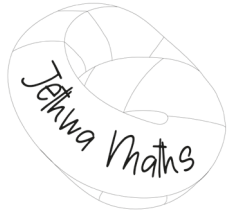


A-Level Starter Activity



Topic: Double Angle Formulae

Chapter Reference: Pure 2, Chapter 7

8

minutes

1. Solve the equation $\cos 2x + \cos x = 0$ for x in the interval $0 \leq x \leq 360^\circ$. Give your answers to 1 decimal place where appropriate. (3)

2. Solve the equation $5 \sin 4x = 2 \sin 2x$ for x in the interval $0 \leq x \leq 360^\circ$. Give your answers to 1 decimal place where appropriate. (4)

3. Prove the identity, $\frac{2 \sin x}{2 \cos x - \sec x} \equiv \tan 2x$ (3)

Solutions

1.

$2 \cos^2 x - 1 + \cos x = 0$ $(2 \cos x - 1)(\cos x + 1) = 0$	M1
$\cos x = -1$ $x = 180^\circ$	M1
$\cos x = \frac{1}{2}$ $x = 60^\circ$ $x = 360 - 60 = 300^\circ$	M1

2.

$10 \sin 2x \cos 2x = 2 \sin 2x$ $2 \sin 2x(5 \cos 2x - 1) = 0$	M1
$\sin 2x = 0$ $2x = 0, 180, 260, 540, 720$	M1
$\cos 2x = \frac{1}{5}$ $2x = 78.463$ $2x = 360 - 78.463 = 281.537$ $2x = 360 + 78.463 = 438.463$ $2x = 720 - 78.463 = 641.537$	M1
$x = 0^\circ, 39.2^\circ, 90^\circ, 140.8^\circ, 180^\circ, 219.2^\circ, 270^\circ, 320.8^\circ, 360^\circ$	M1

3.

$\text{L.H.S} = \frac{2 \sin x \cos x}{\cos x(2 \cos x - \sec x)}$	M1
$= \frac{2 \sin x \cos x}{2 \cos^2 x - 1}$	M1
$= \frac{\sin 2x}{\cos 2x}$	M1
$= \tan 2x \text{ (RHS)}$	M1



A-Level Starter Activity



Topic: Addition Formulae

Chapter Reference: Pure 2, Chapter 7

**8
minutes**

1. Express $\sin 62 \cos 74 + \cos 62 \sin 74$ in the form $\sin x$, where x is acute. (2)

2. Find the maximum value that $\cos x \cos 30 + \sin x \sin 30$ can take and the smallest positive value of x in degrees for which this maximum occurs. (3)

3. Solve the equation $\sin(x + 30) = \cos(x - 45)$ in the interval $0 \leq x \leq 360$. Give your answers to 1 decimal place where appropriate. (4)

4. Prove that $\cos x - \cos\left(x - \frac{\pi}{3}\right) \equiv \cos\left(x + \frac{\pi}{3}\right)$ (3)

Solutions

1.

$= \sin(62 + 74)^\circ$ $= \sin 136^\circ$	M1
$= \sin (180 - 136)^\circ$ $= \sin 44^\circ$	M1

2.

$\cos x \cos 30 + \sin x \sin 30 = \cos (x - 30)$	M1
Therefore, maximum = 1	M1
When $x = 30^\circ$	M1

3.

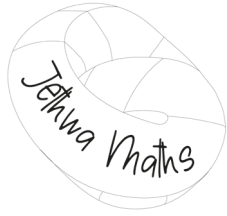
$\sin x \cos 30 + \cos x \sin 30 = \cos x \cos 45 + \sin x \sin 45$	M1
$\frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x = \frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x$ $(\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}) \sin x = (\frac{1}{\sqrt{2}} - \frac{1}{2}) \cos x$	M1
$\tan x = (\frac{1}{\sqrt{2}} - \frac{1}{2}) \div (\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}) = 1.3032$	M1
$x = 52.2$ $x = 180 + 52.5 = 232.5$	M1

4.

