A-Level Starter Activity

Jethwa Maths

Topic: Integrating f(ax + b) Chapter Reference: Pure 2, Chapter 11	8 minutes

1. Integrate $6(1 + 3x)^4$ **(2)**

2. Find $\int e^{2x-3}\,dx$ **(1)**

3. Given that $f'(x) = 2 - \frac{8}{4x-1}$, find an expression for f(x) given that the curve passes through the point $(\frac{1}{2}, 4)$. **(3)**

4. Evaluate, $\int_0^1 \frac{1}{\sqrt[3]{7x+1}}\,dx$ **(3)**

Solutions

1.

$= \frac{1}{3} \times \frac{6}{5}(1+3x)^5 + c$	**M1**
$= \frac{2}{5}(1+3x)^5 + c$	**M1**

2.

$= \frac{1}{2}e^{2x-3} + c$	**M1**

3.

f(x) = $\int 2 - \frac{8}{4x-1}\ dx$ = 2x – 8 x $\frac{1}{4}\ln\lvert 4x-1\rvert + c$ = 2x – 2 ln$\lvert 4x-1\rvert$ + c	**M1**
(0.5, 4) → 4 = 1 + c c = 3	**M1**
f(x) = 2x – 2 ln $\lvert 4x-1\rvert$ + 3	**M1**

4.

$\int_0^1 (7x+1)^{-\frac{1}{3}}\,dx$	**M1**
$= [\frac{1}{7} \times \frac{3}{2}\left(7x+1\right)^{\frac{2}{3}}]\begin{matrix}1\\0\end{matrix}$	**M1**
$= \frac{3}{14}(4-1)$ $= \frac{9}{14}$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Integrating Trig. Terms Chapter Reference: Pure 2, Chapter 11	8 minutes

1. Integrate $3\sin\left(\frac{\pi}{3} - x\right)$ **(1)**

2. Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} sec^2 3x \ dx$ **(2)**

3. Use the identity for cos (A + B) to express $\cos^2 A$ in terms of cos 2A. **(3)**

b. Find $\int cos^2 x \, dx$ **(2)**

4. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\cos 2x}{sin^2 2x} \ dx$ **(4)**

Solutions

1.

$= 3\cos\left(\frac{\pi}{3} - x\right) + c$	**M1**

2.

$= \left[\frac{1}{3}\tan 3x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$	**M1**
$= 0 - \left(-\frac{1}{3}\right) = \frac{1}{3}$	**M1**

3a.

$\cos(A = B) = \cos A \cos B - \sin A \sin B$	**M1**
Let $B = A$, $\cos 2A = \cos^2 A - \sin^2 A$ $\cos 2A = \cos^2 A - (1 - \cos^2 A)$ $\cos 2A = 2\cos^2 A - 1$	**M1**
$\cos^2 A = \frac{1}{2}(1 + \cos 2A)$	**M1**

3b.

$\int cos^2 x\, dx = \int \left(\frac{1}{2} + \frac{1}{2}\cos 2x\right) dx$	**M1**
$= \frac{1}{2}x + \frac{1}{4}\sin 2x + c$	**M1**

4.

$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{1}{\sin 2x} \times \frac{\cos 2x}{\sin 2x}\ dx$	**M1**
$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} cosec\ 2x \cot 2x\ \ dx$	**M1**
$= \left[-\frac{1}{2}\ cosec\ 2x\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$	**M1**
$= -\frac{1}{2} - \left(-\frac{1}{\sqrt{3}}\right)$ $= \frac{1}{3}\sqrt{3} - \frac{1}{2}$	**M1**

Topic: Integration by Parts

Chapter Reference: Pure 2, Chapter 11

9 minutes

1. Find the value of $\int x\, e^{x}\, dx$ **(3)**

2. Evaluate $\int_{0}^{\frac{\pi}{6}} x \cos x\, dx$ **(4)**

3. Find $\int \ln 2x \; dx$ **(4)**

Solutions

1.

