A-Level Starter Activity

Jethwa Maths

Topic: Parametric Equations

Chapter Reference: Pure 2, Chapter 8

6 minutes

1. Find the cartesian equation for, $x = 1 - t^2$ and $y = 4 - t$ **(2)**

2. A curve has parametric equations, $x = 2t + 1$ and $y = t^2$. Find a cartesian equation for the curve. **(2)**

3a. Find the cartesian equation of $x = 2t$ and $y = 4t(t - 1)$ **(2)**

b. Sketch the cartesian curve showing any points of intersection. **(3)**

Solutions

1.

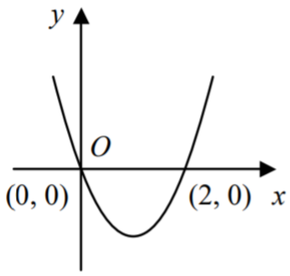
$t = 4 - y$	**M1**
$x = 1 - (4 - y)^2$	**M1**

2.

$t = \frac{1}{2}(x - 1)$	**M1**
$y = \frac{1}{4}(x - 1)^2$	**M1**

3.

$t = \frac{x}{2}$ $y = 4(\frac{x}{2})(\frac{x}{2} - 1)$	**M1**
$y = x(x - 2)$	**M1**
Shape **M1** (0,0) **M1** (2, 0) **M1**	



Topic: Parametric Equations, Points of Intersection Chapter Reference: Pure 2, Chapter 8	8 minutes

1. Find the values of t at the point of intersection of the line $y = x - \ln 3$ and the curve with parametric equation $x = \ln(t-1)$, $y = \ln(2t-5)$, $t > \frac{5}{2}$, and give the exact coordinates of this point. **(6)**

2. The curve C has parametric equations $x = \sin t$ and $y = \cos t$. The straight line l passes through the points A and B where $t = \frac{\pi}{6}$ and $t = \frac{\pi}{2}$ respectively. Find an equation for the line l in the form $ax + by + c = 0$ **(7)**

Solutions

1.

$y = x - \ln 3$ $\ln(2t - 5) = \ln(t - 1) - \ln 3$	**M1**
$\ln(2y - 5) = \ln \frac{t-1}{3}$ $2t - 5 = \frac{t-1}{3}$	**M1**
$6t - 15 = t - 1$ $5t = 14$	**M1**
$t = \frac{14}{5}$	**M1**
$x\left(\frac{14}{5}\right) = \ln\left(\frac{14}{5} - 1\right) = \ln\left(\frac{9}{5}\right)$	**M1**
$y\left(\frac{14}{5}\right) = \ln\left(\frac{28}{5} - 5\right) = \ln\left(\frac{3}{5}\right)$	**M1**

2.

At A, $x\left(\frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$	**M1**
At A, $y\left(\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$	**M1**
At B, $x\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} = 1$	**M1**
At B, $y\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0$	**M1**
As line passes through A and B, $m = \frac{0-\frac{\sqrt{3}}{2}}{1-\frac{1}{2}} = -\sqrt{3}$	**M1**
$y - 0 = -\sqrt{3}\,(x - 1)$ $y = -\sqrt{3}x + \sqrt{3}$	**M1**
$\sqrt{3}x + y - \sqrt{3} = 0$	**M1**

Topic: Sketching Parametric Equations Chapter Reference: Pure 2, Chapter 8	7 minutes

1. For the curve given by parametric equations $x = 2t$ and $y = 4t(t - 1)$, find a cartesian equation and hence, sketch the curve, showing the coordinates of any points where it meets the coordinate axes. **(5)**

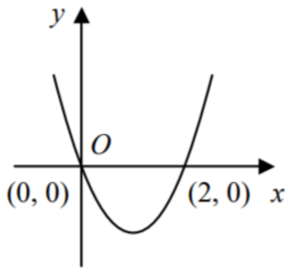
2. For the curve given by parametric equations $x = 1 - \sin t$ and $y = 2 - \cos t$, find a cartesian equation and hence, sketch the curve, showing the coordinates of any points where it meets the coordinate axes. **(4)**

3. For the curve given by parametric equations $x = t + 1$ and $y = \frac{2}{t}$, find a cartesian equation and hence, sketch the curve, showing the coordinates of any points where it meets the coordinate axes and asymptotes. **(4)**

Solutions

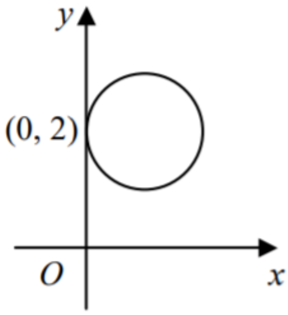
1.

