

## Part 1: Equation of a Line



AS Level

Pt 1: Equation of a Line

Pt. 2: Circles

A-Level

Pt 3: Parametric and Cartesian Equations

1. Find in the form  $y = mx + c$ , the equation of the straight line passing through the pair of co-ordinates  $(-\frac{1}{2}, -2)$  and  $(2, 8)$ . (3)

2. The straight line  $l$  passes through the points  $A(-6, 8)$  and  $B(3, 2)$ .  
a. Find an equation of the line  $l$  (3)

b. Show that the points  $C(9, -2)$  lies on  $l$ . (2)

3. The straight line  $l_1$  passes through the points  $P(-2, 1)$  and  $Q(4, -1)$ .  
a. Find the equation of  $l_1$  in the form  $ax + by + c = 0$ , where  $a, b$ , and  $c$  are integers. (3)

The straight line  $l_2$  passes through the points  $R(2, 4)$  and through the mid-point  $PQ$ .  
b. Find the equation of  $l_2$  in the form  $y = mx + c$ . (3)

4. The straight line  $p$  has the equation  $3x - 4y + 8 = 0$ .  
The straight line  $q$  is parallel to  $p$  and passes through the point with coordinates  $(8, 5)$ .  
a. Find the equation of  $q$  in the form  $y = mx + c$ . (2)

The straight line  $r$  is perpendicular to  $p$  and passes through the point with coordinates  $(-4, 6)$ .  
b. Find the equation of  $r$  in the form  $ax + by + c = 0$ , where  $a, b$  and  $c$  are integers. (3)

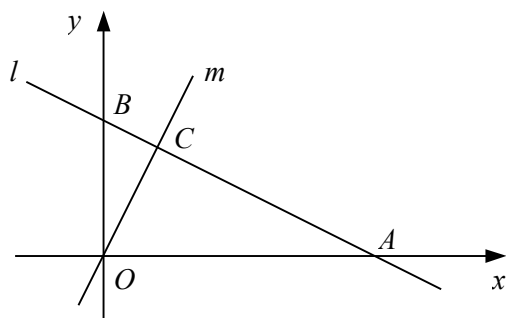
c. Find the coordinates of the point where lines  $q$  and  $r$  intersect. (4)

5. The vertices of a triangle are the points  $A(5, 4)$ ,  $B(-5, 8)$  and  $C(1, 11)$ .  
a. Find the equation of the straight line passing through  $A$  and  $B$ , giving your answer in the form  $ax + by + c = 0$ , where  $a, b$  and  $c$  are integers. (2)

b. Find the coordinates of the point  $M$ , the mid-point of  $AC$ . (1)

c. Show that  $OM$  is perpendicular to  $AB$ , where  $O$  is the origin. (2)

6. The diagram shows the straight line  $l$  with equation  $x + 2y - 20 = 0$  and the straight line  $m$  which is perpendicular to  $l$  and passes through the origin  $O$ .



a. Find the coordinates of the points  $A$  and  $B$  where  $l$  meets the  $x$ -axis and  $y$ -axis respectively. (2)

Given that  $l$  and  $m$  intersect at the point  $C$ ,  
b. find the ratio of the area of triangle  $OAC$  to the area of triangle  $OBC$ . (5)

7. The vertices of a triangle are the points  $P(3, c)$ ,  $Q(9, 2)$  and  $R(3c, 11)$  where  $c$  is a constant.  
Given that  $\angle PQR = 90^\circ$   
a. Find the value of  $c$  (5)

b. Show that the length of  $PQ$  is  $k\sqrt{10}$ , where  $k$  is an integer to be found (3)

c. Find the area of triangle  $PQR$ . (4)

## Mark Scheme

1.

Gradient = $\frac{8+2}{2+0.5} = 4$	<b>M1</b>
$y - 8 = 4(x - 2)$	<b>M1</b>
$y = 4x$	<b>M1</b>

2a.

Gradient = $\frac{2-8}{3+6} = -\frac{2}{3}$	<b>M1</b>
$y - 8 = -\frac{2}{3}(x + 6)$	<b>M1</b>
$2x + 3y - 12 = 0$	<b>M1</b>

2b.

