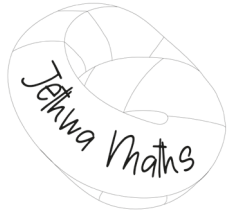


# A-Level Starter Activity



**Topic: Expanding  $(1+x)^n$**

**Chapter Reference: Pure 2, Chapter 4**

**6  
minutes**

1. Expand  $(1 + 4x)^4$  in ascending powers of  $x$ , simplifying the coefficients. (2)

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2a. Expand  $(1 + 3x)^7$  in ascending powers of  $x$  up to and including the term in  $x^4$ , simplifying each coefficient in the expansion. (2)

b. Use your series with a suitable value of  $x$  to estimate the value of  $1.03^7$  correct to 5 decimal places. (2)

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3. Write down the first three terms in the binomial expansion of  $(1 + ax)^n$ , where  $n$  is a positive integer, in ascending powers of  $x$ . (2)

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## Solutions

1.

$1 + 4(4x) + 6(4x)^2 + 4(4x)^3 + (4x)^4$	<b>M1</b>
$= 1 + 16x + 96x^2 + 256x^3 + 256x^4$	<b>M1</b>

2a.

$= 1 + 7(3x) + \frac{7 \times 6}{2} (3x)^2 + \frac{7 \times 6 \times 5}{3 \times 2} (3x)^3 + \frac{7 \times 6 \times 5 \times 4}{4 \times 3 \times 2} (3x)^4$	<b>M1</b>
$= 1 + 21x + 189x^2 + 945x^3 + 2835x^4 + \dots$	<b>M1</b>

2b.

Let $x = 0.01$	
$1.03^7 = 1 + 0.21 + 0.0189 + 0.000945 + 0.00002835$	<b>M1</b>
$= 1.22987$ (5 d.p)	<b>M1</b>

3.

$= 1 + n(ax) + \frac{n(n-1)}{2} (ax)^2 + \dots$	<b>M1</b>
$= 1 + anx + \frac{1}{2} a^2 n(n-1)x^2 + \dots$	<b>M1</b>





## Solutions

1.

$8^{\frac{1}{3}}(1-3x)^{\frac{1}{3}} = 2(1-3x)^{\frac{1}{3}}$	<b>M1</b>
$= 2\left[1 + \left(\frac{1}{3}\right)(-3x) + \frac{\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)}{2}(-3x)^2 + \frac{\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)\left(\frac{-5}{3}\right)}{3 \times 2}(-3x)^3\right]$	<b>M1</b>
$= 2 - 2x - 2x^2 - \frac{10}{3}x^3 + \dots$	<b>M1</b>

1b.

$  -3x   < 1$ $ x  < \frac{1}{3}$	<b>M1</b>
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2a.

$(2x-1)(1+4x)^{-2} = (2x-1)\left[1 + (-2)(4x) + \frac{(-2)(-3)}{2}(4x)^2 + \frac{(-2)(-3)(-4)}{3 \times 2}(4x)^3 + \dots\right]$	<b>M1</b>
$= (2x-1)(1 - 8x + 48x^2 - 256x^3 + \dots)$	<b>M1</b>
$= 2x - 16x^2 + 96x^3 - 1 + 8x - 48x^2 + 256x^3 + \dots$	<b>M1</b>
$= -1 + 10x - 64x^2 + 352x^3 + \dots$	<b>M1</b>

2b.

$ 4x  < 1$ $ x  < \frac{1}{4}$	<b>M1</b>
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## Solutions

1a.

$\frac{x-2}{(1-x)(1-2x)} = \frac{A}{1-x} + \frac{B}{1-2x}$ $x-2 = A(1-2x) + B(1-x)$	<b>M1</b>
Let $x = 1$ , $-1 = -A$ $A = 1$	<b>M1</b>
Let $x = \frac{1}{2}$ $-\frac{3}{2} = \frac{1}{2}B$ $B = -3$	<b>M1</b>
$\frac{x-2}{(1-x)(1-2x)} = \frac{1}{1-x} - \frac{3}{1-2x}$	

1b.

$\frac{1}{1-x} = (1-x)^{-1} = 1 + (-1)(-x) + \frac{(-1)(-2)}{2}(-x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}(-x)^3 + \dots$	<b>M1</b>
$= 1 + x + x^2 + x^3 + \dots$	<b>M1</b>
$\frac{3}{1-2x} = 3(1-2x)^{-1} = 3[1 + (-1)(-2x) + \frac{(-1)(-2)}{2}(-2x)^2 + \frac{(-1)(-2)(-3)}{3 \times 2}(-2x)^3 + \dots]$	<b>M1</b>
$= 3 + 6x + 12x^2 + 24x^3 + \dots$	<b>M1</b>
$\frac{x-2}{(1-x)(1-2x)} = (1 + x + x^2 + x^3 + \dots) - (3 + 6x + 12x^2 + 24x^3 + \dots)$	<b>M1</b>
$= -2 - 5x - 11x^2 - 23x^3$	<b>M1</b>
Valid for $ x  < \frac{1}{2}$	<b>M1</b>

