A-Level Starter Activity

Jethwa Maths

Topic: Arithmetic Sequences and Series Chapter Reference: Pure 2, Chapter 3	**8 minutes**

1. The arithmetic series has common difference -11 and tenth term 101.

a. Find the first term of the series. **(2)**

b. Find the sum of the first 30 terms of the series. **(2)**

2. The sum of the first six terms of an arithmetic series is 213 and the sum of the first ten terms of the series is 295.

a. Find the first term and common difference of the series. **(4)**

b. Find the number of positive terms in the series. **(2)**

c. Hence find the maximum value of S_n, the sum of the first n terms of the series. **(2)**

3. The fifth, sixth and seventh term of an arithmetic series are $(5 - t)$, $2t$ and $(6t - 3)$ respectively. Find the value of the constant t. **(2)**

Solutions

1a.

$a + (9 \times -11) = 101$	**M1**
$a = 200$	**M1**

1b.

$S_{30} = \frac{30}{2}[400 + (29 \times -11)]$	**M1**
$= 1215$	**M1**

2a.

$\frac{6}{2}(2a + 5d) = 213$ $2a + 5d = 71$	**M1**
$\frac{10}{2}(2a + 9d) = 295$ $2a + 9d = 59$	**M1**
$4d = -12$ $d = -3$	**M1**
$a = 43$	**M1**

2b.

$43 - 3(n - 1) > 0$ $n < \frac{46}{3}$	**M1**
Therefore 15 positive terms	**M1**

2c.

max S_n when $n = 15$	**M1**
$S_{15} = \frac{15}{2}[86 + (14 \times -3)] = 330$	**M1**

3.

$2t - (5 - t) = (6t - 3) - 2t$ $3t - 5 = 4t - 3$	**M1**
$t = -2$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Geometric Sequences and Series

Chapter Reference: Pure 2, Chapter 3

8 minutes

1. Find the common ratio and the eight term of, 3 + 9 + 27 + 81 + …. **(2)**

2. The second and third term of a geometric series are 2 and 10 respectively.

a. Find the common ratio of the series **(1)**

b. Find the first term of the series **(1)**

c. Find the sum of the first eight terms of the series. **(2)**

3. All the terms of a geometric series are positive. The sum of the first and second terms of the series is 10.8 and the sum of the third and fourth terms of the series is 43.2.

a. Find the first term and common ratio of the series. **(5)**

b. Find the sum of the first 16 terms of the series. **(2)**

Solutions

1.

$r = 3$	**M1**
$u_8 = 3 \text{ x } 3^7 = 6561$	**M1**

2a.

$r = 10 \div 2 = 5$	**M1**

2b.

$a \times 5 = 2$ $a = 0.4$	**M1**

2c.

$S_8 = \frac{0.4(5^8-1)}{5-1}$	**M1**
$= 39\ 062.4$	**M1**

3a.

$a + ar = a(1 + r) = 10.8$	**M1**
$ar^2 + ar^3 = ar^2(1 + r) = 43.2$	**M1**
$r^2 = 43.2 \div 10.8 = 4$	**M1**
As all the terms are positive, $r = 2$	**M1**
$a = 10.8 \div 3 = 3.6$	**M1**

3b.

$S_{16} = \frac{3.6(2^{16}-1)}{2-1}$	**M1**
$= 235\ 926$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Sum to Infinity Chapter Reference: Pure 2, Chapter 3	8 minutes

1. The sum of the first and third terms of a geometric series is 150. The sum of the second and fourth terms of the series in -75. Find the sum to infinity of the series. **(5)**

2. A geometric series has common ratio r and the nth term of the series is denoted by u_n. Given that $u_1 = 64$ and that $u_3 - u_2 = 20$,

a. Show that $16r^2 - 16r - 5 = 0$ **(2)**

b. Find the two possible values of r **(3)**

c. Find the fourth term of the series corresponding to each possible value of r. **(2)**

d. Taking the values of r such that the series converges, find the sum to infinity of the series. **(3)**

<u>Solutions</u>

1.

$a + ar^2 = 1(1 + r^2) = 150$ $ar + ar^3 = ar(1 + r^2) = -75$	**M1**
$r = -75 \div 150 = -\frac{1}{2}$	**M1**
$a = 150 \div \frac{5}{4} = 120$	**M1**
$S_\infty = \frac{120}{1-(-\frac{1}{2})}$	**M1**
$S_\infty = 80$	**M1**

2a.

$a = 64$ $ar^2 - ar = 20$ $64r^2 - 64r = 20$	**M1**
$16r^2 - 16r - 5 = 0$	**M1**

2b.

$(4r + 1)(4r - 5) = 0$	**M1**
$r = -\frac{1}{4}$	**M1**
$r = \frac{5}{4}$	**M1**

2c.

$r = -\frac{1}{4}$ $u_4 = 64 \text{ x } (-\frac{1}{4})^3 = -1$	**M1**
$r = \frac{5}{4}$ $u_4 = 63 \text{ x } (\frac{5}{4})^3 = 125$	**M1**

2d.

$r = -\frac{1}{4}$	**M1**
$S_\infty = \frac{64}{1-(-\frac{1}{2})}$	**M1**
$S_\infty = 51\frac{1}{5}$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Sigma Notation

Chapter Reference: Pure 2, Chapter 3

8 minutes

1. Evaluate $\sum_{r=1}^{6} 8^{r+1}$ **(3)**

2. Evaluate $\sum_{r=10}^{50} \left(\frac{r+2}{4}\right)$ **(3)**

3a. Evaluate $\sum_{r=3}^{10} 3^r$ **(2)**

b. Show that $\sum_{r=1}^{15}(2^r - 12r) = 64094$ **(5)**

Solutions

1.