$2(9) + 3(-2) - 12 = 0$	<b>M1</b>
Therefore, C lies on C.	<b>M1</b>

3a.

Gradient = $\frac{-1-1}{4+2} = -\frac{1}{3}$	<b>M1</b>
$y - 1 = -\frac{1}{3}(x + 2)$	<b>M1</b>
$3y - 3 = -x - 2$	<b>M1</b>
$x + 3y - 1 = 0$	<b>M1</b>

3b.

Mid-point of PQ = $(\frac{-2+4}{2}, \frac{1-1}{2}) = (1, 0)$	<b>M1</b>
Gradient of $l_2 = \frac{0-4}{1-2} = 4$	<b>M1</b>
$y = 4(x - 1)$ $y = 4x - 4$	<b>M1</b>

4a.

$p \rightarrow y = \frac{3}{4}x = 2$ gradient = $\frac{3}{4}$	<b>M1</b>
$y - 5 = \frac{3}{4}(x - 8)$ $y = \frac{3}{4}x - 1$	<b>M1</b>

4b.

Perpendicular gradient = $-\frac{4}{3}$	<b>M1</b>
$y - 6 = -\frac{4}{3}(x + 4)$ $3y - 18 = -4x - 16$	<b>M1</b>
$4x + 3y - 2 = 0$	<b>M1</b>

4c.

$q \rightarrow 3x - 4y - 4 = 0 \rightarrow 9x - 12y - 12 = 0$	<b>M1</b>
$r \rightarrow 16x + 12y - 8 = 0$	<b>M1</b>
Adding, $25x - 20 = 0$ $x = \frac{4}{5}$	<b>M1</b>
$y = \frac{3(0.8) - 4}{4} = -\frac{2}{5}$ Co-ordinates = $(\frac{4}{5}, -\frac{2}{5})$	<b>M1</b>

5a.

Gradient = $\frac{8-4}{-5-5} = -\frac{2}{5}$	M1
$y - 4 = -\frac{2}{5}(x - 5)$ $5y - 20 = -2x + 10$ $2x + 5y - 30 = 0$	M1

5b.

Midpoint = $(\frac{5+1}{2}, \frac{4+11}{2}) = (3, 3.5)$	M1
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5c.

Gradient of OM = $3.5 \div 3 = \frac{5}{2}$	M1
Gradient OM x Gradient AB = $\frac{5}{2} \times -\frac{2}{5} = -1$ Therefore, OM is perpendicular to AB.	M1

6a.

At A, $y = 0, x = 20 \rightarrow A(20, 0)$	M1
At B, $x = 0, y = 10 \rightarrow B(0, 10)$	M1

6b.

$l \rightarrow y = 10 - 0.5x$ Gradient of $l = -0.5$	M1
Gradient of $m = 2$ Equation of line $m: y = 2x$	M1
At C, $10 - 0.5x = 2x$ $x = 4$ Therefore, $C = (4, 8)$	M1
Area of $\Delta OAC$ : area of $\Delta OBC$ $0.5 \times 20 \times 8 : 0.5 \times 10 \times 4$	M1
4 : 1	M1

7a.

Gradient of PQ = $\frac{2-c}{9-3} = \frac{2-c}{6}$	M1
Gradient of QR = $\frac{11-2}{3c-9} = \frac{3}{c-3}$	M1
$\angle PQR = 90^\circ$ , therefore PQ is perpendicular to QR $\frac{2-c}{6} \times \frac{3}{c-3} = -1$ $3(2-c) = -6(c-3)$ $3c = 12$ $c = 4$	

7b.

$PQ^2 = 6^2 + 2^2 = 40$	M1
$PQ = \sqrt{40} = 2\sqrt{10}$ $k = 2$	M1

7c.

