A-Level Starter Activity

Jethwa Maths

Topic: Small Angle Approximations Chapter Reference: Pure 2, Chapter 5	7 minutes

1. Given that x is small and is measure in radians, use the small angle approximations to find an approximate value of,

$$\frac{1 - \cos 4x}{2x \sin 3x}$$

(3)

2. Show that, for a small angle x, where x is in radians, $1 + \cos x - 3\cos^2 x \approx -1 + \frac{5}{2}x^2$ **(4)**

Solutions

1.

$\frac{1-\cos 4x}{2x \sin 3x} \approx \frac{1-[1-\frac{(4x)^2}{2}]}{2x(3x)}$	**M1**
$= \frac{8x^2}{6x^2}$	**M1**
$= \frac{4}{3}$	**M1**

2.

When x is small, $1 + \cos x - 3\cos^2 x \approx 1 + (1 - \frac{1}{2}x^2) - 3(1 - \frac{1}{2}x^2)^2$	**M1**
$= 1 + (1 - \frac{1}{2}x^2) - 3(1 - x^2 + \frac{1}{4}x^4)$	**M1**
$= 1 + 1 - \frac{1}{2}x^2 - 3 + 3x^2 - \frac{3}{4}x^4$	**M1**
As x is small, we can ignore higher order terms, $1 + \cos x - 3\cos^2 x \approx -1 + \frac{5}{2}x^2$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Inverse Trig Functions Chapter Reference: Pure 2, Chapter 6	8 minutes

1. Find in radians the value of:

a. arc cos (-1) **(1)**

b. arc cos ($\frac{\sqrt{3}}{2}$) **(1)**

c. arc tan (-$\frac{1}{\sqrt{3}}$) **(1)**

2. Solve arcsin $x = \frac{\pi}{4}$ **(1)**

3. Solve the equation 2 + 3 arctan x = 0 giving your answers to 3 significant figures. **(2)**

4. f(x) = arccos x - $\frac{\pi}{3}$

a. State the value of f(-$\frac{1}{2}$) in terms of π **(1)**

b. Solve the equation f(x) = 0 **(1)**

c. Define the inverse function $f^{-1}(x)$ and state its domain. **(3)**

Solutions

1a.

π	**M1**

1b.

$\frac{\pi}{6}$	**M1**

1c.

$-\frac{\pi}{6}$	**M1**

2.

$x = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$	**M1**

3.

arc tan $x = -\frac{2}{3}$	**M1**
$x = \tan(-\frac{2}{3}) = -0.787$	**M1**

4a.

$f(-\frac{1}{2}) = \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3}$	**M1**

4b.

arc cos $x = \frac{\pi}{3}$ $x = \cos\frac{\pi}{3} = \frac{1}{2}$	**M1**

4c.

$y =$ arc cos $x - \frac{\pi}{3}$ $x =$ arc cos $y - \frac{\pi}{3}$	**M1**
$y = \cos(x + \frac{\pi}{3})$	**M1**
$f^{-1}(x) = \cos(x + \frac{\pi}{3})$ Therefore, domain: $-\frac{\pi}{3} \leq x \leq \frac{2\pi}{3}$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Sec, Cosec and Cot Proof Chapter Reference: Pure 2, Chapter 6	8 minutes

1. Prove the identity, $\operatorname{cosec}^2 x - \sec^2 x = \cot^2 x - \tan^2 x$ **(2)**

2. Prove that there are no real values of x for which, $4\sec^2 x - \sec x + 2\tan^2 x = 0$ **(5)**

3. Prove the identity, $(\cos x + \sin x)(\operatorname{cosec} x - \sec x) = 2\cot x$ **(4)**

Solutions

1.

L.H.S = 1 + $\cot^2 x$ – (1+ $\tan^2 x$)	**M1**
= $\cot^2 - \tan^2 x$	**M1**

2.

4 $\sec^2 x - \sec x + 2\tan^2 x = 4\sec^2 x - \sec x + 2(\sec^2 x - 1) = 0$	**M1**
$6\sec^2 x - \sec x - 2 = 0$	**M1**
$(3\sec x - 2)(2\sec x + 1) = 0$	**M1**
Sec $x = -\frac{1}{2}$ Sec $x = \frac{2}{3}$	**M1**
For real values of x, $\lvert\sec x\rvert > 1$, therefore there are no real solutions.	**M1**

3.

$(\cos x + \sin x)(\text{cosec}\, x - \sec x) = \cos x\, \text{cosec}\, x - 1 + 1 - \sin x \sec x$	**M1**
$= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$ $= \frac{cos^2 x - sin^2 x}{\sin x \cos x}$	**M1**
$= \frac{\cos 2x}{\frac{1}{2}\sin 2x}$	**M1**
= 2 cot 2x (RHS)	**M1**

Jetwa Maths

A-Level Starter Activity

Jethwa Maths

Topics: Sec, Cosec, Cot Graphs	8 minutes
Chapter Reference: Pure 1, Chapter 6	

1. Sketch the graph of $y = 3 \sec x$ in the interval $0 \leq x \leq 2\pi$. Show the coordinates of any turning points and the equations of any asymptotes. **(6)**

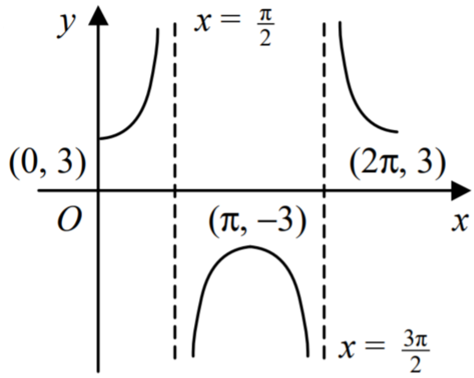
2. Sketch the graph of $y = 1 + \operatorname{cosec} x$ in the interval $0 \leq x \leq 2\pi$. Show the coordinates of any turning points and the equations of any asymptotes. **(6)**

