Practice Exam Paper 1
Time: 2 Hours

P1

1a. Find the value of $16^{-\frac{1}{4}}$ **(2)**

b. Simplify $x(2x^{-\frac{1}{4}})^4$ **(2)**

(Total Marks: 4)

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2. Simplify, $\sqrt{32} + \sqrt{18}$

Giving your answers in the form $a\sqrt{2}$, where a is an integer. **(2)**

b. Simplify,

$$\frac{\sqrt{32} + \sqrt{18}}{3 + \sqrt{2}}$$

Giving your answer in the form $b\sqrt{2} + c$, where b and c are integers. **(4)**

(Total Marks: 6)

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3. The line l_1 has equation $y = -2x + 3$

The line l_2 is perpendicular to l_1 and passes through the point (5, 6).
a. Find an equation for l_2 in the form $ax + by + c = 0$, where a, b, and c are integers **(3)**

The line l_2 crosses the x-axis at the point A and the y-axis at the point B

b. Find the x-coordinate of A and the y-coordinate of B **(2)**

Given that O is the origin,

c. Find the area of the triangle OAB **(2)**

(Total Marks: 7)

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4. Solve the simultaneous equations,

$$x - 2y - 1 = 0$$
$$x^2 + 4y^2 - 10x + 9 = 0$$

(7)

(Total Marks: 7)

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5. Find the first 3 terms in ascending powers of x of

$$(2 - \frac{x}{2})^6$$

Giving each term in its simplest form **(4)**

(Total Marks: 4)

6. Given that a and b are positive constants, solve the simultaneous equations

$$ab = 25$$
$$\text{Log}_4 a - \log_4 b = 3$$

Show each step of your working, giving exact values for a and b. **(6)**

(Total Marks: 6)

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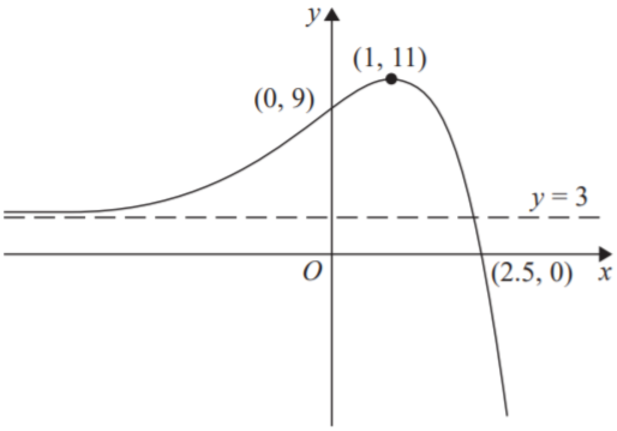
7. The figure shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$. The curve crosses the coordinate axes at the points (2.5, 0) and (0, 9), has a stationary point at (1, 11), and has an asymptote $y = 3$



On separate diagrams, sketch the curve with equation

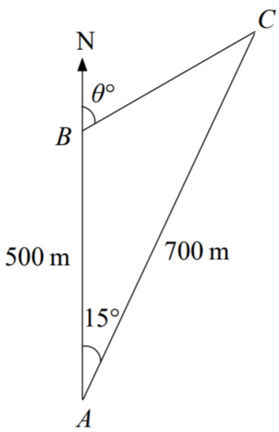
a. $y = 3\mathrm{f}(x)$ **(3)**

b. $y = \mathrm{f}(-x)$ **(3)**

On each diagram show clearly the coordinates of the points of intersection of the curve with the two coordinate axes, the coordinates of the stationary point, and the equation of the asymptote.

(Total Marks: 6)

8.The figure shows 3 yachts *A*, *B* and *C* which are assumed to be in the same horizontal plane. Yacht *B* is 500 m due north of yacht *A* and yacht *C* is 700 m from *A*. The bearing of *C* from *A* is 015°



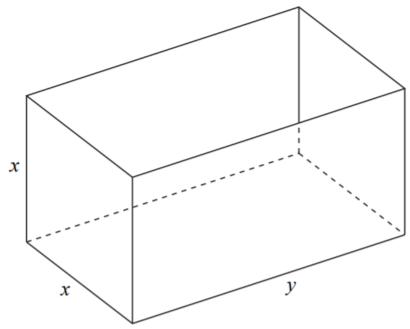
a. Calculate the distance between yacht *B* and yacht *C*, in metres to 3 significant figures. **(3)**

