M1
$\frac{dy}{dx} = 10x^4 - 3x^{-4}$	M1

5b.

$y = 2x^5 + 7 + \frac{1}{x^3} = 2x^5 + 7 + x^{-3}$	
$\int y dx = \frac{2x^6}{6} + 7x - \frac{x^{-2}}{-2} + c$	M1 M1

6.

$\int 12x^5 - 3x^2 + 4x^{\frac{1}{3}} dx = \int 12x^5 - 3x^2 + 4x^{-\frac{1}{3}} dx$	M1
$= \frac{12x^6}{6} - \frac{3x^3}{3} + \frac{4x^{\frac{2}{3}}}{\frac{2}{3}} + c$	M1
$= 2x^6 - x^3 + 6x^{\frac{2}{3}} + c$	M1

7.

$\int 8x^3 + 6x^{\frac{1}{2}} - 5 dx = \frac{8x^4}{4} + \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} - 5x + c$	M1
$= 2x^4 + 4x^{\frac{3}{2}} - 5x + c$	M1

8.

$\int 12x^5 - 8x^3 + 3 dx = \frac{12x^6}{6} - \frac{8x^4}{4} + 3x + c$	M1
$= 2x^6 - 2x^4 + 3x + c$	M1



Integration

Pt. 2: Equation of Curves

AS Level

Pt. 1: Basic Integration

Pt. 2: Equations of Curves

Pt. 3: Definite Integration



1. A curve has equation $y = f(x)$. It is given that $f'(x) = \frac{1}{\sqrt{x+6}} + \frac{6}{x^2}$ and that $f(3) = 1$. Find $f(x)$. (5)

2. $f'(x) = \frac{(3-x^2)^2}{x^2}$, $x \neq 0$.

a. Show that $f'(x) = 9x^{-2} + A + Bx^2$, where A and B are constants to be found. (3)

b. Find $f''(x)$ (2)

Given that the point $(-3, 10)$ lies on the curve with equation $y = f(x)$.

c. Find $f(x)$ (5)

3. $\frac{dy}{dx} = -x^3 + \frac{4x-5}{2x^3}$, $x \neq 0$

Given that $y = 7$ at $x = 1$, find y in terms of x , giving each term in its simplest form (5)

4a. Find $\int (x^2 - 2x + 5) dx$ (2)

b. Hence, find the equation of the curve for which $\frac{dy}{dx} = x^2 - 2x + 5$ and which passes through the point $(3, 11)$ (2)

5. Given that $\frac{6x+3x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $6x^p + 3x^q$.

a. Write down the value of p and the value of q . (2)

Given that $\frac{dy}{dx} = \frac{6x+3x^{\frac{5}{2}}}{\sqrt{x}}$, and that $y = 90$ when $x = 4$,

b. Find y in terms of x , simplifying the coefficient of each term. (5)

6. The curve equation $f(x)$ passes through the point $(-1, 0)$. Given that $f'(x) = 12x^2 - 8x + 1$, find $f(x)$ (4)

7. $\frac{dy}{dx} = 5x^{-\frac{1}{2}} + x\sqrt{x}$, $x > 0$

Given that $y = 35$ and $x = 4$, find y in terms of x , giving each term in its simplest form. (5)

8. A curve has equation $y = f(x)$ and passes through the point $(4, 22)$.

Given that, $f'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7$, use integration to find $f(x)$, giving each term in its simplest form (5)

Mark Scheme

1.

$f'(x) = \frac{1}{\sqrt{x+6}} + \frac{6}{x^2} = (x+6)^{-0.5} + 6x^{-2}$	M1
$f(x) = \frac{(x+6)^{\frac{1}{2}}}{\frac{1}{2}} + \frac{6x^{-1}}{-1} + c$	M1
$= 2(x+6)^{\frac{1}{2}} - \frac{6}{x} + c$	M1
When $x = 3, f(3) = 1$ $1 = 6 - 2 + c$ $c = -3$	M1
$f(x) = 2(x+6)^{\frac{1}{2}} - \frac{6}{x} - 3$	M1

2a.

