Further Maths A-Level Starter Activity

Topic: Imaginary and Complex Numbers

Chapter Reference: Core Pure 1, Chapter 1

8 minutes

1. Write $\sqrt{-49}$ in terms of i. **(1)**

2. Express each of the following in terms $a + bi$,

a) $(3 + 5i) + (2 + 3i)$, **(1)**

b) $\frac{10-8i}{2}$. **(1)**

3. Solve the equation $z^2 + 16 = 0$. **(2)**

4. The solutions to the quadratic equation,

$$z^2 - 10z + 28 = 0,$$

are z_1 and z_2.

Find z_1 and z_2, giving your answers in the form $p \pm i\sqrt{q}$, where p and q are integers. **(3)**

Solutions

1.

$\sqrt{-49} = \sqrt{-49}\sqrt{-1} = 7i$	**M1**

2.

a) $(3 + 5i) + (2 + 3i) = 5 + 8i$	**M1**
b) $\frac{10-8i}{2} = 5 - 4i$	**M1**

3.

$z^2 = -16$ $z = \sqrt{-16} = \sqrt{16 \times -1} = \pm\sqrt{16}\sqrt{-1}$	**M1**
$z = \pm 4i$ $= 4i, -4i$	**M1**

4.

$z = \frac{10 \pm \sqrt{100-4(1)(28)}}{2(1)}$	**M1**
$z = \frac{10 \pm \sqrt{100-112}}{2}$ $= \frac{10 \pm \sqrt{-12}}{2}$ $= \frac{10 \pm 2\sqrt{3}i}{2}$	**M1**
$z = 5 \pm \sqrt{3}i$ ($p = 5$, $q = 3$)	**M1**

Further Maths A-Level Starter Activity

Topic: Multiplying Complex Numbers Chapter Reference: Core Pure 1, Chapter 1	**9 minutes**

1. Simplify $(2 + i\sqrt{5})(\sqrt{5} - i)$ in terms $a + bi$, **(2)**

2. If $z = 5 - 3i$, $w = 2 + 2i$, express the following in the form of a + bi, where a and b are real constants,

a. z^2, **(2)**

b. $\frac{z}{w}$ **(3)**

3. Given that $z_1 = 3 + 2i$ and $z_2 = \frac{12-5i}{z1}$,

find z_2 in the form $a + bi$, where a and b are real. **(2)**

Solutions

1.

$(2 + i\sqrt{5})(\sqrt{5} - i) = 2\sqrt{5} - 2i + 5i - i^2\sqrt{5}$	**M1**
$= 3\sqrt{5} + 3i$	**M1**

2.

a) $z^2 = (5-3i)(5-3i)$ $= 25 - 15i - 15i + 9i^2$ $= 25 - 15i - 15i - 9$	**M1**
$= 16 - 30i$	**M1**
b) $\frac{z}{w} = \frac{5-3i}{2+2i} = \frac{5-3i}{2+2i} = \frac{5-3i}{2+2i} \times \frac{2-2i}{2-2i}$	**M1**
$= \frac{10-10i-6i-6}{4+4} = \frac{4-16i}{8}$	**M1**
$= \frac{1}{2} - 2i$	**M1**

3.

$z_2 = \frac{12-5i}{3+2i} \times \frac{3-2i}{3-2i} = \frac{36-24i-15i-10}{13}$	**M1**
$z_2 = 2 - 3i$	**M1**

Further Maths A-Level Starter Activity

Topic: Complex Conjugation Chapter Reference: Core Pure 1, Chapter 1	9 minutes

1. The complex number $3 - 4i$ is denoted by z.

   Giving your answers in the form $x + iy$, find $2z + 5z^*$. **(2)**

2. Solve the following equations, giving each root in the form $a + bi$:

a. $x^2 + 25 = 0$ **(2)**

b. $x^2 - 2x + 17 = 0$ **(2)**

3. If $z = 2 + 3i$ and $w = 4 - i$, express z^*w in the form $x + iy$, clearly showing how you obtained your answer. **(3)**

Solutions

1.

