A-Level Starter Activity

Jethwa Maths

Topic: Double Angle Formulae Chapter Reference: Pure 2, Chapter 7	8 minutes

1. Solve the equation $\cos 2x + \cos x = 0$ for x in the interval $0 \leq x \leq 360°$. Give your answers to 1 decimal place where appropriate. **(3)**

2. Solve the equation $5 \sin 4x = 2 \sin 2x$ for x in the interval $0 \leq x \leq 360°$. Give your answers to 1 decimal place where appropriate. **(4)**

3. Prove the identity, $\frac{2 \sin x}{2 \cos x - \sec x} \equiv \tan 2x$ **(3)**

Solutions

1.

$2\cos^2 x - 1 + \cos x = 0$ $(2\cos x - 1)(\cos x + 1) = 0$	**M1**
$\cos x = -1$ $x = 180^{\circ}$	**M1**
$\cos x = \frac{1}{2}$ $x = 60^{\circ}$ $x = 360 - 60 = 300^{\circ}$	**M1**

2.

$10 \sin 2x \cos 2x = 2 \sin 2x$ $2 \sin 2x(5 \cos 2x - 1) = 0$	**M1**
$\sin 2x = 0$ $2x = 0, 180, 260, 540, 720$	**M1**
$\cos 2x = \frac{1}{5}$ $2x = 78.463$ $2x = 360 - 78.463 = 281.537$ $2x = 360 + 78.463 = 438.463$ $2x = 720 - 78.463 = 641.537$	**M1**
$x = 0^{\circ}, 39.2^{\circ}, 90^{\circ}, 140.8^{\circ}, 180^{\circ}, 219.2^{\circ}, 270^{\circ}, 320.8^{\circ}, 360^{\circ}$	**M1**

3.

L.H.S $= \frac{2 \sin x \cos x}{\cos x(2 \cos x - \sec x)}$ $= \frac{2 \sin x \cos x}{2cos^2 x - 1}$	**M1**
$= \frac{\sin 2x}{\cos 2x}$	**M1**
$= \tan 2x$ (RHS)	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Addition Formulae

Chapter Reference: Pure 2, Chapter 7

8 minutes

1. Express sin 62 cos 74 + cos 62 sin 74 in the form sin x, where x is acute. **(2)**

2. Find the maximum value that cos x cos 30 + sin x sin 30 can take and the smallest positive value of x in degrees for which this maximum occurs. **(3)**

3. Solve the equation sin (x + 30) = cos (x – 45) in the interval $0 \leq x \leq 360$. Give your answers to 1 decimal place where appropriate. **(4)**

4. Prove that $\cos x - \cos (x - \frac{\pi}{3}) \equiv \cos (x + \frac{\pi}{3})$ **(3)**

Solutions

1.

= sin(62 + 74)^o = sin 136^o	**M1**
= sin (180 – 136)^o = sin 44^o	**M1**

2.

cos x cos 30 + sin x sin 30 = cos (x – 30)	**M1**
Therefore, maximum = 1	**M1**
When x = 30^o	**M1**

3.

sin x cos 30 + cos x sin 30 = cos x cos 45 + sin x sin 45	**M1**
$\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x$ $(\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}})$ sin x = $(\frac{1}{\sqrt{2}} - \frac{1}{2})$ cos x	**M1**
tan x = $(\frac{1}{\sqrt{2}} - \frac{1}{2}) \div (\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}})$ = 1.3032	**M1**
x = 52.2 x = 180 + 52.5 = 232.5	**M1**

4.

LHS = cos x – (cos x cos $\frac{\pi}{3} + \sin x \sin\frac{\pi}{3}$) = cos x - $\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$	**M1**
= $\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$ = cos x cos $\frac{\pi}{3}$ – sin x sin $\frac{\pi}{3}$	**M1**
= cos ($x + \frac{\pi}{3}$) (RHS)	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Proving Trig Identities Chapter Reference: Pure 2, Chapter 7	8 minutes

1. Prove the identity, $\cot 2x + \operatorname{cosec} 2x \equiv \cot x$ **(3)**

2. Prove that for all real values of x, $\cos (x + 30)^{\circ} + \sin x^{\circ} \equiv \cos (x - 30)^{\circ}$ **(3)**

3. Prove the identity, $\frac{1-\cos x}{1+\cos x} \equiv tan^2 \frac{x}{2}$ **(3)**

Solutions

1.