The bearing of yacht *C* from yacht *B* is θ°, as shown in the figure

b. Calculate the value of θ. **(4)**

(Total Marks: 7)

9. The figure shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is x metres.



The capacity of the tank is 100 m^3.

a. Show that the area A m^2 of the sheet metal used to make the tank is given by,

$A = \frac{300}{x} + 2x^2$ **(4)**

b. Use calculus to find the value of x for which A is stationary. **(4)**

c. Prove that this value of x gives a minimum value of A. **(2)**

d. Calculate the minimum area of sheet metal needed to make the tank **(2)**

(Total Marks: 12)

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10. Given that $0 < x < 4$ and,

$$\log_5(4 - x) - 2\log_5 x = 1$$

Find the value of x **(6)**

(Total Marks: 6)

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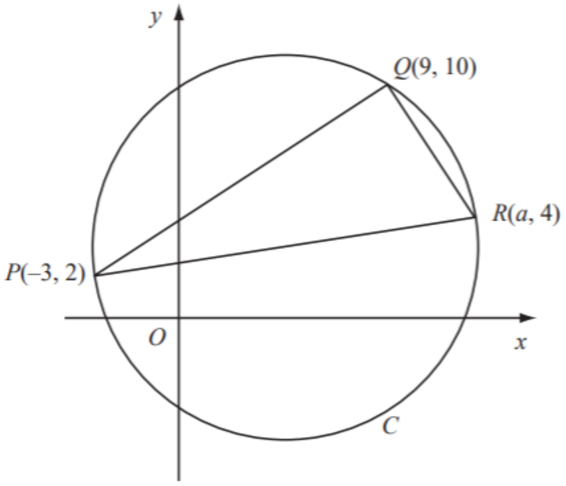
11. The points $P(-3, 2)$, $Q(9, 10)$ and $R(a, 4)$ lie on the circle C, as shown in the figure.



Given that PR is a diameter of C,

a. Show that $a = 13$ **(3)**

b. Find an equation for C. **(5)**

(Total Marks: 8)

12a. Show that the equation

$$4\sin^2 x + 9\cos x - 6 = 0$$

Can be written as,

$$4\cos^2 x - 9\cos x + 2 = 0$$

(2)

12b. Hence solve, for $0 \leq x < 720°$,

$$4\sin^2 x + 9\cos x - 6 = 0$$

Giving your answers to 1 decimal place **(6)**

(Total Marks: 8)

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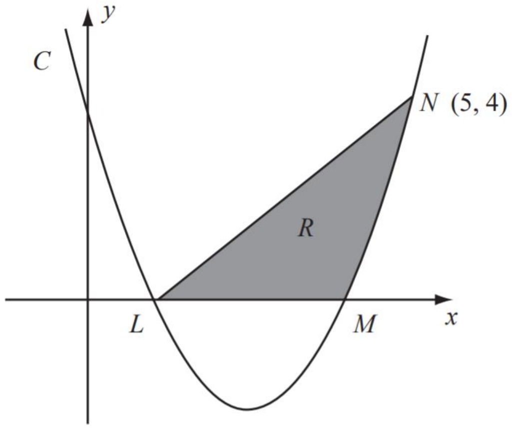
13. The curve C has equation $y = x^2 - 5x + 4$. It cuts the x-axis at the points L and M as shown in the figure.



a. Find the coordinates of the point L and the point M **(2)**

b. Show that the point N (5, 4) lies on C **(1)**

c. Find $\int (x^2 - 5x + 4)\, dx$ **(2)**

The finite region R is bounded by LN, LM and the curve C as shown in the figure.

d. Use your answer to part (c) to find the exact value of the area of R. **(5)**

(Total Marks: 10)

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14. $f(x) = x^4 + x^3 + 2x^2 + ax + b$
Where a and b are constants.

