Practice Exam Paper J
Time: 2 Hours

P1

1. Simplify

a. $(2\sqrt{5})^2$, **(1)**

b. $\frac{\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}$, giving your answer in the form $a + \sqrt{b}$, where a and b are integers. **(4)**

(Total Marks: 5)

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2. Factorise completely $x - 4x^3$. **(3)**

(Total Marks: 3)

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3. Express 8^{2x+3} in the form 2^y, stating y in terms of x. **(2)**

(Total Marks: 2)

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4. The curve C has equation $y = x(5 - x)$ and the line L has equation $2y = 5x + 4$.

a. Use algebra to show that C and L do not intersect. **(4)**

b. Sketch C and L on the same diagram, showing the coordinates of the points at which C and L meet the axes. **(4)**

(Total Marks: 8)

5a. By eliminating y from the equations

$$y = x - 4$$
$$2x^2 - xy = 8$$

Show that,

$$x^2 + 4x - 8 = 0$$

(2)

b. Hence, or otherwise, solve the simultaneous equations,

$$y = x - 4$$
$$2x^2 - xy = 8$$

Giving your answers in the form $a \pm b\sqrt{3}$ **(5)**

(Total Marks: 7)

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6. The straight line with equation $y = 3x - 7$ does not cross or touch the curve with equation $y = 2px^2 - 6px + 4p$, where p is a constant.

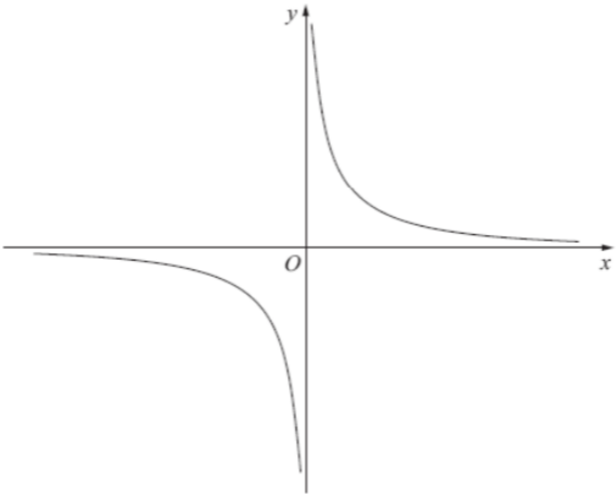
a. Show that $4p^2 - 20p + 9 < 0$. **(4)**

b. Hence find the set of possible values of p. **(4)**

(Total Marks: 8)

7. The figure shows a sketch of the curve C with equation,

$$y = 2 - \frac{1}{x}, x \neq 0$$



a. Find the coordinates of A **(1)**

b. Show that the equation of the normal to C at A can be written as $2x + 8y - 1 = 0$. **(6)**

The normal to C at A meets C again at the point B, as shown in the figure.

c. Find the coordinates of B. **(4)**

(Total Marks: 11)

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8. Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(3 - 2x)^5$, giving each term in its simplest form. **(4)**

(Total Marks: 4)

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9. $y = x^2 - k\sqrt{x}$, where k is a constant

a. Find $\frac{dy}{dx}$ **(2)**

b. Given that y is deceasing at $x = 4$, find the possible values of k **(2)**

(Total Marks: 4)

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10. The circle C has equation

$$x^2 + y^2 - 6x + 4y = 12$$

a. Find the centre and the radius of C **(5)**

The point $P(-1, 1)$ and the point $Q(7, -5)$ both lie on C.

b. Show that PQ is a diameter of C. **(2)**

The point R lies on the positive y-axis and the angle $PRQ = 90°$

c. Find the coordinates of R **(4)**

(Total Marks: 11)

11a. Solve, for $0 \leq \theta < 360°$, the equation $9 \sin(\theta + 60°) = 4$, giving your answers to 1 decimal place. You must show each step of your working. **(4)**

b. Solve, for $-180 \leq x < 180$, the equation $2 \tan x - 3 \sin x = 0$, giving your answers to 2 decimal places where appropriate. **(5)**