L.H.S = $\frac{\cos 2x}{\sin 2x} + \frac{1}{\sin 2x}$ $= \frac{\cos 2x + 1}{\sin 2x}$	**M1**
$= \frac{2\cos^2 x - 1 + 1}{2 \sin x \cos x}$ $= \frac{2\cos^2 x}{2 \sin x \cos x}$	**M1**
$= \frac{\cos x}{\sin x}$ = cot x (R.H.S)	**M1**

2.

LHS = cos x cos 30 – sin x sin 30 + sin 30	**M1**
$= \frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x + \sin x$ $= \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x$	**M1**
= cos x cos 30 + sin x sin 30 = cos $(x – 30)^o$ = R.H.S	**M1**

3.

L.H.S = $\frac{1-(1-2sin^2\frac{x}{2})}{1+(2\ cos^2\frac{x}{2}-1)}$	**M1**
$= \frac{2sin^2\frac{x}{2}}{2cos^2\frac{x}{2}}$	**M1**
$= \tan^2 \frac{x}{2}$ = R.H.S	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Solving Trig. Equations

Chapter Reference: Pure 2, Chapter 6

8 minutes

1a. Prove the identity $\cot 2x + \operatorname{cosec} 2x \equiv \cot x$ **(3)**

b. Hence, for x in the interval $0 \leq x \leq 2\pi$, solve the equation $\cot 2x + \operatorname{cosec} 2x = 6 - \cot^2 x$, giving your answers correct to 2 decimal places. **(4)**

2. Find the values of x in the interval $0 \leq x \leq 2\pi$ for which $\cot^2 x - \cot x + \operatorname{cosec}^2 x = 4$ **(4)**

Solutions

1a.

LHS = $\frac{\cos 2x}{\sin 2x} + \frac{1}{\sin 2x}$ $= \frac{\cos 2x + 1}{\sin 2x}$	**M1**
$= \frac{2cos^2 x - 1 + 1}{2\sin x \cos x}$ $= \frac{2cos^2 x}{2\sin x \cos x}$	**M1**
$= \frac{\cos x}{\sin x}$ $= \cot x$ (= R.H.S)	**M1**

1b.

$\cot x = 6 - \cot^2 x$ $\cot^2 x + \cot x - 6 = 0$	**M1**
$(\cot x + 3)(\cot x - 2) = 0$	**M1**
$\cot x = -3$ $\tan x = -\frac{1}{3}$ $x = \pi - 0.3218 = 2.82$ $x = 2\pi - 0.3218 = 5.96$	**M1**
$\cot x = 2$ $\tan x = \frac{1}{2}$ $x = 0.4636$ $x = \pi + 0.4636 = 3.61$	**M1**

2.

$\cot^2 x - \cot x + 1 + \cot^2 x = 4$	**M1**
$2\cot^2 x - \cot x - 3 = 0$ $(2 \cot x - 3)(\cot x - 3) = 0$	**M1**
$\cot x = -1$ $\tan x = -1$ $x = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ $x = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$	**M1**
$\cot x = \frac{3}{2}$ $\tan x = \frac{2}{3}$ $x = 0.5880$ $x = \pi + 0.5880 = 3.73$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: a cos x $\pm$ b sin x Chapter Reference: Pure 2, Chapter 7	8 minutes

1. Express $5\cos x^{o} - 12\sin x^{o}$ as $R\cos(x+\alpha)$. Give the exact value of R and the value of α to 1 decimal place.

2a. Express $3\sin x - 2\cos x$ as $R\sin(x - \alpha)$. Give the exact value of R, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$, and the value of α to 2 decimal places. **(3)**

b. State the maximum value of $3\sin x - 2\cos x$ and the value of x at which this maximum occurs. Give the exact value for the maximum and the value of x to 1 decimal place. **(2)**

3. Given that $y = 12\cos x + 5\sin x = 13\cos(x - 22.6)$, sketch the graph of y in the interval $0 \leq x \leq 360$, showing the coordinates of any turning points. **(3)**

Solutions

1.

$5 \cos x - 12 \sin x = R \cos x \cos \alpha - R \sin x \sin \alpha$	**M1**
$R \cos \alpha = 5$ $R \sin \alpha = 12$	**M1**
$R = \sqrt{12 + 144} = 13$	**M1**
$\tan \alpha = \frac{12}{5}$ $\alpha = 67.4^{\circ}$	**M1**
$5 \cos x - 12 \sin x = 13 \cos (x + 67.4)^{\circ}$	

2a.

