A-Level Starter Activity

Jotwa Maths

Topic: Iteration

Chapter Reference: Pure 2, Chapter 10

6 minutes

1. Use the iterative formula $x_{n+1} = \sqrt{3 + \frac{5.5}{x_n}}$ and $x_0 = 0.8$, to find the values of x_1, x_2 and x_3 to 4 decimal places **(2)**

2. Rearrange $e^x - 8x + 5 = 0$ to make an iterative formula. Use the formula and $x_0 = 2$ to find x_1, x_2 and x_3 to 4 decimal places. **(3)**

3a. For the equation for $\frac{2}{x} + \cos x - 3 = 0$, find the iterative formula in the form $x_{n+1} = \frac{a}{b - \cos x_n}$, where a and b are constants. **(3)**

b. Using $x_0 = 0.8$, find the value of x_3 to 4 decimal places. **(1)**

Solutions

1.

$x_1 = 2.397916\ldots$ $x_2 = 2.300795\ldots$	**M1**
$x_3 = 2.321740\ldots = 2.322$ (3 d.p)	**M1**

2.

$e^x - 8x + 5 = 0$ $e^x = 8x - 5$	**M1**
$x = \ln(8x - 5)$ $x_{n+1} = \ln(8x_n - 5)$	**M1**
$x_1 = 2.944439\ldots = 2.9444$ (to 4 d.p) $x_2 = 2.920767\ldots = 2.9208$ (to 4 d.p) $x_3 = 2.910508\ldots = 2.9105$ (to 4 d.p)	**M1**

3a.

$\frac{2}{x} + \cos x - 3 = 0$ $\frac{2}{x} = 3 - \cos x$	**M1**
$2 = x(3 - \cos x)$ $x = \frac{2}{3-\cos x}$	**M1**
$x_{n+1} = \frac{2}{3-2\cos x_n}$ $a = 2$, $b = 3$	**M1**

3b.

$x_1 = 0.868322\ldots$ $x_2 = 0.849657\ldots$ $x_3 = 0.854789\ldots = 0.855$ (3 d.p)	**M1**

A-Level Starter Activity

Jotnwa Maths

Topic: Locating Roots

Chapter Reference: Pure 2, Chapter 10

5 minutes

1. For the function, $f(x) = 2e^x + x + 5$, show that there is a root between $x = -6$ and $x = -5$ when $f(x) = 0$ **(2)**

2. Show that there is a root in the interval $x = -6.5$ and $x = -6$, when $\sin 4x = 7e^x$ **(3)**

3. Given that $x = \frac{1}{2}$ is a root of the equation, $2x^3 - 9x^2 + kx - 13 = 0$. Find the value of k. **(2)**

Solutions

1.

f(-6) = -0.995 f(-5) = 0.0135	**M1**
As there is a change in sign and f(x) is continuous, there is a root in the interval.	**M1**

2.

Let f(x) = sin $4x$ – $7e^x$	**M1**
f(-6.5) = -0.773 f(-6) = 0.888	**M1**
As there is a change in sign and f(x) is continuous, there is a root in the interval.	**M1**

3.

$2(0.5)^3 - 9(0.5)^2 + k(0.5) - 13 = 0$ $-15 + \frac{k}{2} = 0$	**M1**
$k = 30$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Modelling in Numerical Methods

Chapter Reference: Pure 2, Chapter 10

10 minutes

1. The depth of a stream is modelled by the function,

$$d(x) = e^{-0.6x}(x^2 - 3x),\ 0 \leq x \leq 3$$

where x is the distance in metres from the left bank of the stream and $d(x)$ is the depth of the stream in metres. The diagram shows a sketch of $y = d(x)$.

a. Explain the condition $0 \leq x \leq 3$ (3)

b. Show that $d'(x) = -\frac{1}{5}e^{-0.6x}(ax^2 + bx + c)$, where a, b and c, are constants to be found. (3)

c. Dhow that $d'(x)$ can be written as $x = \frac{3x^2+15}{19}$ (2)

d. Let $x_0 = 1$. Determine whether the iteration converges or diverges. (2)

e. Find the maximum depth of the river in metres to 2 decimal places. **(2)**

Solutions

1a.

$d(x) = e^{-0.6x}(x^2 - 3x)$ $d(x) = 0$	**M1**
$x^2 - 3x = 0$ $x(x - 3) = 0$ $x = 0$ $x = 3$	**M1**
The stream is 3 metres side so the function is only valid for $0 \leq x \leq 3$	**M1**

1b.

$d'(x) = e^{-0.6x}(2x - 3) - \frac{3}{5}e^{-0.6x}(x^2 - 3x)$	**M1**
$= 2xe^{-0.6x} - 3e^{-0.6x} - \frac{3}{5}x^2e^{-0.6x} + \frac{9}{5}xe^{-0.6x}$ $= e^{-0.6x}(-\frac{3}{5}x^2 + \frac{19}{5}x - \frac{15}{5})$	**M1**
$d'(x) = -\frac{1}{5}e^{-0.6x}(3x^2 - 19x + 15)$ $a = 3$, $b = -19$, $c = 16$	**M1**

1c.

$3x^2 - 19x + 15 = 0$ $19x = 3x^2 + 15$	**M1**
$x = \frac{3x^2+15}{19}$	**M1**

1d.

For $x_0 = 1$, $x_1 = 0.94736\ldots$ $x_2 = 0.93118\ldots$ $x_3 = 0.92638\ldots$ $x_4 = 0.9249\ldots$ $x_5 = 0.9245\ldots$ $x_6 = 0.9244\ldots$	**M1**
Iteration converges to 0.924 after 6 iterations.	**M1**

1e.

$d(0.924) = e^{-0.6 \times 0.924}(0.924^2 - 3 \times 0.924)$ $= -1.1018$	**M1**
The maximum depth of the river is 1.10m, correct to 2 d.p.	**M1**

A-Level Starter Activity

Jotwa Maths

Topic: Newton-Raphson Method

Chapter Reference: Pure 2, Chapter 10

6 minutes

1. $f(x) = x^3 - \frac{7}{x} + 2, x > 0$

a. Show that $f(x) = 0$ has a root α between 1.4 and 1.5 **(2)**

b. Starting with the interval [1.4, 1.5], use interval bisection twice to find an interval of width 0.025 that contains α **(3)**

c. Taking 1.45 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x) = x^3 - \frac{7}{x} + 2$ to obtain a second approximation to α, giving your answer to 3 decimal places. **(5)**

Solutions

1a.

f(1.4) = -0.256 f(1.5) = 0.783…	**M1**
As there is a change in sign, there must be a root [1.4, 1.5] as the curve is continuous.	**M1**

1b.

$x_1 = 1.45$ f(1.45) = 0.22103 Therefore, root lies in [1.4, 1.45] $x_2 = \frac{1.4+1.45}{2} = 1.425$	**M1**
f(1.425) = -0.0186	**M1**
Therefore root α lies in [1.425, 1.45]	**M1**

1c.

$x_1 = 1.45$ $f(x) = x^3 - 7x^{-1} + 2$ $f'(x) = 3x^2 + 7x^{-2}$	**M1**
$x_2 = 1.45 - \frac{(1.45)^3 - \frac{7}{1.45} + 2}{3(1.45)^2 + \frac{7}{1.45^2}} = 1.42706….$	**M1**
$x_2 = 1.427$ (to 3 d.p)	**M1**