(Total Marks: 9)

12a. Find the value of y such that $\log_2 y = -3$ **(2)**

12b. Find the values of x such that $\frac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x$ **(5)**

(Total Marks: 7)

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13. Find the solutions of the equation $\sin(3x - 15°) = \frac{1}{2}$, for which $0 \leq x \leq 180°$. **(6)**

(Total Marks: 6)

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14. Find the exact solutions to the equations,

a. $\ln x + \ln 3 = \ln 6$ **(2)**

b. $e^x + 3e^{-x} = 4$ **(4)**

(Total Marks: 6)

15a. Express

$$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$$

As a single fraction in its simplest form **(4)**

Given that $f(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2$

b. Show that $f(x) = \frac{3}{2x-1}$ **(2)**

(Total Marks: 6)

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16. Use calculus to find the exact value of $\int_1^2 \left(3x^2 + 5 + \frac{4}{x^2}\right) dx$ **(5)**

(Total Marks: 5)

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17. $f(x) = 2x^3 + 3x^2 - 29x - 60$

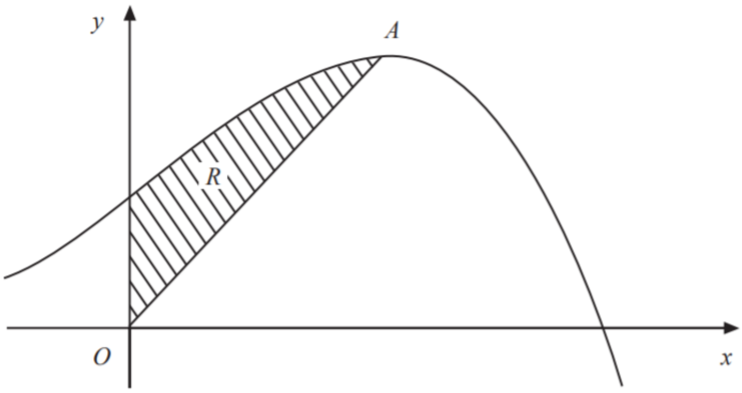
a. Find the remainder when $f(x)$ is divided by $(x + 2)$ **(2)**

b. Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$. **(2)**

c. Factorise $f(x)$ completely **(4)**

(Total Marks: 8)

18. The figure shows a sketch of part of the curve with equation $y = 10 + 8x + x^2 - x^3$



The curve has a maximum turning point *A*.

a. Using calculus, show that the *x*-coordinate of *A* is 2. **(3)**

The region *R*, shown shaded in the figure, is bounded by the curve, the *y*-axis and the line from *O* to *A*, where *O* is the origin

b. Using calculus, find the exact area of *R*. **(7)**

(Total Marks: 10)

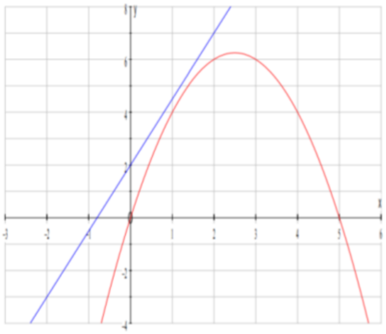
Total Marks: 120

Mark Scheme

1a	20	**B1**
1b	$\frac{\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} \times \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}}$	**M1**
	$= \frac{2\sqrt{10}+6}{2}$	**M1** **A1**
	$= 3 + \sqrt{10}$	**A1**

2	$x(1 - 4x^2)$	**B1**
	$x(1 - 2x)(1 + 2x)$	**M1** **A1**

3	$8^{2x+3} = 2^{3(2x+3)}$	**M1**
	2^{6x+9}	**A1**

4a	$x(5 - x) = \frac{1}{2}(5x + 4)$	**M1**
	$2x^2 - 5x + 4 = 0$	**A1**
	$b^2 - 4ac = (-5)^2 - 4 \times 2 \times 4$	**M1**
	$25 - 32 = -7$ $-7 < 0$ Therefore, no roots	**A1**
4b		
	Shape of curve and passing through (0, 0)	**B1**
	Shape of curve and passing through (5, 0)	**B1**
	Positive gradient of line and no intersections with curves.	**B1**
	(-0.8, 0) marked on axes	**b1**