When $f(x)$ is divided by $(x - 1)$, the remainder is 7.

a. Show that $a + b = 3$ **(2)**

When $f(x)$ is divided by $(x + 2)$, the remainder is -8

b. Find the value of a and the value of b **(5)**

(Total Marks: 7)

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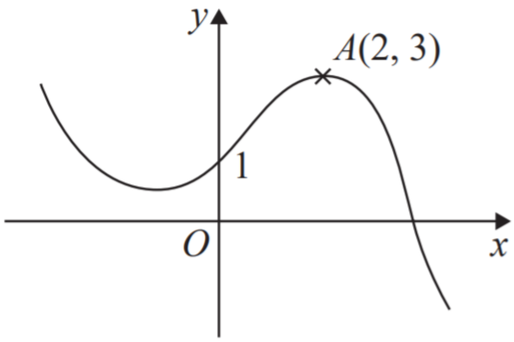
15. Express $\frac{x+1}{3x^2-3} - \frac{1}{3x+1}$ as a single fraction in its simplest form **(4)**

(Total Marks: 4)

16. The figure shows a sketch of the graph of $y = f(x)$



The graph intersects the y-axis at the point (0, 1) and the point $A(2, 3)$ is the maximum turning point.

Sketch, on separate axes, the graphs of

a. $y = f(-x) + 1$,
b. $y = f(x + 2) + 3$,
c. $y = 2f(2x)$.

On each sketch, show the coordinates of the point at which your graph intersects the y-axis and the coordinates of the point to which A is transformed. **(9)**

(Total Marks: 9)

17a. Sketch for $0 \leq x < 360$, the graph of $y = \sin(x + 60)$ **(2)**

b. Write down the exact coordinates of the points where the graph meets the coordinate axes **(2)**

c. Solve for $0 \leq x \leq 360$, the equation,

$$\sin(x + 60) = 0.65$$

giving tour answer in degrees to 2 decimal places **(5)**

(Total Marks: 9)

Total Marks: 120

Jethwa Maths

<u>Mark Scheme</u>

1a	$16^{\frac{1}{4}} = 2$	**M1**
	$(16^{-\frac{1}{4}}) = \frac{1}{2}$	**A1**
1b	$(2x^{-\frac{1}{4}})^4 = 2^4 x^{-\frac{4}{4}}$	**M1**
	$x(2x^{-\frac{1}{4}})^4 = 2^4$	**A1**

2a	$\sqrt{32} = 4\sqrt{2}$	**M1**
	$\sqrt{32} + \sqrt{18} = 7\sqrt{2}$	**B1**
2b	$\frac{\sqrt{32}+\sqrt{18}}{3+\sqrt{2}} \text{ x } \frac{3-\sqrt{2}}{3-\sqrt{2}}$	**M1**
	$= \frac{7\sqrt{2}(3-\sqrt{2})}{9-2}$	**M1**
	$3\sqrt{2} - 2$	**A1** **A1**

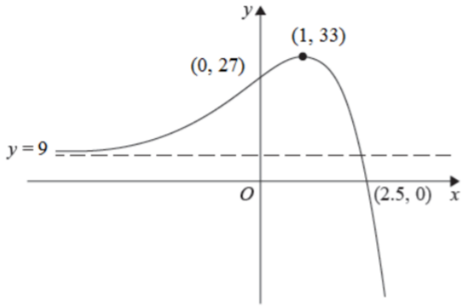
3a	Gradient of l_2 is $\frac{1}{2}$	**B1**
	$y - 6 = \frac{1}{2}(x - 5)$	**M1**
	$x - 2y + 7 = 0$	**A1**
3b	$x = 0$ $0 - 2y + 7 = 0$	**M1**
	$y = \frac{7}{2}$	**A1**
3c	Area $OAB = \frac{1}{2}(7)\left(\frac{7}{2}\right) = \frac{49}{4}$ unit^2	**M1** **A1**

4	$x = 2y + 1$ $(2y + 1)^2 + 4y^2 - 10(2y + 1) + 9 = 0$	**M1**
	$8y^2 - 16y = 0$	**M1** **A1**
	$8y(y - 2) = 0$	**M1**
	$y = 0, x = 1$	**M1**
	$y = 2, x = 5$	**A1** **A1**

5	$(2 - \frac{x}{2})^6 = 2^6 + \binom{6}{1}2^5(\frac{-x}{2}) + \binom{6}{2}2^4(\frac{-x}{2})^2$	**M1**
	$= 64 - 96x + 60x^2 + \ldots.$	**B1** **A1** **A1**