3. Sketch the graph of $y = 2 \cot (x + \frac{\pi}{2})$ in the interval $0 \leq x \leq 2\pi$. Show the coordinates of any turning points and the equations of any asymptotes. **(3)**

Solutions

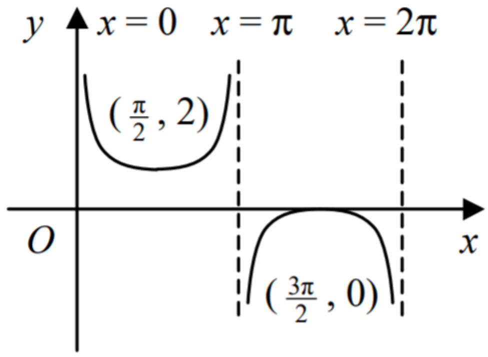
1.

Shape **M1**
(0, 3) **M1**
(π, -3) **M1**
(2π, 3) **M1**
Asymptotes at $x = \frac{\pi}{2}$ **M1**
Asymptotes at $x = \frac{3\pi}{2}$ **M1**



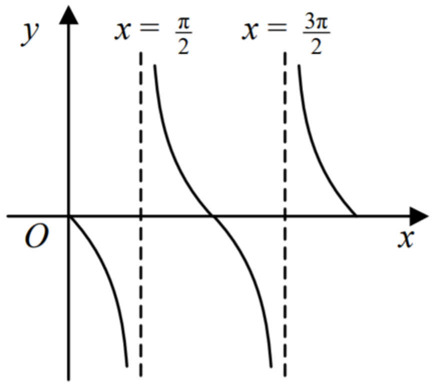
2.

Shape **M1**
($\frac{\pi}{2}$, 2) **M1**
($\frac{3\pi}{2}$, 0) **M1**
Asymptotes at $x = 0$ **M1**
Asymptotes at $x = \pi$ **M1**
Asymptotes at $x = 2\pi$ **M1**



2.

Shape **M1**
Asymptotes at $x = \frac{\pi}{2}$ **M1**
Asymptotes at $x = \frac{3\pi}{2}$ **M1**



A-Level Starter Activity

Jethwa Maths

Topic: Sec, Cosec, Cotangent Chapter Reference: Pure 2, Chapter 6	8 minutes

1. Solve the equation sec $x = 1.8$ in the interval $0 \leq x \leq 360^{\circ}$ giving your answers to 1 decimal place. **(3)**

2. Solve the equation $2 \cos x = \cot x$, in the interval $0 \leq x \leq 360^{\circ}$ giving your answers to 1 decimal place **(4)**

3. Prove the identity, $(1 + \cos x)(\operatorname{cosec} x - \cot x) = \sin x$ **(4)**

Solutions

1.

$\cos x = 0.5556$	**M1**
$x = 56.3^{\circ}$	**M1**
$x = 360 - 56.3 = 303.7^{\circ}$	**M1**

2.

$2\cos x = \frac{\cos x}{\sin x}$ $2\cos x \sin x = \cos x$ $2\cos x \sin x - \cos x$	**M1**
$\cos x\,(2\sin x - 1) = 0$	**M1**
$\cos x = 0$ $x = 90^{\circ}$ $x = 360 - 90 = 270^{\circ}$	**M1**
$\sin x = \frac{1}{2}$ $x = 30^{\circ}$ $x = 180 - 30 = 150^{\circ}$	**M1**

3.

L.H.S = $\operatorname{cosec} x - \cot x + \cot x - \cos x \cot x$ $= \frac{1}{\sin x} - \cos x \times \frac{\cos x}{\sin x}$	**M1**
$= \frac{1 - \cos^2 x}{\sin x}$	**M1**
$= \frac{\sin^2 x}{\sin x}$	**M1**
$= \sin x$ (R.H.S)	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Solving Trig. Equations Chapter Reference: Pure 2, Chapter 5	8 minutes

1. Sketch the graph of $y = \sin\left(x - \frac{\pi}{3}\right)$ in the interval $0 \le x \le 2\pi$. Show the coordinates of any turning points.

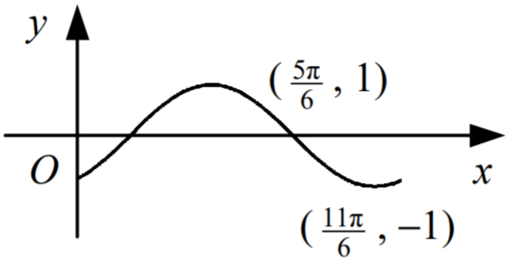
2. Solve the equation $\sin x = -\frac{1}{\sqrt{2}}$ in the interval $0 \le x \le 2\pi$. **(2)**

3. Solve the equation $3\sin^2 x - \cos^2 x = 0$ in the interval $-\pi \le x \le \pi$ **(6)**

Solutions

1.

Shape **M1**
$(\frac{5\pi}{6}, 1)$ **M1**
$(\frac{5\pi}{6}, -1)$ **M1**



2.

$x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$	**M1**
$x = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$	**M1**

3.

$3\sin^2 x - (1 - \sin^2 x) = 0$	**M1**
$4\sin^2 x = 1$ $\sin x = \pm 0.5$	**M1**
$x = \frac{\pi}{6}$ $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$	**M1** **M1**
$x = -\frac{\pi}{6}$ $x = -\frac{\pi}{6} + \pi = -\frac{5\pi}{6}$	**M1** **M1**