A-Level Starter Activity

Jethwa Maths

Topic: Differentiating Logarithms and Exponentials

Chapter Reference: Pure 2, Chapter 9

8 minutes

1. Find the value of f'(x) when,

a. $f(x) = 3x + e^x$ **(1)**

b. $f(x) = x^{\frac{1}{2}} + 2\ln x$ **(1)**

c. $f(x) = 4\sqrt{x} + \frac{1}{4}\ln x$ **(1)**

2. Find the value of x for which $f'(x) = -1$ when $f(x) = \frac{x^2}{8} - 2x + \ln x$ **(3)**

3. Find the equation for the normal to the curve $y = 4 + 3e^x$ when $x = 0$ **(4)**

4. Find he equation for the tangent to the curve $y = x^{\frac{1}{3}} - 3e^x$ at the point when $x = 1$. **(4)**

Solutions

1a.

$f'(x) = 3 + e^x$	**M1**

1b.

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{2}{x}$	**M1**

1c.

$f'(x) = 2x^{-\frac{1}{2}} + \frac{1}{4x}$	**M1**

2.

$f'(x) = \frac{1}{4}x - 2 + \frac{1}{x}$ $\frac{1}{4}x - 2 + \frac{1}{x} = -1$	**M1**
$x^2 - 4x + 4 = 0$ $(x - 2)^2 = 0$	**M1**
$x = 2$	**M1**

3.

When $x = 0$, $y = 7$	**M1**
$\frac{dy}{dx} = 3e^x$ Gradient = 3	**M1**
Gradient of normal = $-\frac{1}{3}$	**M1**
$y = 7 - \frac{1}{3}x$	**M1**

4.

$x = 1$ $y = 1 - 3e$	**M1**
$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} - 3e^x$ Gradient = $\frac{1}{3} - 3e$	**M1**
$y - (1 - 3e) = (\frac{1}{3} - 3e)(x - 1)$	**M1**
$y = (\frac{1}{3} - 3e)x + \frac{2}{3}$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Differentiating sin and cos

Chapter Reference: Pure 2, Chapter 9

8 minutes

1. Differentiate:

a. $y = \cos(3x - 2)$ **(1)**

b. $y = 4\sin(\frac{\pi}{3} - x)$ **(1)**

2. Find the coordinate of any stationary points on the curve $y = x + 2\sin x$ in the interval $0 \leq x \leq 2\pi$ **(4)**

3. Find the equation for the tangent to the curve $y = \cos x$ at the point $x = \frac{\pi}{3}$ **(5)**

Solutions

1a.

$\frac{dy}{dx} = -3\sin(3x-2)$	**M1**

1b.

$\frac{dy}{dx}$ = -4 cos $(\frac{\pi}{3} - x)$	**M1**

2.

$\frac{dy}{dx} = 1 + 2\cos x$	**M1**
S.P: $1 + 2\cos x = 0$ $\cos x = -\frac{1}{2}$	**M1**
$x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ $y = \frac{2\pi}{3} + \sqrt{3}$	**M1**
$x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$ $y = \frac{4\pi}{3} - \sqrt{3}$	**M1**

3.

$x = \frac{\pi}{3}$ $y = \frac{1}{2}$	**M1**
$\frac{dy}{dx} = -\sin x$	**M1**
gradient = $-\sin(\frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$	**M1**
$y - \frac{1}{2} = -\frac{\sqrt{3}}{2}(x - \frac{\pi}{3})$	**M1**
$3\sqrt{3}x + 6y - 3 - \sqrt{3}\pi = 0$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Differentiating tan, sec, cosec, and cot Chapter Reference: Pure 2, Chapter 9	7 minutes

1. Differentiate 3 cosec $(x + \frac{\pi}{6})$ **(1)**

2. Differentiate $\sec^2 2x$ **(1)**

3. Differentiate ln (tan $4x$) **(2)**

4. Differentiate 2 cot x^2 **(1)**

5. Find the equation for the tangent to the curve $y = \text{cosec } x - 2 \sin x$ at the point $x = \frac{\pi}{6}$ **(4)**

Solutions

1.