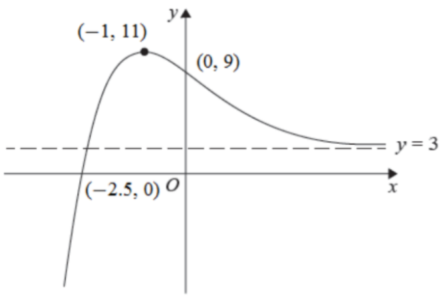
6	$\log_4 \frac{a}{b} = 3$	**M1**
	$\log_4 64 = 3$	**B1**
	Elimination of one variable	**M1**
	$a = 40$ or $b = \frac{5}{8}$	**A1**
	Substitutes to give second variable or solves again from start	**M1**
	$a = 40$ or $b = \frac{5}{8}$	**A1**

7a



7a	Shape - similar to before but with indication of stretch in y direction by at least one correct from the three traits:	**B1**
	y intercept, (0, 27) maximum point (1, 33) or asymptote indicated at 9	**B1**
	Intercept (0,27), max (1,33) and x intercept (2.5,0) all three of these seen	**B1**

7b



7b	Shape (reflection in y axis)	**B1**
	(–1,11), (0,9) and (–2.5,0) seen	**B1**
	y = 3 (must be equation)	**B1**

8a	$BC^2 = 700^2 + 500^2 - 2 \times 500 \times 700 \cos 15$	**M1** **A1**
	$BC = 253$	**A1**
8b	$\frac{\sin B}{700} = \frac{\sin 15}{253}$	**M1**
	$\sin B = \sin 15 \times \frac{700}{253} = 0.716\ldots$ $B = 134.2^{\circ}$	**M1**
	$\theta = 180 - 134.2$	**M1**
	$\theta = 045.8$	**A1**

9a	Total area: $3xy + 2x^2$	**B1**
	Volume: $x^2 y = 100$ $y = \frac{100}{x^2}$ $xy = \frac{100}{x}$	**B1**
	Deriving expression for area in terms of x only	**M1**
	Area = $\frac{300}{x} + 2x^2$	**A1**
9b	$\frac{dA}{dx} = \frac{300}{x^2} + 4x$	**M1** **A1**
	$\frac{dA}{dx} = 0$	**M1**
	$x^3 = 75$ $x = 3.2172$	**A1**
9c	$\frac{d^2A}{dx^2} = \frac{600}{x^3} + 4$	**M1**
	$\frac{d^2A}{dx^2}$ = positive, therefore minimum	**A1**
9d	Substituting found value of x into (a)	**M1**

	$y = \frac{100}{4.2172^2} = 5.6228$ Area = 106.707	**A1**

10	$2\log_5 x = \log_5 (x^2)$ $\log_5(4 - x) - \log_5(x^2) = \log_5 \left(\frac{4-x}{x^2}\right)$	**B1** **M1**
	$\log \left(\frac{4-x}{x^2}\right) = \log 5$ $5x^2 + x - 4 = 0$	**M1** **A1**
	$(5x - 4)(x + 1) = 0$ $(x = -1)$ $x = \frac{4}{5}$	**M1** **A1**

11a	PQ: $m_1 = \frac{10-2}{9-(-3)} = \frac{2}{3}$ QR: $m_2 = \frac{10-4}{9-a}$	**M1**
	$m_1 m_2 = -1$ $\frac{8}{12} \times \frac{6}{9-a} = -1$	**M1**
	$a = 13$	**A1**
11b	Centre is at (5, 3)	**B1**
	$(r^2) = (10 - 3)^2 + (9 - 5)^2$	**M1** **A1**
	$(x - 5)^2 + (y - 2)^2 = 65$	**M1** **A1**