$u = x$ $\frac{du}{dx} = 1$ $\frac{dv}{dx} = e^x$ $v = e^x$	**M1**
$\int x\, e^x\, dx = xe^x - \int e^x dx$	**M1**
$= xe^x - e^x + c$ $= e^x(x - 1) + c$	**m1**

2.

$u = x$ $\frac{du}{dx} = 1$ $\frac{dv}{dx} = \cos x$ $v = \sin x$	**M1**
$\int_0^{\frac{\pi}{6}} x \cos x\, dx = [x \sin x + \cos x]_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}} \sin x\, dx$	**M1**
$= [x \sin x + \cos x]_0^{\frac{\pi}{6}}$ $= (\frac{\pi}{12} + \frac{\sqrt{3}}{2}) - (0 + 1)$	**M1**
$= \frac{1}{12}(\pi + 6\sqrt{3} - 12)$	**M1**

3.

$u = \ln 2x$ $\frac{du}{dx} = \frac{1}{x}$ $\frac{dv}{dx} = 1$ $v = x$	**M1**
$\int \ln 2x\, dx = x \ln 2x - \int \frac{1}{x} \times x\, dx$	**M1**
$= x \ln 2x - \int 1\ dx$ $= x \ln 2x - x + c$	**M1**
$= x(\ln 2x - 1) + c$	**M1**

Topic: Integration by Substitution

Chapter Reference: Pure 2, Chapter 11

10 minutes

1. Using u = sec x, find $\int sec^3 x \tan dx$ **(3)**

2. Using the substitution $u = x^2 + 3$, $\int_0^1 2x(x^2+3)^2\, dx$ **(4)**

3. Using the substitution $u = 1 + e^{2x}$ to find the $\int_0^1 e^{2x}(1+e^{2x})^3\, dx$ **(5)**

Solutions

1.

$u = \sec x$ $\frac{du}{dx} = \sec x \tan x$	**M1**
$\int sec^3 x \tan x \, dx = \int u^2 \, du$	**M1**
$= \frac{1}{3}u^3 + c$ $= \frac{1}{3}\sec^3 x + c$	**M1**

2.

$u = x^2 + 3$ $\frac{du}{dx} = 2x$	**M1**
$\int_0^1 2x(x^2+3)^2 \, dx = \int_3^4 u^2 \times \frac{du}{dx} dx$ $= \int_3^4 u^2 \, du$	**M1**
$\int_3^4 u^2 \, du = [\frac{1}{3}u^3]_3^4$	**M1**
$= \frac{64}{3} - 9$ $= 12\frac{1}{3}$	**M1**

3.

$u = 1 + e^{2x}$ $\frac{du}{dx} = 2e^{2x}$	**M1**
$x = 0$, u = 2 $x = 1$, u = 1 + e^2	**M1**
$= \int_2^{1+e^2} \frac{1}{2}u^3 du$	**M1**
$= [\frac{1}{8}u^4]_2^{1+e^2}$ $= \frac{1}{8}[(1+e^2)^4 - 16]$	**M1**
$= \frac{1}{8}(1+e^2)^4 - 2$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Integration Using Partial Fractions

Chapter Reference: Pure 2, Chapter 11

8 minutes

1a. Express $\frac{3x+5}{(x+1)(x+3)}$ in partial fractions. **(3)**

b. Hence, find $\int \frac{3x+5}{(x+1)(x+3)}\,dx$ **(2)**

2a. Express $\frac{x^2-x-4}{(x+2)(x+1)^2}$ in partial fractions. **(4)**

b. Hence, find $\int \frac{x^2-x-4}{(x+2)(x+1)^2}\,dx$ **(2)**

Solutions

1a.

$\frac{3x+5}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}$ $3x + 5 = A(x + 3) + B(x + 1)$	**M1**
Let $x = -3$ $2 = 2A$ $A = 1$	**M1**
Let $x = -1$ $-4 = -2B$ $B = 2$	**M1**
$\frac{3x+5}{(x+1)(x+3)} = \frac{1}{x+1} + \frac{2}{x+3}$	

1b.

