A-Level Starter Activity

Jonwa Maths

Topic: Expanding $(1+x)^n$

Chapter Reference: Pure 2, Chapter 4

6 minutes

1. Expand $(1 + 4x)^4$ in ascending powers of x, simplifying the coefficients. **(2)**

2a. Expand $(1 + 3x)^7$ in ascending powers of x up to and including the term in x^4, simplifying each coefficient in the expansion. **(2)**

b. Use your series with a suitable value of x to estimate the value of 1.03^7 correct to 5 decimal places. **(2)**

3. Write down the first three terms in the binomial expansion of $(1 + ax)^n$, where n is a positive integer, in ascending powers of x. **(2)**

Solutions

1.

$1 + 4(4x) + 6(4x)^2 + 4(4x)^3 + (4x)^4$	**M1**
$= 1 + 16x + 96x^2 + 256x^3 + 256x^4$	**M1**

2a.

$= 1 + 7(3x) + \frac{7\times6}{2}(3x)^2 + \frac{7\times6\times5}{3\times2}(3x)^3 + \frac{7\times6\times5\times4}{4\times3\times2}(3x)^4$	**M1**
$= 1 + 21x + 189x^2 + 945x^3 + 2835x^4 + \ldots.$	**M1**

2b.

Let $x = 0.01$ $1.03^7 = 1 + 0.21 + 0.0189 + 0.000945 + 0.00002835$	**M1**
$= 1.22987$ (5 d.p)	**M1**

3.

$= 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$	**M1**
$= 1 + anx + \frac{1}{2}a^2n(n-1)x^2 + \ldots$	**M1**

A-Level Starter Activity

Jotwa Maths

Topic: Expanding $(a + bx)^n$

Chapter Reference: Pure 2, Chapter 4

7 minutes

1a. Expand $(8 - 24x)^{\frac{1}{3}}$ in ascending powers of x up to and including the term in x^3 **(3)**

b. State the values of x for which the expansion is valid for. **(1)**

2a. Find the first four terms in the series expansion in ascending powers of x of $\frac{2x-1}{(1+4x)^2}$ **(4)**

b. State the set of values of x for which the expansion is valid. **(1)**

Solutions

1.

$8^{\frac{1}{3}}(1-3x)^{\frac{1}{3}} = 2(1-3x)^{\frac{1}{3}}$	**M1**
$= 2[1 + (\frac{1}{3})(-3x) + \frac{(\frac{1}{3})(-\frac{2}{3})}{2}(-3x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3\times2}(-3x)^3$	**M1**
$= 2 - 2x - 2x^2 - \frac{10}{3}x^3 + \ldots$	**M1**

1b.

$\lvert -3x \rvert < 1$ $\lvert x \rvert < \frac{1}{3}$	**M1**

2a.

$(2x - 1)(1 + 4x)^{-2} = (2x - 1)[1 + (-2)(4x) + \frac{(-2)(-3)}{2}(4x)^2 + \frac{(-2)(-3)(-4)}{3\times2}(4x)^3 + \ldots]$	**M1**
$= (2x - 1)(1 - 8x + 48x^2 - 256x^3 + \ldots)$	**M1**
$= 2x - 16x^2 + 96x^3 - 1 + 8x - 48x^2 + 256x^3 + \ldots$	**M1**
$= -1 + 10x - 64x^2 + 352x^3 + \ldots$	**M1**

2b.

$\lvert 4x \rvert < 1$ $\lvert x \rvert < \frac{1}{4}$	**M1**

A-Level Starter Activity

Jotnwa Maths

Topic: Binomial Expansion using Partial Fractions

Chapter Reference: Pure 2, Chapter 4

7 minutes

1. Express $\frac{x-2}{(1-x)(1-2x)}$ in partial fractions. **(3)**

b. Hence, find the series expansion of $\frac{x-2}{(1-x)(1-2x)}$ in ascending powers of x up to and including the term in x^3 and state the set of values of x for which the expansion is valid. **(7)**

Solutions

1a.

$\frac{x-2}{(1-x)(1-2x)} = \frac{A}{1-x} + \frac{B}{1-2x}$ $x - 2 = A(1 - 2x) + B(1 - x)$	**M1**
Let $x = 1$, $-1 = -A$ $A = 1$	**M1**
Let $x = \frac{1}{2}$ $-\frac{3}{2} = \frac{1}{2}B$ $B = -3$	**M1**
$\frac{x-2}{(1-x)(1-2x)} = \frac{1}{1-x} - \frac{3}{1-2x}$	

1b.

$\frac{1}{1-x} = (1 - x)^{-1} = 1 + (-1)(-x) + \frac{(-1)(-2)}{2}(-x)^2 + \frac{(-1)(-2)(-3)}{3\times2}(-x)^3 + \ldots$	**M1**
$= 1 + x + x^2 + x^3 + \ldots$	**M1**
$\frac{3}{1-2x} = 3(1 - 2x)^{-1} = 3[1 + (-1)(-2x) + \frac{(-1)(-2)}{2}(-2x)^2 + \frac{(-1)(-2)(-3)}{3\times2}(-2x)^3 + \ldots$	**M1**
$= 3 + 6x + 12x^2 + 24x^3 + \ldots$	**M1**
$\frac{x-2}{(1-x)(1-2x)} = (1 + x + x^2 + x^3 + \ldots.) - (3 + 6x + 12x^2 + 24x^3 + \ldots.)$	**M1**
$= -2 - 5x - 11x^2 - 23x^3$	**M1**
Valid for $\lvert x\rvert < \frac{1}{2}$	**M1**