A-Level Starter Activity

Jethwa Maths

Topic: Modulus Function

Chapter Reference: Pure 2, Chapter 2

7 minutes

1a. Sketch on the same set of axes the graph of $y = |x|$ and $y = |2x - 3|$ **(3)**

b. Hence, or otherwise, solve the equation, $|x| = |2x - 3|$ **(2)**

3. The function f is defined by $f : x \rightarrow |3x - a|$, where a is a positive constant.

a. Find $ff(-2a)$ **(1)**

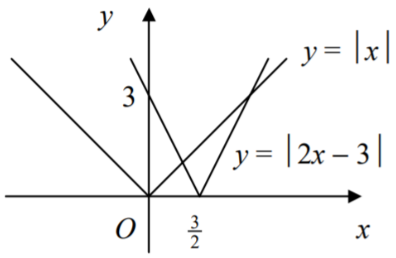
b. Sketch the graph $y = f(x)$, showing the coordinates of any points where the graph meets the coordinate axes. **(3)**

c. Solve the equation $f(x) = x$, giving tour answers in terms of a **(3)**

Solutions

1a.

Sketch $y = |x|$ **M1**
Sketch $y = |2x - 3|$ **M1**
One 2 intersections **M1**



1b.

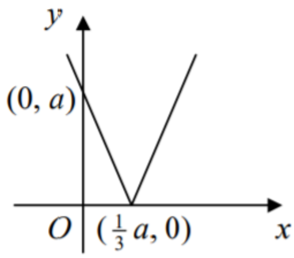
$x = 2x - 3$ $x = 3$	**M1**
$x = -(2x - 3)$ $x = 1$	**M1**
$x = 1, 3$	

2a.

$f(-2a) = \|-7a\| = 7a$ $ff(-2a) = f(7a) = \|20a\| = 20a$	**M1**

2a.

Shape **M1**
$(0, a)$ **M1**
$(\frac{1}{3}a, 0)$ **M1**



2c.

$3x - a = x$ $x = \frac{1}{2}a$	**M1**
$-(3x - a) = x$ $x = \frac{1}{4}a$	**M1**
$x = \frac{1}{4}a, \frac{1}{2}a$	**M1**

A-Level Starter Activity

Jethwa Maths

Topic: Composite Functions Chapter Reference: Pure 2, Chapter 2	8 minutes

1. Given that $f : x \rightarrow 5x - 3$ and $g : x \rightarrow 3x^2 + 1$,

a. Find $fg(x)$ **(2)**

b. Solve $fg(x) = 32$ **(2)**

2. Given $f : x \rightarrow \sqrt{x+4}$, $g : x \rightarrow e^{1 + 2x}$ and $h : x \rightarrow \frac{x+1}{3}$

a. Find $fh(x) = 3$ **(3)**

b. $ghh(x) = \frac{1}{2}$ **(4)**

Solutions

1a.

$fg(x) = f(3x^2 + 1)$	**M1**
$= 5(3x^2 + 1) - 3$ $= 15x^2 + 2$	**M1**

1b.

$15x^2 + 2 = 32$ $x^2 = 32$	**M1**
$x = \pm\sqrt{2}$	**M1**

2a.

$fh(x) = f(\frac{x+1}{3})$ $= \sqrt{\frac{x+1}{3} + 4}$ $= \sqrt{\frac{x+13}{3}}$	**M1**
$\sqrt{\frac{x+13}{3}} = 3$ $\frac{x+13}{3} = 9$	**M1**
$x + 13 = 27$ $x = 14$	**M1**

2b.

$ghh(x) = g\left(\frac{x+4}{9}\right)$ $= e^{1+\frac{2(x+4)}{9}}$ $= e^{\frac{2x+17}{9}}$	**M1**
$e^{\frac{2x+17}{9}} = \frac{1}{2}$	**M1**
$\frac{2x+17}{9} = \ln\frac{1}{2}$ $x = \frac{1}{2}(9\ln\frac{1}{2} - 17)$	**M1**
$x = -11.6$ (3 s.f)	**M1**

A-Level Starter Activity	Topic: Inverse Functions Chapter Reference: Pure 2, Chapter 2	8 minutes

1. For the function $f(x) = \frac{1}{x-5}$, find $f^{-1}(x)$ and state its domain. **(3)**

2. The function f is defined by $f : x \rightarrow 4 - 2x$.

a. Sketch $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes. **(2)**

b. Find the coordinates of the point where the lines $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ intersect. **(3)**

3. The functions $f : x \rightarrow 5x + 2$ and $g : x \rightarrow \frac{1}{x}$

a. Find $(fg)^{-1}$ **(3)**

b. Solve the equation $f^{-1}(x) = fg(x)$ giving your answers correct to 2 decimal places. **(3)**

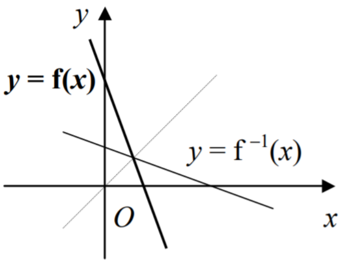
Solutions

1.