$QR = \sqrt{3^2 + 9^2} = \sqrt{90} = 3\sqrt{10}$	M1
Area = $\frac{1}{2} \times PQ \times QR = 30$	M1





### AS Level

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Pt. 2: Circles

### A-Level

Pt 3: Parametric and Cartesian Equations

1. Find the coordinates of the centre and the radius of the circles  $9x^2 + 9y^2 + 6x - 24y + 8 = 0$  (3)
  
2. Find whether the  $(7, -3)$  lies inside or outside the circle  $x^2 + y^2 + 10x - 4y = 140$  (3)
  
3. Find the equation of the normal to the circle with equation  $x^2 + y^2 + 4x = 13$  at the point  $(-1, 4)$ . (3)
  
4. The line with equation  $y = 1 - x$  intersects the circle with equation  $x^2 + y^2 + 6x + 2y = 27$  at the points  $A$  and  $B$ . Find the length of the chord  $AB$ , giving your answer in the form  $k\sqrt{2}$  (3)
  
5. The circle  $C$  has centre  $(3, -2)$  and radius 5.
  - a. Write down an equation of  $C$  in cartesian form. (1)
  - The line  $y = 2x - 3$  intersects  $C$  at the points  $A$  and  $B$ .
  - b. Show that  $AB = 4\sqrt{5}$ . (5)
  
6. The circle  $C$  touches the  $y$ -axis at the point  $A(0, 3)$  and passes through the point  $B(2, 7)$ .
  - a. Find an equation of the perpendicular bisector of  $AB$ . (4)
  - b. Find an equation for  $C$ . (3)
  - c. Show that the tangent to  $C$  at  $B$  has equation  $3x - 4y + 22 = 0$ . (4)
  
7. The circle  $C$  has equation  $x^2 + y^2 - 8x + 4y + 12 = 0$ .
  - a. Find the coordinates of the centre of  $C$  and the radius of  $C$ . (2)
  - The point  $P$  has coordinates  $(3, 5)$  and the point  $Q$  lies on  $C$ .
  - b. Find the largest and smallest values of the length  $PQ$ , giving your answers in the form  $k\sqrt{2}$ . (3)
  - c. Find the length of  $PQ$  correct to 3 significant figures when the line  $PQ$  is a tangent to  $C$ . (2)
  
8. Circle  $C_1$  has the equation  $x^2 + y^2 - 2ay = 0$ , where  $a$  is a positive constant.
  - a. Find the coordinates of the centre and the radius of  $C_1$ . (2)
  - Circle  $C_2$  has the equation  $x^2 + y^2 - 2bx = 0$ , where  $b$  is a constant and  $b > a$ .
  - b. Sketch  $C_1$  and  $C_2$  on the same diagram. (3)
  
9. The circle  $C$  has equation  $x^2 + y^2 - 8x - 16y + 72 = 0$ .
  - a. Find the coordinates of the centre and the radius of  $C$ . (2)
  - b. Find the distance of the centre of  $C$  from the origin in the form  $k\sqrt{5}$ . (2)
  - The point  $A$  lies on  $C$  and the tangent to  $C$  at  $A$  passes through the origin  $O$ .
  - c. Show that  $OA = 6\sqrt{2}$ . (2)

## Mark Scheme

1.

$x^2 + y^2 + \frac{2}{3}x - \frac{8}{3}y + \frac{8}{9} = 0$	<b>M1</b>
$(x + \frac{1}{3})^2 - \frac{1}{9} + (y - \frac{4}{3})^2 - \frac{16}{9} + \frac{8}{9} = 0$ $(x + \frac{1}{3})^2 + (y - \frac{4}{3})^2 = 1$	<b>M1</b>
Centre $(-\frac{1}{3}, 0)$ Radius 1	<b>M1</b>

2.

$(x + 5)^2 - 25 + (y - 2)^2 - 4 = 140$ $(x + 5)^2 + (y - 2)^2 = 169$	<b>M1</b>
Centre $(-5, 2)$ Radius 13	<b>M1</b>
Distance to centre = $\sqrt{144 + 25} = 13$ Therefore point is on circle.	<b>M1</b>

3.

$(x + 2)^2 - 4 + y^2 = 13$ Therefore, centre $(-2, 0)$	<b>M1</b>
Gradient = $\frac{0-4}{-2+1} = 4$	<b>M1</b>
Therefore, $y - 4 = 4(x + 1)$ $y = 4x + 8$	<b>M1</b>

4.