Chain Rule $\frac{dy}{dx} = -3 \text{ cosec } 3x \cot 2x$	**M1**

2.

Chain Rule $\frac{dy}{dx} = 2 \sec 2x \times 2 \sec 2x \tan 2x$ $= 4 \sec^2 2x \tan 2x$	**M1**

3.

Chain Rule $\frac{1}{\tan 4x} \times 4sec^2 4x$	**M1**
$= \frac{\cos 4x}{\sin 4x} \times \frac{4}{cos^2 4x}$ $= 4 \sec 4x \text{ cosec } 4x$	**M1**

4.

Chain Rule $-2 \text{ cosec}^2 x^2 \times 2x$ $= -4x \text{ cosec}^2 x^2$	**M1**

5.

$x = \frac{\pi}{6}$ $y = 1$	**M1**
$\frac{dy}{dx} = -cosec\, x \cot x - 2 \cos x$	**M1**
Gradient = $-3\sqrt{3}$	**M1**
$y - 1 = -3\sqrt{3}\ (x - \frac{\pi}{6})$ $6\sqrt{3}x + 2y - 2 - \sqrt{3}\pi = 0$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: The Chain Rule

Chapter Reference: Pure 2, Chapter 9

8 minutes

1. Differentiate $\frac{4}{(2x+3)^3}$ **(2)**

2. Differentiate with respect x, $3\ln(4 - \sqrt{x})$ **(2)**

3. Find the coordinates of any station points on the curve $y = 8x - e^{2x}$ **(4)**

4. Find an equation for the normal to each curve at the point on the curve when $y = e^{4-x^2} - 10$ when $x = -2$ **(4)**

Solutions

1.

$-12\ (2x + 3)^{-4}\ x\ 2$	**M1**
$= -24(2x + 3)^{-4}$	**M1**

2.

$\frac{3}{4-\sqrt{x}} \times (-\frac{1}{2}x^{-\frac{1}{2}})$	**M1**
$= \frac{3}{2x-8\sqrt{x}}$	**M1**

3.

$\frac{dy}{dx} = 8 - 2e^{2x}$ S.P: $8 - 2e^{2x} = 0$	**M1**
$e^{2x} = 4$	**M1**
$x = \frac{1}{2}\ln 4 = \ln 2$	**M1**
$y = 8\ln 2 - 4$	**M1**
$(\ln 2, 8\ln 2 - 4)$	

4.

$x = -2$ $y = -9$	**M1**
$\frac{dy}{dx} = e^{4-x^2} \times (-2x) = -2xe^{4-x^2}$	**M1**
Gradient = 4 Gradient of normal = $-\frac{1}{4}$	**M1**
$y + 9 = -\frac{1}{4}(x + 2)$ $y = -\frac{1}{4}x - \frac{19}{2}$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: The Product Rule

Chapter Reference: Pure 2, Chapter 9

8 minutes

1. Find $\frac{dy}{dx}$ when $y = (x + 1) \ln (x^2 - 1)$ **(3)**

2. Find the values of f'(x) at the point $x = \frac{1}{4}$ when $f(x) = x^{\frac{1}{2}}(1 - 2x)^3$ **(4)**

3. Find the stationary points on the curve $y = 2 + x^2e^{-4x}$ **(4)**

5. Find the value of f'(x) at the point $x = \frac{\pi}{4}$ when $f(x) = \sin 3x \cos 5x$ **(3)**

Solutions

1.

$1\ x\ \ln(x^2 - 1) + (x + 1)\ x\ \frac{1}{x^2-1}\ x\ 2x$	**M1**
$= \ln(x^2 - 1) + \frac{2x(x+1)}{(x+1)(x-1)}$	**M1**
$= \ln(x^2 - 1) + \frac{2x}{x-1}$	**M1**

2.

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} \times (1-2x)^3 + x^{\frac{1}{2}} \times 3(1-2x)^2 \times -2$	**M1**
$= \frac{1}{2}x^{-\frac{1}{2}}(1-2x)^2[(1-2x) - 12x]$ $= \frac{1}{2}x^{-\frac{1}{2}}(1-14x)(1-2x)^2$	**M1**
$f'(\frac{1}{4}) = \frac{1}{2} \times 2 \times (-\frac{5}{2}) \times \frac{1}{4}$	**M1**
$f'(\frac{1}{4}) = -\frac{5}{8}$	**M1**

3.