GP: $a = 64$ $r = 8$ $n = 6$	**M1**
$S_6 = \frac{64(8^6-1)}{8-1}$	**M1**
$S_6 = 2396736$	**M1**

2.

AP: $a = 3$ $l = 13$ $n = 41$	**M1**
$S_{41} = \frac{41}{2}(3 + 13)$	**M1**
$S_{41} = 328$	**M1**

3a.

$\sum_{r=3}^{10} 3^r$ $a = 27$ $r = 3$	**M1**
$S_8 = \frac{27(3^8-1)}{3-1} = 88560$	**M1**

3b.

$\sum_{r=1}^{15}(2^r)$ G.P, $a = 2$ $r = 2$	**M1**
$S_{15} = \frac{2(2^{15}-1)}{2-1} = 65534$	**M1**
$\sum_{r=1}^{15}(12r)$ A.P, $a = 12$ $d = 12$	**M1**
$S_{15} = \frac{15}{2}[24 + (14 \times 12)] = 1440$	**M1**
$\sum_{r=1}^{15}(2^r - 12r) = 65534 - 1440 = 64\,094$	**M1**

Jethwa Maths

Topic: Recurrence Relations Chapter Reference: Pure 2, Chapter 3	8 minutes

1. A sequence can be described by the recurrence formula,

$$u_{n+1} = 2u_n + 1, n \geq 1, u_1 = 1$$

Find u_2 and u_3 **(2)**

2. A sequence is defined by the recurrence relation

$$u_{n+1} = 0.6u_n + 4, u_0 = 7.$$

Calculate the smallest value of n for which $u_n > 9.7$. **(3)**

3. The deer population in a forest is estimated to drop by 7.3% each year. Each year, 20 deer are introduced to the forest. The initial deer population is 200.

a. How many deer will there be in the forest after 3 years. **(2)**

b. What is the long-term effect on the population. **(3)**

Solutions

1.

$u_{n+1} = 2u_n + 1$, $n \geq 1$, $u_1 = 1$ $u_2 = 2(1) + 1 = 4$	**M1**
$u_3 = 2(4) + 1 = 9$	**M1**

2.

$u_0 = 7$	**M1**
$u_1 = 0.6 \text{ x } 7 + 4 = 8.2$	
$u_2 = 0.6 \text{ x } 8.2 + 4 = 8.92$	
$u_3 = 0.6 \text{ x } 8.92 + 4 = 9.352$	**M1**
$u_4 = 0.6 \text{ x } 0.9352 + 4 = 0.96112$	
$u_5 = 0.6 \text{ x } 0.96112 + 4 = 9.76672$	
The smallest value of n for which $u_n > 9.7$ is 5	**M1**

3a.

$u_0 = 200$ $u_1 = 0.927 \text{ x } 200 + 20 = 205.4$ $u_2 = 0.927 \text{ x } 205.4 + 20 = 210.4058$ $u_3 = 0.927 \text{ x } 210.4058 + 20 = 215.0461$	**M1**
There are 215 deer living in the forest after 3 years.	**M1**

3b.

$l = \frac{b}{l-a}$ where $a = 0.927$ and $b = 0.20$	**M1**
$= \frac{20}{1-0.927}$ $= 273.97$ (to 2 d.p)	**M1**
Therefore, the number will settle around 273.	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Modelling with Series

Chapter Reference: Pure 2, Chapter 3

8 minutes

1. Tom's parents decide to pay him an allowance each month beginning on his 12th birthday. The allowance is to be £40 for each of the first three months, £42 for each of the next three months and so on, increasing by £2 per month after each three-month period.

a. Find the total amount that Tom will receive in allowances before his 14th birthday. **(3)**

b. Show that the total amount, in pounds, that Tom will received in allowances in the n years after his 12th birthday, where n is a positive integer, is given by $12n(4n + 39)$. **(4)**

2. Ginny opens a savings account and decide to pay £200 into the account at the start of each month. At the end of each month, interest of 0.5% is paid into the account.

a. Find, to the nearest penny, the interest paid into the account at the end of the third month. **(2)**

b. Show that the total interest paid into the account over the first 12 months is £79.45 to the nearest penny. **(5)**

Solutions

1a.

2 years = 8 x 3 months Total = 3 x S_8 AP: $a = 40$ $d = 2$	**M1**
Total = 3 x $\frac{8}{2}[80 + (7 \times 2)$ = 3 x 376	**M1**
= £1128	**M1**

1b.

n years = $4n$ x 3 months	**M1**
Total = 3 x S_{4n} = 3 x $\frac{4n}{2}\{80 + [(4n - 1) \times 2]\}$	**M1**
= $6n(80 + 8n - 2)$	**M1**
= $12n(4n + 39)$	**M1**

2a.

Amount in account after 3rd payment in = 200 + (1.005 x 200) + (1.005^2 x 200) = 603.005	**M1**
Interest paid at end of 3rd month, = 0.005 x 603.005 = £3.02 (nearest penny)	**M1**

2b.

Amount paid in = 12 x 200 = £2400	**M1**
Amount in account after 12 months, = $200(1.005 + 1.005^2 + \ldots + 1.005^{12})$ = 200 x S_{12}	**M1**
G.P: $a = 1.005$ $r = 1.005$	**M1**
= 200 x $\frac{1.005(1.005^{12}-1)}{1.005-1} = 2479.45$	**M1**
Total interest = 2479.45 – 2400 = £79.45	**M1**