$y = \frac{1}{x-5}$ $x = \frac{1}{y-5}$	**M1**
$y = \frac{1}{x} + 5$ $f^{-1}(x) = \frac{1}{x} + 5$	**M1**
Domain, x is any real number, $x \neq 0$	**M1**

2a.

f(*x*) **M1**
$f^{-1}(x)$ **M1**



2b.

$4 - 2x = x$	**M1**
$x = \frac{4}{3}$	**M1**
Therefore, point of intersection is $(\frac{4}{3}, \frac{4}{3})$	**M1**

3a.

$y = \frac{5}{x} + 2$ $x = \frac{5}{y} + 2$	**M1**
$y = \frac{5}{x-2}$	**M1**
$(fg)^{-1} : x \rightarrow \frac{5}{x-2}$	**M1**

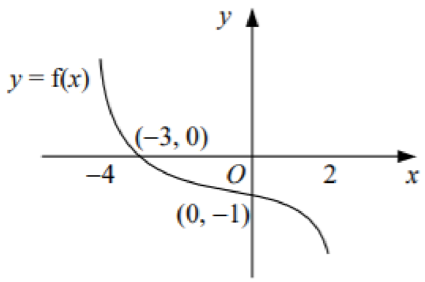
3b.

$\frac{x-2}{5} = \frac{5}{x} + 2$ $x(x-2) = 25 + 10x$	**M1**
$x^2 - 12x - 25 = 0$	**M1**
$x = -1.81$ $x = 13.81$	**M1**

A-Level Starter Activity

Topic: Modulus Function and Graphs Chapter Reference: Pure 2, Chapter 2	7 minutes

1. The diagram shows the curve $y = f(x)$. The domain of f is $-4 < x < 2$ and the curve intersects the coordinate aces at the points (-3, 0) and (0, -1).



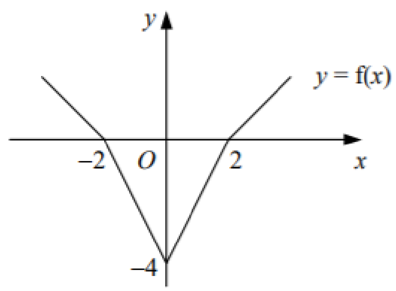
a. Explain how the graph shows that f is one-to-one **(1)**

b. Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the graphs of

i. $y = |f(x)|$ **(3)**

ii. $y = f^{-1}(x)$ **(3)**

2. The diagram shows the graph of $y = f(x)$ which meets the coordinate axes at the points (-2, 0), (0, -4) and (2, 0). Showing the coordinates of any points of intersection with the coordinate axes, sketch on diagrams the graph of,



a. $y = \frac{1}{2}|f(x)|$ **(4)**

b. $y = 4 + f(x + 2)$ **(3)**

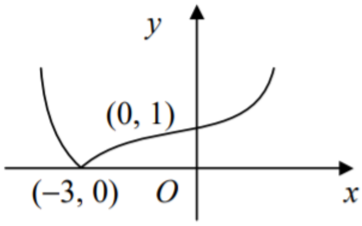
<u>**Solutions**</u>

1a.

Each value of $f(x)$ corresponds to a unique value of x	**M1**

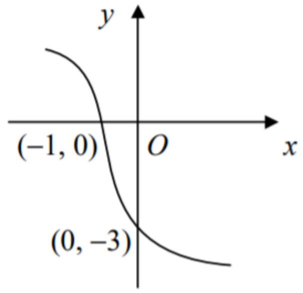
1bi.

Shape **M1**
(0, 1) **M1**
(-3, 0) **M1**



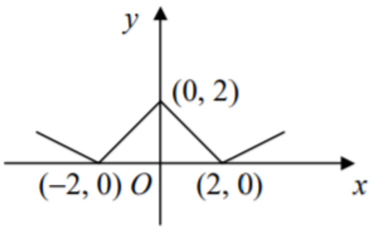
1bii.

Shape **M1**
(-1, 0) **M1**
(0, -3) **M1**



2a.

Shape **M1**
(-2, 0) **M1**
(0, 2) **M1**
(2, 0) **M1**



2b.

Shape **M1**
(-2, 0) **M1**
(0, 4) **M1**