$\frac{dy}{dx} = 1 \times (x-4)^3 + x \times 3(x-4)^2$ $= (x-4)^2[(x-4) + 3x]$	**M1**
$4(x - 1)(x-4)^2 = 0$	**M1**
$x = 1$ $y = -27$	**M1**
$x = 0$ $y = 4$	**M1**

4.

$f'(x) = 3\cos 3x \times \cos 5x + \sin 3x \times (-5\sin 5x)$	**M1**
$f'(x) = 3\cos 3x\cos 5x - 5\sin 3x\sin 5x$	**M1**
$f'(\frac{\pi}{4}) = 3(-\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}) - 5 \times \frac{1}{\sqrt{2}}(-\frac{1}{\sqrt{2}})$ $= 4$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: The Quotient Rule

Chapter Reference: Pure 2, Chapter 9

7 minutes

1. Find $\frac{dy}{dx}$ when $y = \frac{1-x}{x^3+2}$ **(2)**

2. Find the coordinates of any stationary points on the curve $y = \frac{e^{4x}}{2x-1}$ **(5)**

3. Find the value of f'(x) when f(x) $= \frac{\ln(2\cos x)}{\sin x}$ at the point $x = \frac{\pi}{3}$ **(3)**

Solutions

1.

$\frac{dy}{dx} = = \frac{-1\times(x^3+2)-(1-x)\times 3x^2}{(x^3+2)^2}$	**M1**
$= \frac{2x^3-3x^2-2}{(x^3+2)^2}$	**M1**

2.

$\frac{dy}{dx} = \frac{4e^{4x}\times(2x-1)-e^{4x}\times 2}{(2x-1)^2}$	**M1**
$\frac{dy}{dx} = \frac{2e^{4x}(4x-3)}{(2x-1)^2}$	**M1**
$\frac{2e^{4x}(4x-3)}{(2x-1)^2} = 0$	**M1**
$x = \frac{3}{4}$ $y = 2e^3$	**M1**
$(\frac{3}{4}, 2e^3)$	**M1**

3.

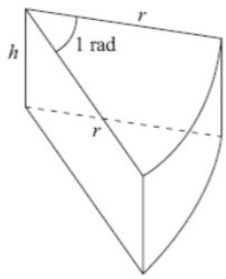
$f'(x) = \frac{\frac{1}{2\cos x}\times(-2\sin x)\times(\sin x) - \ln(2\cos x)\times\cos x}{\sin^2 x}$	**M1**
$= -\sec x - \frac{\cos x \ln(2\cos x)}{\sin^2 x}$	**M1**
$f'(\frac{\pi}{3}) = -2 - 0 = -2$	**M1**

Topic: Radians

Chapter Reference: Pure 2, Chapter 9

8 minutes

1. The diagram below shows a closed box used by a show for packing pieces of cake. The box is a right prism of height *h* cm. The cross section is a sector of a circle. The sector has radius *r* cm and angle 1 radian.



The volume of the box is 300 cm^3

a. Show that the surface area of the box, *S* cm^2, is given by,

$S = r^2 + \frac{1800}{r}$ **(5)**

b. Use calculus to find the value of r for which *S* is stationary. **(4)**

c. Prove that this value of r gives a minimum value of *S*. **(2)**

d. Find, to the nearest cm^2, this minimum value of *S*. **(2)**

Solutions

1a.

Arc length = $r\theta = r \text{ x } 1 = r$	**M1**
Sector area = $\frac{1}{2}r^2\theta = \frac{1}{2}(1)^2\theta = \frac{r^2}{2}$	**M1**
Surface area = 2 sectors + 2 rectangles + curved face. $= r^2 + 3rh$	**M1**
Volume = $300 = \frac{1}{2}r^2h$	**M1**
$S = r^2 + 3 \times 3 \times \frac{600}{r}$ $S = r^2 + \frac{1800}{r}$	**M1**

1b.