LHS = $\cos x - (\cos x \cos \frac{\pi}{3} + \sin x \sin \frac{\pi}{3})$ $= \cos x - \frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x$	M1
$= \frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x$ $= \cos x \cos \frac{\pi}{3} - \sin x \sin \frac{\pi}{3}$	M1
$= \cos (x + \frac{\pi}{3})$ (RHS)	M1



A-Level Starter Activity



Topic: Proving Trig Identities

Chapter Reference: Pure 2, Chapter 7

8
minutes

1. Prove the identity, $\cot 2x + \operatorname{cosec} 2x \equiv \cot x$

(3)

2. Prove that for all real values of x , $\cos(x + 30)^\circ + \sin x^\circ \equiv \cos(x - 30)^\circ$

(3)

3. Prove the identity, $\frac{1 - \cos x}{1 + \cos x} \equiv \tan^2 \frac{x}{2}$

(3)

Solutions

1.

$\begin{aligned} \text{L.H.S} &= \frac{\cos 2x}{\sin 2x} + \frac{1}{\sin 2x} \\ &= \frac{\cos 2x + 1}{\sin 2x} \end{aligned}$	M1
$\begin{aligned} &= \frac{2\cos^2 x - 1 + 1}{2 \sin x \cos x} \\ &= \frac{2\cos^2 x}{2 \sin x \cos x} \end{aligned}$	M1
$\begin{aligned} &= \frac{\cos x}{\sin x} \\ &= \cot x \text{ (R.H.S)} \end{aligned}$	M1

2.

$\text{LHS} = \cos x \cos 30 - \sin x \sin 30 + \sin 30$	M1
$\begin{aligned} &= \frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x + \sin x \\ &= \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x \end{aligned}$	M1
$\begin{aligned} &= \cos x \cos 30 + \sin x \sin 30 \\ &= \cos (x - 30)^\circ \\ &= \text{R.H.S} \end{aligned}$	M1

3.

$\text{L.H.S} = \frac{1 - (1 - 2\sin^2 \frac{x}{2})}{1 + (2\cos^2 \frac{x}{2} - 1)}$	M1
$= \frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}}$	M1
$\begin{aligned} &= \tan^2 \frac{x}{2} \\ &= \text{R.H.S} \end{aligned}$	M1



Solutions

1a.

$\begin{aligned} \text{LHS} &= \frac{\cos 2x}{\sin 2x} + \frac{1}{\sin 2x} \\ &= \frac{\cos 2x + 1}{\sin 2x} \end{aligned}$	M1
$\begin{aligned} &= \frac{2\cos^2 x - 1 + 1}{2 \sin x \cos x} \\ &= \frac{2\cos^2 x}{2 \sin x \cos x} \end{aligned}$	M1
$\begin{aligned} &= \frac{\cos x}{\sin x} \\ &= \cot x \\ & (= \text{R.H.S}) \end{aligned}$	M1

1b.

$\begin{aligned} \cot x &= 6 - \cot^2 x \\ \cot^2 x + \cot x - 6 &= 0 \end{aligned}$	M1
$(\cot x + 3)(\cot x - 2) = 0$	M1
$\begin{aligned} \cot x &= -3 \\ \tan x &= -\frac{1}{3} \\ x &= \pi - 0.3218 = 2.82 \\ x &= 2\pi - 0.3218 = 5.96 \end{aligned}$	M1
$\begin{aligned} \cot x &= 2 \\ \tan x &= \frac{1}{2} \\ x &= 0.4636 \\ x &= \pi + 0.4636 = 3.61 \end{aligned}$	M1

2.

$\cot^2 x - \cot x + 1 + \cot^2 x = 4$	M1
$\begin{aligned} 2\cot^2 x - \cot x - 3 &= 0 \\ (2\cot x - 3)(\cot x - 3) &= 0 \end{aligned}$	M1
$\begin{aligned} \cot x &= -1 \\ \tan x &= -1 \\ x &= \pi - \frac{\pi}{4} = \frac{3\pi}{4} \\ x &= 2\pi - \frac{\pi}{4} = \frac{7\pi}{4} \end{aligned}$	M1
$\begin{aligned} \cot x &= \frac{3}{2} \\ \tan x &= \frac{2}{3} \\ x &= 0.5880 \\ x &= \pi + 0.5880 = 3.73 \end{aligned}$	M1

Solutions

1.