$x^2 + (1 - x)^2 + 6x + 2(1 - x) = 27$	<b>M1</b>
$x^2 + x - 12 = 0$ $(x + 4)(x - 3) = 0$ $x = -4, y = 1 - (-4) = 5$ $x = 3, y = 1 - 3 = -2$	<b>M1</b>
Therefore, $AB = \sqrt{49 + 49} = 7\sqrt{2}$	<b>M1</b>

5a.

$(x - 3)^2 + (y + 2)^2 = 25$	<b>M1</b>
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5b.

$(x - 3)^2 + [(2x - 3) + 2]^2 = 25$	<b>M1</b>
$(x - 3)^2 + (2x - 1)^2 = 25$ $x^2 - 2x - 3 = 0$	<b>M1</b>
$(x + 1)(x - 3) = 0$ $x = -1, y = 2(-1) - 3 = -5$ $x = 3, y = 2(3) - 3 = 3$	<b>M1</b>
$AB^2 = 4^2 + 8^2 = 80$	<b>M1</b>
$AB = \sqrt{80} = 4\sqrt{5}$	<b>M1</b>

6a.

Midpoint AB = $(\frac{0+2}{2}, \frac{3+7}{2}) = (1, 5)$	<b>M1</b>
Gradient AB = $\frac{7-3}{2-0} = 2$	<b>M1</b>
Therefore perpendicular gradient = $-\frac{1}{2}$	<b>M1</b>
$y - 5 = -\frac{1}{2}(x - 1)$	<b>M1</b>

$y = \frac{11}{2} - \frac{1}{2}x$	
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6b.

Circle touches $y$ -axis at $(0, 3)$ Therefore $y$ -coordinate of centre = 3	<b>M1</b>
$3 = \frac{11}{2} - \frac{1}{2}x$ $x = 5$	<b>M1</b>
Centre $(5, 3)$ radius 5. $(x - 5)^2 + (y - 3)^2 = 25$	<b>M1</b>

6c.

Gradient of radius = $\frac{7-3}{2-5} = -\frac{4}{3}$	<b>M1</b>
Therefore gradient of tangent = $\frac{3}{4}$	<b>M1</b>
$y - 7 = \frac{3}{4}(x - 2)$ $4y - 28 = 3x - 6$ $3x - 4y + 22 = 0$	<b>M1</b>

7a.

$(x - 4)^2 - 16 + (y + 2)^2 - 4 + 12 = 0$ $(x - 4)^2 + (y + 2)^2 = 8$	<b>M1</b>
Centre: $(4, -2)$ Radius: $2\sqrt{2}$	<b>M1</b>

7b.

Distance P to centre = $\sqrt{1 + 49} = \sqrt{50} = 5\sqrt{2}$	<b>M1</b>
Therefore, max PQ = $5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$	<b>M1</b>
Minimum PQ = $5\sqrt{2} - 2\sqrt{2} = 3\sqrt{2}$	<b>M1</b>

7c.

Tangent perpendicular to radius: $PQ^2 = (5\sqrt{2})^2 - (5\sqrt{2})^2 = 50 - 8 = 42$	<b>M1</b>
$PQ = \sqrt{42} = 6.48$	<b>M1</b>

8a.

$x^2 + (y - a)^2 - a^2 = 0$ $x^2 + (y - a)^2 = a^2$	<b>M1</b>
Centre: $(0, a)$ Radius: $a$	<b>M1</b>

8b.

$C_2: (x - b)^2 - b^2 + y^2 = 0$ $(x - b)^2 + y^2 = b^2$	<b>M1</b>
Centre: $(b, 0)$ Radius: $b$	<b>M1</b>



9a.

$(x-4)^2 - 16 + (y-8)^2 - 64 + 72 = 0$ $(x-4)^2 + (y-8)^2 = 8$	<b>M1</b>
Centre: (4, 8) Radius: $2\sqrt{2}$	<b>M1</b>

9b.

$\sqrt{16 + 64} = \sqrt{80}$	<b>M1</b>
$= 4\sqrt{5}$	<b>M1</b>

9c.

Tangent perpendicular to radius:	<b>M1</b>
$OA^2 = (\sqrt{80})^2 - (2\sqrt{2})^2 = 72$ $OA = \sqrt{72} = 6\sqrt{2}$	<b>M1</b>