$\frac{dS}{dr} = 2r - \frac{1800}{r^2}$	**M1**
Maximum when $\frac{dS}{dr} = 0$	**M1**
$2r - \frac{1800}{r^2} =$ $r^3 = 900$	**M1**
$r = 9.7$	**M1**

1c.

$\frac{d^2S}{dr^2} = 2 + \frac{3600}{r^3} > 0$	**M1**
Therefore, a minimum point.	**M1**

1d.

$S_{min} = (9.65)^2 + \frac{1800}{9.65}$	**M1**
$S_{min} = 279.65$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Parametric Differentiation Chapter Reference: Pure 2, Chapter 9	8 minutes

1. Find an expression for $\frac{dy}{dx}$ when $x = t^2 - 1$, $y = 2t^3 + t^2$ **(2)**

2. Find an expression for $\frac{dy}{dx}$ when $x = e^{t+1}$ and $y = e^{2t-1}$ **(3)**

3. Find in the form $y = mx + c$, find an equation for the tangent to the curve at the point $t = \frac{\pi}{3}$ when $x = 2 \sin t$ and $y = 1 - 4 \cos t$. **(4)**

Solutions

1.

$\frac{dx}{dt} = 2t$ $\frac{dy}{dt} = 6t^2 + 2t$	**M1**
$\frac{dy}{dx} = \frac{6t^2+2t}{2t} = 3t + 1$	**M1**

2.

$\frac{dx}{dt} = e^{t+1}$ $\frac{dy}{dt} = 2e^{2t-1}$	**M1**
$\frac{dy}{dx} = \frac{2e^{2t-1}}{e^{t+1}}$	**M1**
$\frac{dy}{dx} = 2e^{t-2}$	**M1**

3.

$t = \frac{\pi}{3}$ $x = \sqrt{3}$ $y = -1$	**M1**
$\frac{dx}{dt} = 2\cos t$ $\frac{dy}{dt} = 4\sin t$	**M1**
$\frac{dy}{dx} = \frac{4\sin t}{2\cos t} = 2\tan t$ $\frac{dy}{dx}(\sqrt{3}) = 2\sqrt{3}$	**M1**
$y + 1 = 2\sqrt{3}(x - \sqrt{3})$ $y = 2\sqrt{3}x - 7$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Implicit Differentiation Chapter Reference: Pure 2, Chapter 9	8 minutes

1. Find $\frac{dy}{dx}$ in terms of x and y for the curve, $2e^{3x} + e^{-2y} + 7 = 0$. Leave your answers in the form $\frac{dy}{dx} = ae^{bx + cy}$ where a, b, and c are constants. **(3)**

2. Find $\frac{dy}{dx}$ in terms of x and y given $x \sin y + x^2 \cos y = 1$ **(3)**

3. Find an equation for the tangent to the curve at $(\frac{\pi}{3}, \frac{\pi}{6})$ on the curve, $4 \sin y - \sec x = 0$ **(5)**

Solutions

1.

$6e^{3x} - 2e^{2y}\frac{dy}{dx} = 0$	**M1**
$6e^{3x} = 2e^{-2}y\frac{dy}{dx}$ $\frac{dy}{dx} = \frac{3e^{3x}}{e^{-2y}}$	**M1**
$\frac{dy}{dx} = 3e^{3x+2y}$	**M1**

2.

$1 \times \sin y + x \times \frac{dy}{dx}\cos y + 2x \times \cos y + x^2 \times (-\sin y)\frac{dy}{dx} = 0$	**M1**
$\sin y + 2x\cos y = \frac{dy}{dx}(x^2 \sin y - x\cos y)$	**M1**
$\frac{dy}{dx} = \frac{\sin y + 2xcos\, y}{x^2 \sin y - x\cos y}$	**M1**

3.