$5 \cos x - 12 \sin x = R \cos x \cos \alpha - R \sin x \sin \alpha$	M1
$R \cos \alpha = 5$	M1
$R \sin \alpha = 12$	M1
$R = \sqrt{12^2 + 144} = 13$	M1
$\tan \alpha = \frac{12}{5}$ $\alpha = 67.4^\circ$	M1
$5 \cos x - 12 \sin x = 13 \cos (x + 67.4)^\circ$	

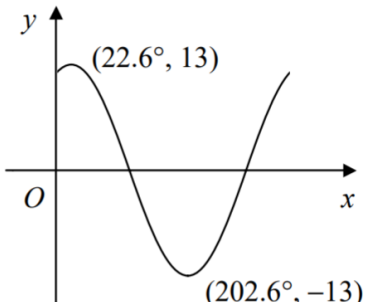
2a.

$3 \sin x - 2 \cos x = R \sin x \cos \alpha - R \cos x \sin \alpha$	M1
$R \cos \alpha = 3$	M1
$R \sin \alpha = 2$	M1
$R = \sqrt{9 + 4} = \sqrt{13}$	M1
$\tan \alpha = \frac{2}{3}$ $\alpha = 0.59$	M1
$3 \sin x - 2 \cos x = \sqrt{13} \sin (x - 0.59)^\circ$	

2b.

$3 \sin x - 2 \cos x = \sqrt{13} \sin (x - 0.59)^\circ$ Maximum value = $\sqrt{13} \times 1 = \sqrt{13}$	M1
Value of x , $x - 0.59 = \frac{\pi}{2}$ $x = 2.16$	M1

3.

<p>Shape M1 $(22.6^\circ, 13)$ M1 $(202.6^\circ, -13)$ M1</p>	
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Solutions

1a.

Set $200 \sin x - 150 \cos x \equiv R \sin(x - \alpha) \equiv R \sin x \cos \alpha - R \cos x \sin \alpha$	M1
$R \cos \alpha = 200$ $R \sin \alpha = 150$	M1
$\tan \alpha = \frac{150}{200}$ $\alpha = 0.6425$ (to 4 d.p)	M1
$R = \sqrt{40000 + 22500} = 250$	M1
$200 \sin x - 150 \cos x \equiv 259 \sin(x - 0.6435)$	M1

1b.

$1700 + 200 \sin\left(\frac{4\pi x}{25}\right) - 150 \cos\left(\frac{4\pi x}{25}\right)$	M1
$= 1700 + 250 \sin\left(\frac{4\pi x}{25} - 0.6435\right)$	M1
The maximum value of E is when $\sin\left(\frac{4\pi x}{25} - 0.6435\right) = 1$	M1
Therefore, maximum value of E is, $1700 + 250 = 1950$ V/m	M1

1c.

The maximum occurs when, $\sin\left(\frac{4\pi x}{25} - 0.6435\right) = 1$	M1
$\frac{4\pi x}{25} - 0.6435 = \frac{\pi}{2}, \frac{5\pi}{2}$	M1
$\frac{4\pi x}{25} = 2.2143$ and 8.4975	M1
$x = 4.41$ cm, 16.91 cm (to 2 d.p)	M1

1d.

$1700 + 250 \sin\left(\frac{4\pi x}{25} - 0.6435\right) = 1800$	M1
$\sin\left(\frac{4\pi x}{25} - 0.6435\right) = \frac{100}{250} = 0.4$	M1
$\frac{4\pi x}{25} - 0.6435 = 0.4155 = 0.42$ (2 d.p) $\frac{4\pi x}{25} - 0.6435 = \pi - 0.4115 = 2.7301 = 2.73$ (2 d.p) $2\pi + 0.4115 = 6.6947 = 6.70$ (2 d.p) $3\pi - 0.4115 = 19.0133 = 19.01$ (2 d.p)	M1