$f'(x) = \frac{(3-x^2)^2}{x^2} = \frac{(3-x^2)(3-x^2)}{x^2} = \frac{9-6x^2+x^4}{x^2}$	M1
$= \frac{9}{x^2} - \frac{6x^2}{x^2} + \frac{x^4}{x^2}$	M1
$f'(x) = 9x^{-2} - 6 + x^2$ Where $A = -6, B = 1$	M1

2b.

$f'(x) = -18x^{-3} + 2x$	M1 M1
--------------------------	--------------

2c.

$y = f(x) = \int 9x^{-2} - 6 + x^2 dx$	M1
$= \frac{9x^{-1}}{-1} - 6x + \frac{x^3}{3} + c$	M1
$= -\frac{9}{x} - 6x + \frac{x^3}{3} + c$	M1
When $x = -3, y = 10$ $10 = 3 + 18 - 9 + c$	M1
$c = -2$	M1
$f(x) = \frac{9x^{-1}}{-1} - 6x + \frac{x^3}{3} - 2$	

3.

$y = \int (-x^3 + \frac{4x-5}{2x^3}) dx = \int (-x^3 + \frac{4x}{2x^3} - \frac{5}{2x^3}) dx$	M1
$= \int (-x^3 + \frac{2}{x^2} - \frac{5}{2x^3}) dx$	M1
$= -\frac{x^4}{4} + \frac{2x^{-1}}{-1} - \frac{5x^{-2}}{2(-2)} + c$	M1
$= -\frac{x^4}{4} - \frac{2}{x} - \frac{5}{4x^2} + c$	
When $y = 7, x = 1$ $7 = \frac{1}{4} - 2 + \frac{5}{4} + c$	M1
$c = 8$	M1
$y = -\frac{x^4}{4} - \frac{2}{x} - \frac{5}{4x^2} + 8$	



4a.

$\int (x^2 - 2x + 5) dx = \frac{x^3}{3} - \frac{2x^2}{2} + 5x + c$	M1
$= \frac{x^3}{3} - x^2 + 5x + c$	M1

4b.

$y = \frac{x^3}{3} - x^2 + 5x + c$ When $x = 3, y = 11$	M1
$11 = 15 + c$ $c = -4$	M1
$y = \frac{x^3}{3} - x^2 + 5x - 4$	

5a.

$\frac{6x+3x^{\frac{5}{2}}}{\sqrt{x}} = \frac{6x+3x^{\frac{5}{2}}}{x^{\frac{1}{2}}}$	M1
$\frac{6x}{x^{\frac{1}{2}}} + \frac{3x^{\frac{5}{2}}}{x^{\frac{1}{2}}} = 6x^{0.5} + 3x^2$	M1
$p = \frac{1}{2}$ $q = 2$	

5b.

$y = \int \frac{6x+3x^{\frac{5}{2}}}{\sqrt{x}} dx = \int (6x^{\frac{1}{2}} + 3x^2) dx$	M1
$y = \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{3x^3}{3} + c$ $y = 4x^{\frac{3}{2}} + x^3 + c$	M1
When $x = 4, y = 90$ $90 = 32 + 64 + c$	M1
$c = -6$	M1
$y = 4x^{\frac{3}{2}} + x^3 - 6$	

6.

$y = f(x) = \int f'(x) dx = \int 12x^2 - 8x + 1 dx$	M1
$y = \frac{12x^3}{3} - \frac{8x^2}{2} + x + c$ $y = 4x^3 - 4x^2 + x + c$	M1
When $x = -1, y = 0$ $0 = -4 - 4 - 1 + c$	M1
$c = 9$	M1
$y = 4x^3 - 4x^2 + x + 9$	

7.

$y = \int (5x^{\frac{1}{2}} + x^{\frac{3}{2}}) dx$	M1
$= \frac{5x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c$	M1
$= 10x^{\frac{3}{2}} + \frac{2x^{\frac{5}{2}}}{5} + c$	M1
When $x = 4, y = 35$ $35 = 20 + \frac{64}{5} + c$	M1

$c = \frac{11}{5}$	M1
$y = 10x^{\frac{1}{2}} + \frac{2x^{\frac{5}{2}}}{5} + \frac{11}{5}$	

8.