$\int \frac{3x+5}{(x+1)(x+3)}\, dx = \int \frac{1}{x+1} + \frac{2}{x+3}\, dx$	**M1**
$= \ln\|x + 1\| + 2\ln\|x + 3\| + c$	**M1**

3a.

$\frac{x^2-x-4}{(x+2)(x+1)^2} = \frac{A}{x+2} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$ $x^2 - x - 4 = A(x + 1)^2 + B(x + 2)(x + 1) + C(x + 2)$	**M1**
Let $x = -2$, $A = 2$	**M1**
Let $x = -1$, $C = -2$	**M1**
Coefficients of x^2: $1 = A + B$ $B = -1$	**M1**
$\frac{x^2-x-4}{(x+2)(x+1)^2} = \frac{2}{x+2} - \frac{1}{x+1} - \frac{2}{(x+1)^2}$	

3b.

$= \int \frac{2}{x+2} - \frac{1}{x+1} - \frac{2}{(x+1)^2}\, dx$	**M1**
$= 2\ln\|x + 2\| - \ln\|x + 1\| + 2(x + 1)^{-1} + c$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Areas Under Graphs Chapter Reference: Pure 2, Chapter 11	**8 minutes**

1. $f(x) = x^2$ and $g(x) = 3x - x^2$

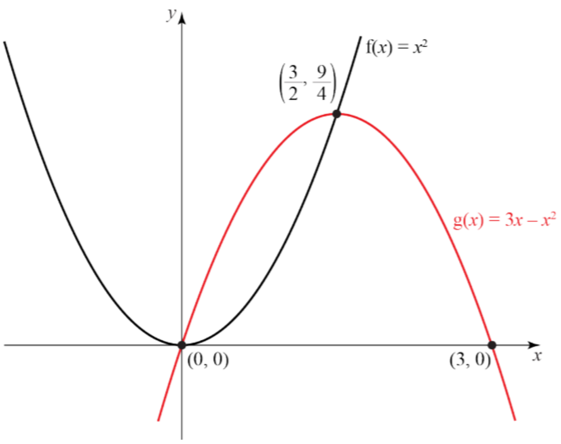
a. On the same set of axes, sketch the graphs of $y = \mathrm{f}(x)$ and $y = g(x)$, and find the coordinates of any points of intersection of the two curves. **(5)**

b. Find the area pf the finite region bounded by the two curves. **(7)**

Solutions

1.

Shape $f(x)$ **M1** Shape $g(x)$ **M1**	



$f(x) = g(x)$ $x^2 = 3x - x^2$ $2x^2 - 3x = 0$ $x(2x - 3) = -0$	**M1**
$x = 0$ $y = 0$	**M1**
$x = \frac{3}{2}$ $y = \frac{9}{4}$	**M1**
Points of intersection, (0, 0) and $(\frac{3}{2}, \frac{9}{4})$	

2.

Area under f(x) between 0 and under f(x) between 0 and $\frac{3}{2}$ $\int_0^{\frac{3}{2}} x^2 \; dx$	**M1**
$= [\frac{x^3}{3}]_0^{\frac{3}{2}}$	**M1**
$= \frac{27}{24} = \frac{9}{8}$	**M1**
Area under g(x) between 0 and $\frac{3}{2}$ $\int_0^{\frac{3}{2}} 3x - x^2 \; dx$	**M1**
$= [\frac{3x^2}{2} - \frac{x^3}{3}]_0^{\frac{3}{2}}$	**M1**
$= \frac{27}{8} - \frac{27}{24} = \frac{9}{4}$	**M1**
Area between two curves, $\frac{9}{4} - \frac{9}{8} = \frac{9}{8}$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Trapezium Rule Chapter Reference: Pure 2, Chapter 11	**5 minutes**

1. The diagram shows the finite region, *R*, which is bounded by the curve $y = 2 - 3x^{-\frac{1}{2}}$, the *x*-axis and the lines $x = 3$ and $x = 7$.

a. Use the trapezium rule with 5 intervals of equal width to estimate the area of *R*. **(4)**

b. Use integration to find the exact area of *R*. **(3)**

Solutions

1a.

x	3	3.8	4.6	5.4	6.2	7	**M1**
$2 - 3x^{-1/2}$	0.2679	0.4610	0.6012	0.7090	0.7952	0.8661	**M1**
Area $= \frac{1}{2} \times 0.8 \times [0.2670 + 0.8661 + 2(0.4610 + 0.6012 + 0.7090 + 0.7952)]$							**M1**
= 2.51							**M1**

1b.