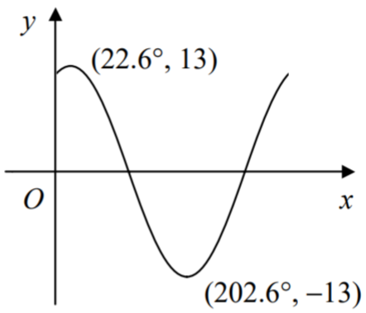
$3 \sin x - 2 \cos x = R \sin x \cos \alpha - R \cos x \sin \alpha$	**M1**
$R \cos \alpha = 3$ $R \sin \alpha = 2$	**M1**
$R = \sqrt{9 + 4} = \sqrt{13}$	**M1**
$\tan \alpha = \frac{2}{3}$ $\alpha = 0.59$	**M1**
$3 \sin x - 2 \cos x = \sqrt{13} \sin (x - 0.59)^{\circ}$	

2b.

$3 \sin x - 2 \cos x = \sqrt{13} \sin (x - 0.59)^{\circ}$ Maximum value = $\sqrt{13}$ x 1 = $\sqrt{13}$	**M1**
Value of x, $x - 0.59 = \frac{\pi}{2}$ $x = 2.16$	**M1**

3.

Shape **M1**
(22.6°, 13) **M1**
(202.6°, -13) **M1**



A-Level Starter Activity

Jethwa Maths

Topic: Modelling with Trigonometry Chapter Reference: Pure 2, Chapter 7	12 minutes

1a. Express 200 sin x – 150 cos x in the form R sin (x – α), where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the value of α to 4 decimal places. **(5)**

The electric field strength, E V/m in a microwave of width 25 cm can be modelled using the equation,

$$E = 1700 + 200 \sin(\frac{4\pi x}{25}) - 150\, cos(\frac{4\pi x}{25})$$

Where x is the distance in cm from the left-hand edge of the microwave oven.

b. Calculate the maximum value of E predicted by this model. **(4)**

c. Find the values of x for $0 \leq x < 25$, where this maximum occurs. **(4)**

d. Food in the microwave will heat best when the electric field strength at the centre of the food is above 1800 V/m. Find the range of possible locations for the centre of the food. **(3)**

Solutions

1a.

Set 200 sin x – 150 cos $x \equiv R \sin(x-\alpha) \equiv R \sin x \cos\alpha - R\cos x \sin\alpha$	**M1**
$R\cos\alpha = 200$ $R\sin\alpha = 150$	**M1**
$\tan\alpha = \frac{150}{200}$ $\alpha = 0.6425$ (to 4 d.p)	**M1**
$R = \sqrt{40000 + 22500} = 250$	**M1**
200 sin x – 150 cos x ≡ 259 sin (x – 0.6435)	**M1**

1b.

$1700 + 200 \sin(\frac{4\pi x}{25}) - 150\cos(\frac{4\pi x}{25})$	**M1**
$= 1700 + 250 \sin(\frac{4\pi x}{25} - 0.6435)$	**M1**
The maximum value of E is when $\sin(\frac{4\pi x}{25} - 0.6435) = 1$	**M1**
Therefore, maximum value of E is, 1700 + 250 = 1950 *V/m*	**M1**

1c.

The maximum occurs when, $\sin(\frac{4\pi x}{25} - 0.6435) = 1$	**M1**
$\frac{4\pi x}{25} - 0.6435 = \frac{\pi}{2}, \frac{5\pi}{2}$	**M1**
$\frac{4\pi x}{25}$ = 2.2143 and 8.4975	**M1**
x = 4.41cm, 16.91 cm (to 2 d.p)	**M1**

1d.

$1700 + 250\sin(\frac{4\pi x}{25} - 0.6435) = 1800$	**M1**
$\sin(\frac{4\pi x}{25} - 0.6435) = \frac{100}{250} = 0.4$	**M1**
$\frac{4\pi x}{25} - 0.6435$ = 0.4155 = 0.42 (2 d.p) $\frac{4\pi x}{25} - 0.6435 = \pi$ – 0.4115 = 2.7301 = 2.73 (2 d.p) 2π + 0.4115 = 6.6947 = 6.70 (2 d.p) 3π - 0.4115 = 19.0133 = 19.01 (2 d.p)	**M1**

Range = 19.01 - 0.42

= 18.59