$z^* = 3 + 4i$ $2(3 - 4i) + 5(3 + 4i)$ $6 - 8i + 15 + 20i$	**M1**
$21 + 12i$	**M1**

2.

a) $x^2 = -25$ $x = \sqrt{-25} = \sqrt{25 \times -1} = \pm\sqrt{25}\sqrt{-1}$	**M1**
$x = \pm 5i$ $= 5i, -5i$	**M1**
b) $(x - 1)^2 - 1 + 17 = 0$ $(x - 1)^2 = -16$ $x - 1 = \sqrt{-16}$	**M1**
$x = 1 \pm 4i$ $x = 1 + 4i, 1 - 4i$	**M1**

3.

$z^* = 2 - 3i$	**M1**
$(2 - 3i)(4 - i)$ $8 - 2i - 12i + 3i^2$ $8 - 2i - 12i - 3$	**M1**
$5 - 14i$	**M1**

Further Maths A-Level Starter Activity

Topic: Roots of Equations with Complex Numbers Chapter Reference: Core Pure 1, Chapter 1	8 minutes

1. One root of the quadratic equation $x^2 + ax + b = 0$, where a and b are real, is $16 - 30i$.

a) Write down the other root of the quadratic equation. **(1)**

b) Find the values of a and b. **(4)**

2. Given that $1 + 3i$ is a root of the equation $z^3 + 6z + 20 = 0$, find the other two roots of the equation. **(3)**

Solutions

1.

a) $16 + 30i$	**M1**
b) $(x - (16 - 30i))(x - (16 + 30i))$	**M1**
$x^2 - (16 + 30i)x - (16 - 30i)x + (16 - 30i)(16 + 30i)$ $x^2 - 32x + 256 + 900$ $x^2 - 32x + 1156$	**M1**
$a = -32$	**M1**
$b = 1156$	**M1**

2.

$1 - 3i$ is a root	**M1**
$(z - (1 - 3i))(z - (1 + 3i))(z + \alpha)$ $(z^2 - 2z + 10)(z + \alpha) = z^3 + 6z + 20$ $10\alpha = 20$ $\alpha = 2$	**M1**
-2 is a root	**M1**

Further Maths A-Level Starter Activity

Topic: Roots of Equations with Complex Numbers

Chapter Reference: Core Pure 1, Chapter 1

9 minutes

1. $x^3 - 27 = 0$

Show that two of the roots of the above equation satisfy the quadratic equation $x^2 + 3x + 9 = 0$, by using factorisation.

(2)

2. $f(z) = z^4 - 12z^3 + 31z^2 + 108z - 360$

Write $f(z)$ in the form $(z^2 - 9)(z^2 + bz + c)$, where b and c are real constants to be found **(2)**

3. $f(x) = (x^2 + 4)(x^2 + 8x + 25)$

Find the four roots of $f(x) = 0$. **(5)**

Solutions

1.

$x^3 - 27 = (x-3)(x^2+3x+9)$	**M1**
$x = 3$ is one root Two roots satisfy $x^2 + 3x + 9$	**M1**

2.

$z^4 - 12z^3 + 31z^2 + 108z - 360$ $= (z^2 - 9)(z^2 + bz + c)$ $= z^4 - bz^3 + (-9 + c)z^2 + 9bz - 9c$	**M1**
$b = -12$ $-9c = -360$ $c = 40$	**M1**

3.

$x^2 + 4 = 0$ $x = \pm\sqrt{-4}$	**M1**
$x = 4i, -4i$	**M1**
$x = \frac{-8\pm\sqrt{64-4(1)(25)}}{2(1)}$ $= \frac{-8\pm\sqrt{64-100}}{2}$ $= \frac{-8\pm\sqrt{-36}}{2}$ $= -4 \pm \frac{6i}{2}$	**M1**
$x = -4 + 3i$	**M1**
$x = -4 - 3i$	**M1**