$f'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7 = 3x^2 - 3x^{0.5} - 7$	M1
$f(x) = \frac{3x^3}{3} - \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - 7x + c$	M1
$f(x) = x^3 - 2x^{\frac{3}{2}} - 7x + c$	M1
When $x = 4$, $y = 22$ $22 = 64 - 16 - 28 + c$	M1
$c = 2$	M1
$f(x) = x^3 - 2x^{\frac{3}{2}} - 7x + 2$	



Integration

Pt. 3: Equation of Curves

AS Level

Pt. 1: Basic Integration
Pt. 2: Equations of Curves
Pt. 3: Definite Integration



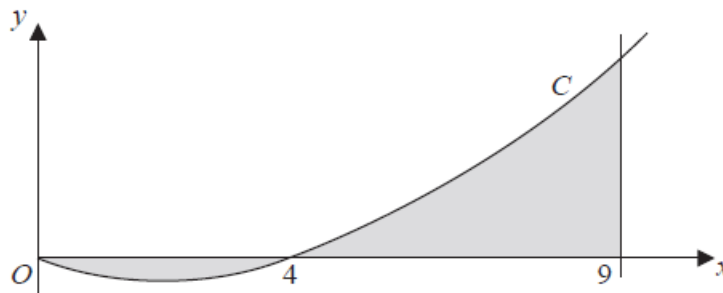
1. Given that A is constant and $\int_1^4 (3\sqrt{x} + A) dx = 2A^2$
Show that there are exactly two possible values for A (6)

2. Find the exact value of $\int_1^6 (12x^3 - 9x^2 + 2) dx$ (4)

3. Find the exact value of $\int_1^4 \left(\frac{8}{\sqrt{x}} - 12\sqrt{x^3} \right) dx$ (5)

4. Find the exact value of $\int_1^2 \frac{\sqrt[3]{x^2}}{4} - \frac{1}{2x^3} dx$ (4)

5a. Find $\int 10x \left(x^{\frac{1}{2}} - 2 \right) dx$, giving each term in its simplest form (3)



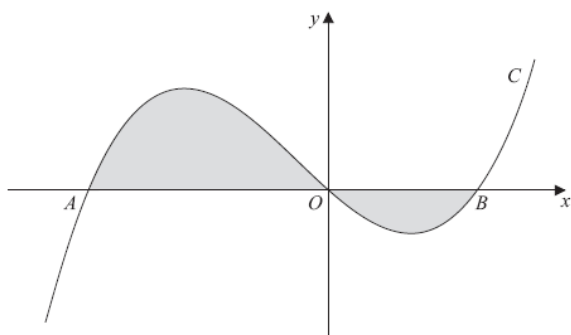
The figure shows a sketch of part of the curve C with equation, $y = 10x(x^{\frac{1}{2}} - 2)$, $x \geq 0$.

The curve C starts at the origin and crosses the x -axis at the point $(4, 0)$.

The area, shown shaded in the figure, consists of two finite regions and is bounded by the curve C , the x -axis and the line $x = 9$.

b. Use your answer from part a, to find the total area of the shaded regions (5)

6. The figure shows a sketch of part of the curve C with equation
 $y = x(x + 4)(x - 2)$



The curve C crosses the x -axis at the origin O and at the point A and B .

a. Write down the x -coordinates of the points A and B (1)

The finite region, shown shaded in the figure, is bounded by the curve C and the x -axis.

b. Use integration to find the total area of the finite region shown shaded in Figure 3. (6)

Mark Scheme

1.

$\int_1^4 (3\sqrt{x} + A) dx = \int_1^4 (3x^{\frac{1}{2}} + A) dx$	M1
$= [2\left(\frac{3x^{\frac{3}{2}}}{3}\right) + Ax]_1^4$	M1
$= 16 + 4A - (2 + A)$ $= 14 + 3A$	M1
$2A^2 = 14 + 3A$ $2A^2 - 4A - 14 = 0$	M1
$(2A - 7)(A + 2) = 0$	M1
$A = \frac{7}{2}$ $A = -2$	M1

2.

$\int_1^6 (12x^3 - 9x^2 + 2) dx = \left[\frac{12x^4}{4} - \frac{9x^3}{3} + 2x\right]_1^6$	M1
$= [3x^4 - 3x^3 + 2x]_1^6$	M1
$= [3(6)^4 - 3(6)^3 + 2(6)] - [3(1)^4 - 3(1)^3 + 2(1)]$	M1
$= 3252 - 2 = 3250$	M1

3.