$4\frac{dy}{dx}\cos y - \sec x \tan x = 0$	**M1**
$4\frac{dy}{dx}\cos y = \sec x \tan x$ $\frac{dy}{dx} = \frac{\sec x \tan x}{4\cos y}$	**M1**
Gradient $= \frac{2\times\sqrt{3}}{4\times\frac{\sqrt{3}}{2}} = 1$	**M1**
$y - \frac{\pi}{6} = x - \frac{\pi}{3}$	**M1**
$y = x - \frac{\pi}{6}$	**M1**

Topic: Connected Rates of Change

Chapter Reference: Pure 2, Chapter 9

10 minutes

1. The area of a circle is decreasing at a constant rate of 0.5 $cm^2\ s^{-1}$.

a. Find the rate at which the radius of the circle is decreasing when the radius is 8 cm. **(4)**

b. Find the rate at which the perimeter of the circle is decreasing when the radius is 8 cm. **(3)**

2. The volume of a cube is increasing at a constant rate of 3.5 $cm^3\ s^{-1}$. Find,

a. The rate at which the length of one side of the cube is increasing when the volume is 200 cm^3. **(5)**

b. The volume of the cube when the length of one side is increasing at a rate of 2 $mm\ s^{-1}$. **(4)**

Solutions

1a.

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$	**M1**
$A = \pi r^2$ $\frac{dA}{dr} = 2\pi r$	**M1**
$\frac{dA}{dr}$ = -0.5, r = 8 $\frac{dA}{dr} = 16\pi$	**M1**
-0.5 = 16π x $\frac{dr}{dt}$ $\frac{dr}{dt} = -\frac{1}{32\pi}$ = -0.00995 cm s^{-1}	**M1**
Therefore, the radius is decreasing at 0.00995cm s^{-1} (3 s.f)	

1b.

$\frac{dP}{dt} = \frac{dP}{dr} \times \frac{dr}{dt}$	**M1**
$P = 2\pi r$ $\frac{dP}{dr} = 2\pi$	**M1**
$\frac{dP}{dt} = 2\pi \times -\frac{1}{32\pi} = -\frac{1}{16}$ Therefore, perimeter decreases at a rate of 0.0625 cm s^{-1}	**M1**

2a.

$\frac{dV}{dt} = \frac{dV}{dl} \times \frac{dl}{dt}$ $\frac{dV}{dt} = 3.5$, $V = l^3$ $\frac{dV}{dl} = 3l^2$	**M1**
$200 = l^3$ $l = 5.848$	**M1**
$\frac{dV}{dl} = 3 \times 5.848^2 = 102.6$	**M1**
3.5 = 102.6 x $\frac{dl}{dt}$	**M1**
$\frac{dl}{dt} = 3.5 \div 102.6 = 0.0341\ cms^{-1}$	**M1**

2b.

2 mm s^{-1} = 0.2 cm s^{-1}	**M1**
$3.5 = \frac{dV}{dl} \times 0.2$ $\frac{dV}{dl} = 17.5$	**M1**
$17.5 = 3l^2$ $l = 2.415$	**M1**
$V = l^3 = 14.1$ cm^3	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Using Second Derivatives

Chapter Reference: Pure 2, Chapter 9

6 minutes

1. Given that $y = x(3x - 1)^3$

a. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ **(3)**

b. Find the points of inflection of y **(3)**

Solutions

1.

$y = x(3x - 1)^5$ $\frac{dy}{dx} = 15x(3x - 1)^4 + (3x - 1)^5$	**M1**
$\frac{d^2y}{dx^2} = 15(3x - 1)^4 + 15(3x - 1)^4 + 180x(3x - 1)^3$ $= 30(3x - 1)^4 + 180x(3x - 1)^3$	**M1**
$= 30(3x - 1)^3(9x - 1)$	**M1**

2.

At points of inflection, $\frac{d^2y}{dx^2} = 0$ $30(3x - 1)^3(9x - 1) = 0$	**M1**
$x = \frac{1}{3}$ $y = \frac{1}{3} \times (1 - 1)^5 = 0$	**M1**
$x = \frac{1}{9}$ $y = \frac{1}{9}(\frac{1}{3} - 1)^5 = -\frac{32}{2187}$	**M1**
Points of inflection are: $(\frac{1}{9}, -\frac{32}{2187})$ and $(\frac{1}{3}, 0)$	