5a	$2x^2 - x(x - 4) = 8$	**M1**
	$x^2 + 4x - 8 = 0$	**A1**
5b	Use of quadratic formula or use of calculator	**M1**
	$x = -2 \pm \ ...$	**A1**
	$\sqrt{48} = 4\sqrt{3}$	**B1**
	$y = (-2 \pm 4\sqrt{3}) - 4$	**M1**
	$x = -2 + 2\sqrt{3}$ $y = -6 + 2\sqrt{3}$	**M1**

6a	$2px^2 - 6px + 4p = 3x - 7$	**M1**
	$y = 2p(\frac{y+7}{3})^2 - 6p(\frac{y+7}{3}) + 4p$	**M1**
	$b^2 - 4ac = (-6p - 3)^2 - 4(2p)(4p+7)$	**M1**
	$y = 2px^2 - 6px + 4p - 3x + 7$	**A1**
6b	$(2p - 9)(2p - 1) = 0$	**M1**
	$p = \frac{9}{2}, \frac{1}{2}$	**A1**
	$\frac{1}{2} < p < \frac{9}{2}$	**M1** **A1**

7a	$y = \frac{2}{x}$ is translated up or down	**M1**
	$y = \frac{2}{x} - 5$ is in the correct position	**A1**
	Intersection with x-axis at $(\frac{2}{5}, 0)$ only	**B1**
	Independent mark $y = 4x + 2$ in a straight line	**B1**
	Intersection with x-axis at $(-\frac{1}{2}, 0)$ and y-axis at $(0, 2)$	**B1**
7b	Asymptotes at $x = 0$	**B1**
	$y = -5$	**B1**
7c	$\frac{2}{x} - 5 = 4x + 2$	**M1**
	$4x^2 + 7x - 2 = 0$	**M1**
	$x = -2$, $x = \frac{1}{4}$	**A1**
	When $x = -2$, $y = -6$ when $x = \frac{1}{4}$, $y = 3$	**M1** **A1**

8	$(3 - 2x)^5 = 243 + 5 \text{ x } (3)^4 (-2x) = -810x$	**B1** **B1**
	$+ \frac{5 \times 4}{2}(3)^3 (-2x)^2 = 1080x^2$	**B1** **A1**

9a	$\frac{dy}{dx} = 2x - \frac{1}{2}kx^{-\frac{1}{2}}$	**M1** **A1**
9b	Substituting $x = 4$ in	**M1**
	$8 - \frac{k}{4} < 0$ $k > 32$	**A1**

10a	$(x - 3)^2 - 9 + (y + 2)^2 - 4 = 12$	**M1** **A1** **A1**
	$(x - 3)^2 + (y + 2)^2 = 5$	**M1** **A1**
10b	$PQ = \sqrt{(7 - 1)^2 + (-5 - 1)^2}$	**M1**
	$= 10$	**A1**
10c	R must lie in the circle	**B1**
	$x = 0$ $y^2 + 4y - 12 = 0$	**M1**
	$(y - 2)(y + 6) = 0$	**M1**
	$y = -6$ or 2	**A1**

11a	$\sin(\theta + 60) = \frac{4}{9}$ Let $x = (\theta + 60)$ $x = 26.38….$	**M1**
	$\theta + 60 = 153.6…$ $\theta + 360 = 386.3…$	**M1**
	$\theta = 93.6^\circ$ $\theta = 326.4^\circ$	**A1** **A1**
11b	$2\left(\frac{\sin x}{\cos x}\right) - 3\sin x = 0$	**M1**
	$2\sin x - 2\sin x \cos x = 0$ $\sin x(2 - 3\cos x) = 0$ $\cos x = \frac{2}{3}$	**A1**

	$\sin x = 0$	
	$x = 48.1^{o}$ $x = -48.1^{o}$ $x = 0^{o}$ $x = -180^{o}$	**A1** **A1** **A1**