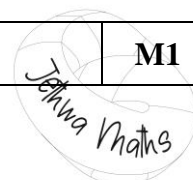
$\int_1^4 \left(\frac{8}{\sqrt{x}} - 12\sqrt{x^3}\right) dx = \int_1^4 (8x^{-\frac{1}{2}} - 12(x^{\frac{1}{2}})^3) dx$	M1
$\int_1^4 (8x^{-\frac{1}{2}} - 12x^{\frac{3}{2}}) dx$	M1
$= \left[\frac{8x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{12x^{\frac{5}{2}}}{\frac{5}{2}}\right]_1^4$ $= \left[16x^{\frac{1}{2}} - \frac{24}{5}x^{\frac{5}{2}}\right]_1^4$	M1
$= (16(4)^{\frac{1}{2}} - \frac{24}{5}(4)^{\frac{5}{2}}) - (16(1)^{\frac{1}{2}} - \frac{24}{5}(1)^{\frac{5}{2}})$ $= -\frac{608}{5} - \frac{56}{5}$	M1
$= -\frac{664}{5}$	M1

4.

$\int_1^2 \left(\frac{\sqrt[3]{x^2}}{4} - \frac{1}{2x^3}\right) dx = \int_1^2 \left(\frac{1}{4}(x^2)^{\frac{1}{3}} - \frac{1}{2}x^{-3}\right) dx = \int_1^2 \left(\frac{1}{4}x^{\frac{2}{3}} - \frac{1}{2}x^{-3}\right) dx$	M1
$= \left[\frac{1x^{\frac{5}{3}}}{4 \times \frac{5}{3}} - \frac{1x^{-2}}{2(-2)}\right]_1^2$ $= \left[\frac{3x^{\frac{5}{3}}}{20} + \frac{x^{-2}}{4}\right]_1^2$	M1
$= \left(\frac{3(2)^{\frac{5}{3}}}{20} + \frac{(2)^{-2}}{4}\right) - \left(\frac{3(1)^{\frac{5}{3}}}{20} + \frac{(1)^{-2}}{4}\right)$	M1
$\frac{3(2)^{\frac{5}{3}}}{20} + \frac{1}{16} - \frac{3(1)^{\frac{5}{3}}}{20} - \frac{1}{4}$ $= \frac{3(2)^{\frac{5}{3}}}{20} - \frac{3(1)^{\frac{5}{3}}}{20} - \frac{3}{16}$	M1

5a.

$\int 10x \left(x^{\frac{1}{2}} - 2\right) dx = \int 10x^{\frac{3}{2}} - 20x dx$	M1
--	-----------



$= \frac{10}{\frac{5}{2}} x^{\frac{5}{2}} + \frac{-20x^2}{2} + c$	M1
$= 4x^{\frac{5}{2}} - 10x^2 + c$	M1

5b.

$\int_0^4 y dx$	M1
$= (4(4)^{\frac{5}{2}} - 10(4)^2) - (4(0)^{\frac{5}{2}} - 10(0)^2)$	
$= 128 - 160$	M1
$= -32$	
$\int_4^9 y dx$	M1
$= (4(9)^{\frac{5}{2}} - 10(9)^2) - (4(4)^{\frac{5}{2}} - 10(4)^2)$	
$= 972 - 810 - (128 - 160)$	M1
$= 194$	
$\int_0^4 y dx + \int_4^9 y dx = 32 + 194 = 226$	M1

6a.

$y = x(x+4)(x-2) = 0$ $x = 0, x = -4$ and $x = 2$	M1
--	-----------

6b.

$y = x(x+4)(x-2) = x^3 + 2x^2 - 8x$	M1
$\int x^3 + 2x^2 - 8x dx = \frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2} + c$	M1
Area = $\int_{-4}^0 y + \int_0^2 y$	M1
$[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2}]_{-4}^0 = (0) - (64 - \frac{128}{3} - 64) = \frac{128}{3}$	M1
$[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2}]_0^2 = (4 + \frac{16}{3} - 16) - 0 = \frac{20}{3}$	M1
$= \frac{128}{3} + \frac{20}{3} = \frac{148}{3}$	M1