$t = \frac{x}{2}$ $y = 4(\frac{x}{2})(\frac{x}{2} - 1)$	**M1**
$y = x(x - 2)$	**M1**
Shape **M1** (0,0) **M1** (2, 0) **M1**	



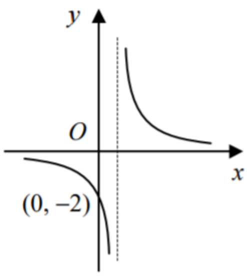
2.

$\sin t = 1 - x$ $\cos t = 2 - y$	**M1**
$\cos^2 t + \sin^2 t = 1$ $(2 - y)^2 + (1 - x)^2 = 1$	**M1**
Shape **M1** (0, 2) **M1**	



3.

$t = x - 1$ $y = \frac{2}{x-1}$	**M1**
Shape **M1** (0, -2) **M1** Asymptote $x = 1$ **M1**	



A-Level Starter Activity

Jethwa Maths

Topic: Parametric Equations Using Trig Identities

Chapter Reference: Pure 2, Chapter 8

7 minutes

1. Find the cartesian equation of a curve, given its parametric equations are $x = \cos t$ and $y = \sin t$. **(2)**

2. Find the cartesian equation of a curve, given its parametric equations are $x = 3 + 2\cos t$ and $y = 1 + 2\sin t$. **(3)**

3. Find the cartesian equation of a curve, given its parametric equations are $x = \sin t$ and $y = \sin^2 2t$. **(3)**

3. Find the cartesian equation of a curve, given its parametric equations are $x = \cos t$ and $y = \tan^2 t$. **(3)**

Solutions

1.

$\cos^2 t + \sin^2 t = 1$	**M1**
$x^2 + y^2 = 1$	**M1**

2.

$\cos t = \frac{x-3}{2}$ $\sin t = \frac{y-1}{2}$	**M1**
$(\frac{x-3}{2})^2 + (\frac{y-1}{2})^2 = 1$	**M1**
$(x-3)^2 + (y-1)^2 = 4$	**M1**

3.

$\sin 2t = 2 \sin t \cos t$ $y = 4\sin^2 t \cos^2 t$	**M1**
$y = 4\sin^2 t\,(1 - \sin^2 t)$	**M1**
$y = 4x^2(1 - x^2)$	**M1**

4.

$\sec t = \frac{1}{x}$	**M1**
$1 + \tan^2 t = \sec^2 t$ $1 + y = (\frac{1}{x})^2$	**M1**
$y = \frac{1}{x^2} - 1$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Modelling with Parametric Equations Chapter Reference: Pure 2, Chapter 8	10 minutes

1. The path of a ski jumper from the point of leaving the ramp to the point of landing is modelled using the parametric equations,

$$x = 18t \qquad y = -4.9t^2 + 4t - 10 \qquad 0 \le t \le k$$

Where x is the horizontal distance is metres from the point of leaving the ramp and y is the height in metres above ground level of the ski jumper, after t seconds.

a. Find the initial height of the ski jumper. **(2)**

b. Find the value of k and hence state the time taken for the ski jumper to complete her jump. **(3)**

c. Find the horizontal distance the ski jumper jumps. **(1)**

d. Show that the ski jumper's path is a parabola and find the maximum height above ground level of the ski jumper. **(6)**

Solutions

1a.

$y(2) = -4.9(2)^2 + 4(2) + 10$	**M1**
$y = 10$ The initial height is 10m.	**M1**

1b.

$y = 0$ $0 = -4.9t^2 + 4t + 10$ $0 = 4.9t^2 - 4t - 10$	**M1**
$t = 1.89$ or 1.08	**M1**
As $t > 0$, $t = 1.08$s.	**M1**

1c.

$x(1.89) = 18\ (1.08) = 34.1$ The horizontal distance travelled is 34.1 m (3 s.f)	**M1**

1d.

$x = 18t \rightarrow t = \frac{x}{18}$	**M1**
$y = -4.9(\frac{x}{18})^2 + 4(\frac{x}{18}) + 10$ $= -\frac{4.9}{324}x^2 + \frac{4}{18}x + 10$ This shows the path is a parabola	**M1**
At the maximum height, $\frac{dy}{dt} = 0$ $y = -4.9t^2 + 4t + 10$ $\frac{dy}{dt} = -9.8t + 4$	**M1**
$-9.8t + 4 = 0$ $t = \frac{20}{49}$	**M1**
$y = -4.9(\frac{20}{49})2 + 4(\frac{20}{49}) + 10$	**M1**
$y = 10.8$m (3 s.f) The maximum height above the ground is 10.8m	**M1**