12a	$\log_2 y = -3$ $y = 2^{-3}$	**M1**
	$y = \frac{1}{8}$	**A1**
12b	$32 = 2^5$	**M1**
	$\log_2 32 + \log_2 16 = 9$	**A1**
	$(\log x)^2 = \ldots$	**M1**
	$\log_2 x = 3$ $x = 2^3 = 8$	**A1**
	$\log_2 x = -3$ $x = 2^{-3} = \frac{1}{8}$	**A1**

13	$\sin(3x - 15) = \frac{1}{2}$ $3x - 15 = 30$ $x = 15$	**M1** **A1**
	$3x - 15 = 180 - \alpha$	**M1**
	$3x - 15 = 360 + \alpha$ $3x - 15 = 540 - \alpha$	**M1**
	$x = 55$ or 175	**A1**
	$x = 55, 135, 175$	**A1**

14a	$\ln 3x = \ln 6$	**M1**
	$x = 2$	**A1**
14b	$(e^x)^2 - 4e^x + 3 = 0$	**M1**
	$(e^x - 3)(e^x + 1) = 0$ $e^x = 3$ $e^x = 1$	**M1**
	$x = \ln 3$ $x = 0$	**M1** **A1**

15a	$\frac{(4x-1)(2x-1)-3}{2(x-1)(2x-1)}$	**M1**
	$\frac{8x^2-6x-2}{2(x-1)(2x-1)}$	**A1**
	$\frac{2(x-1)(4x+1)}{2(x-1)(2x-1)}$	**M1**
	$= \frac{4x+1}{2x-1}$	**A1**
15b	$f(x) = \frac{4x+1}{2x-1} - 2$ $= \frac{4x+1-2(2x-1)}{(2x-1)}$	**M1**
	$= \frac{3}{2x-1}$	**A1**

16	$\int (3x^2 + 5 + 4x^{-2})dx = \frac{3x^3}{3} + 5x + \frac{4x^{-1}}{-1} = (x^3 + 5x - 4x^{-1})$	**M1** **A1** **A1**
	$= [x^3 + 5x - 4x^{-1}]_1^2 = (8 + 10 - 2) - (1 + 5 - 4) = 14$	**M1** **A1**

17a	$f(-2) = 2(-2)^3 + 3(-2)^2 - 29(-2) - 60$	**M1**

	= -16 + 12 + 58 – 60 = -6	**A1**
17b	f(-3) = 2(-3)^3 + 3(-3)^2 – 29(-3) – 60	**M1**
	= -54 + 27 + 87 – 60 = 0 Therefore, (x + 3) is a factor	**A1**
17c	(x + 3)(2x^2 – 3x – 20)	**M1** **A1**
	(x+ 3)(2x + 5)(x – 4)	**M1** **A1**

18a	$\frac{dy}{dx} = 8 + 2x - 3x^2$	**M1** **A1**
	3x^2 – 2x – 8 = 0 (3x + 4)(x – 2) = 0 x = 2	**A1**
18b	Area of triangle = $\frac{1}{2} \times 2 \times 22 = 22$	**M1** **A1**
	$\int 10 + 8x + \ x^2 - x^3 \ dx = 10 + \frac{8x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$	**M1** **A1**
	$[10 + \frac{8x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}]_0^2$ = 20 + 16 + $\frac{8}{3}$ – 4	**M1**
	Area of *R* = 34 $\frac{2}{3}$ – 22 = $\frac{38}{3}$	**M1** **A1**

<u>Topic List</u>

Q1	Surds
Q2	Factorising
Q3	Indices
Q4	Equations of lines
Q5	Simultaneous equations
Q6	Roots
Q7	Equation of lines and normal
Q8	Binomial expansion
Q9	Differentials
Q10	Circles
Q11	Solving trig equations
Q12	Logarithms
Q13	Solving trig equations
Q14	Logarithms and exponentials
Q15	Simplifying algebraic fractions
Q16	Definite integrals
Q17	Factor theorem
Q18	Areas of shaded regions