$\int_3^7 (2 - 3x^{-\frac{1}{2}})\,dx$	**M1**
$[2x - 6x^{\frac{1}{2}}]_3^7$	**M1**
$= (14 - 6\sqrt{7}) - (6 - 6\sqrt{3}) = 8 + 6(\sqrt{3} - \sqrt{7})$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Differential Equations

Chapter Reference: Pure 2, Chapter 11

8 minutes

1. Find the general solution of $\frac{dy}{dx} = 2y + 3$ **(3)**

2. Find the general solution of $(y - 2)\frac{dy}{dx} = xy(y - 1)$ **(6)**

3. Find the general solution of $\frac{dy}{dx} = y^2 \ln x$ **(4)**

Solutions

1.

$\int \frac{1}{2y+3}\,dy = \int 1\ dx$	**M1**
$\frac{1}{2}\ln\lvert 2y+3\rvert = x + c$	**M1**
$y = \frac{1}{2}(ke^{2x} - 3)$	**M1**

2.

$\int \frac{y-3}{y(y-1)}\,dy = \int x\ dx$	**M1**
$\frac{y-3}{y(y-1)} = \frac{A}{y} + \frac{B}{y-1}$ $y - 3 = A(y - 1) + By$	**M1**
$y = 0$ $A = 3$	**M1**
$y = 1$ $B = -2$	**M1**
$\int \frac{3}{y} - \frac{2}{y-1}\,dy = \int x\ dx$	**M1**
$3\ \ln\lvert y\rvert - 2\ \ln\lvert y-1\rvert = \frac{1}{2}x^2 + c$	**M1**

3.

$\int y^{-2}\,dy = \int \ln x\,dx$	**M1**
$u = \ln x$ $\frac{du}{dx} = \frac{1}{x}$ $\frac{dv}{dx} = 1$ $v = x$	**M1**
$-y^{-1} = x \ln x - \int 1\ \mathrm{dx}$ $-y^{-1} = x \ln x - x + c$	**M1**
$y = \frac{1}{x - x\ln x + k}$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Modelling with Differential Equations Chapter Reference: Pure 2, Chapter 11	7 minutes

1. A quantity has the value P at time t seconds and is decreasing at a rate proportional to $\sqrt{P}$.
a. By forming and solving a suitable differential equation, show that,

$$P = (a - bt)^2$$

Where a and b are constants. **(3)**
Given that when $t = 0$, $P = 400$,
b. Find the value of a. **(1)**
Given also that when $t = 30$, $P = 100$.
c. Find the value of P when $t = 50$. **(3)**

Solutions

1a.

$-\frac{dP}{dt} = k\sqrt{P}$ $\int P^{-\frac{1}{2}}\,dP = \int -k\,dt$	**M1**
$2P^{\frac{1}{2}} = -kt + c$ $\sqrt{P} = \frac{1}{2}c - \frac{1}{2}kt = a - bt$	**M1**
Therefore, $P = (a - bt)^2$	**M1**

1b.

$t = 0$, $P = 400$ $\sqrt{400} = a - 0$ $a = 20$	**M1**

1c.

$t = 30$ $P = 100$ $\sqrt{100} = 20 - 30b$ $b = \frac{1}{3}$	**M1**
Therefore, $P = (20 - \frac{1}{3}t)^2$	**M1**
$t = 50$, $P = (20 - \frac{50}{3})^2$ $= 11\frac{1}{9}$	**M1**