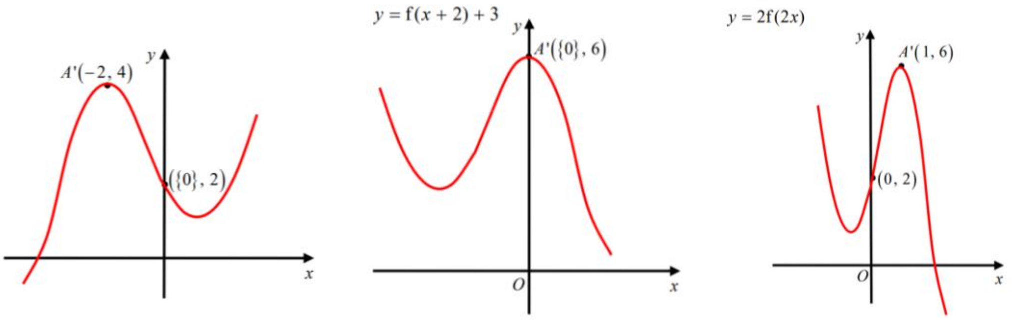
12a	$4(1 - \cos^2 x) + 9\cos x - 6 = 0$ $4\cos^2 x - 9\cos x + 2 = 0$	**M1** **A1**
12b	$(4\cos x - 1)(\cos x - 2) = 0$ $\cos x = 2$ (no solutions) $\cos x = \frac{1}{4}$	**M1** **A1**
	$x = 75.7^{o}$	**B1**
	$x = 360 - 75.7$ $x = 360 + 75.7$ $x = 720 - 75.5$	**M1** **M1**
	$x = 75.7^{o}$ $x = 284.5^{o}$ $x = 435.5^{o}$ $x = 644.5^{o}$	**A1**

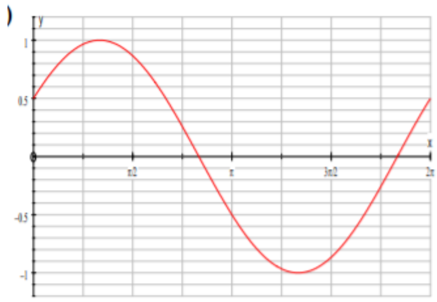
13a	$y = 0$ $(x - 4)(x - 1) = 0$	**M1**
	(1, 0) and (4, 0)	**A1**
13b	$x = 5$ $y = 25 - 25 + 4 = 4$ Therefore (5, 4) lies on the curve	**B1**
13c	$\int(x^2 - 5x + 4)dx = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x\ (+c)$	**M1** **A1**
13d	Area of triangle = $\frac{1}{2} \times 4 \times 4 = 8$	**B1**
	Area under curve = $\int_4^5 (x^2 - 5x + 4)dx$ $= \frac{1}{3}(5)^3 - \frac{5}{2}(5)^2 + 4(5) = -\frac{5}{6}$	**M1**
	$= \frac{1}{3}(4)^3 - \frac{5}{2}(4)^2 + 4(4) = -\frac{8}{3}$	**M1**
	$= -\frac{5}{6} - -\frac{8}{3} = \frac{11}{6}$	**A1**

	Area of R = $8 - \frac{11}{6} = \frac{37}{6}$	**A1**

14a	$f(x) = x^4 + x^3 + 2x^2 + ax + b$ Attempting f(1) or f(-1)	**M1**
	$f(1) = 1 + 1 + 2 + a + b = 7$ Therefore, $a + b = 3$	**A1**
14b	Attempting f(-2)	**M1**
	$f(-2) = 16 – 8 + 8 – 2a + b = -8$ $-2a + b = -24$	**A1**
	Solving both equations a = …. b = …	**M1**
	$a = 9$ $b = -6$	**A1** **A1**

15	$\frac{x+1}{3x^2-3} - \frac{1}{3x+1} = \frac{x+1}{3(x+1)(x-1)} - \frac{1}{3x+1}$	**M1**
	$\frac{3x+1-3(x-1)}{3(x-1)(3x+1)}$	**A1** **M1**
	$= \frac{4}{3(x-1)(3x+1)}$	**A1**

16a **16b** **16c**		**B3** **Shape** **B3** **1 coordinate** **B3** **2 coordinates**

17a		**M1** **(Shape)** **A1** **(axes)**
17b	(0, 0.5) (150, 0) (330, 0)	**B1** **B1** **B1**
17c	40.5°	**B1** **A1**
	180 – 40.5 = 138.5°	**M1**
	40.5 – 30 = 10.5°	**M1**
	139 – 30 = 109	**A1**

Topic List

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