Practice Exam Paper H
Time: 2 Hours

P1

1. The line L has equation $y = 5 - 2x$

a. Show that the point P (3, –1) lies on L **(1)**

b. Find an equation of the line perpendicular to L, which passes through P. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

(Total Marks: 5)

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2. Write $\sqrt{45}$ in the form $a\sqrt{5}$, where a is an integer. **(1)**

b. Express $\frac{2(3+\sqrt{5})}{(3-\sqrt{5})}$ in the form $b + c\sqrt{5}$, where b and c are integers. **(5)**

(Total Marks: 6)

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3. Given that $f(x) = \frac{1}{x}$, $x \neq 0$
a. Sketch the graph of $y = f(x) + 3$ and state the equations of the asymptotes. **(4)**
b. Find the coordinates of the point where $y = f(x) + 3$ crosses a coordinate axis. **(2)**

(Total Marks: 6)

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4. The equation $2x^2 - 3x - (k + 1) = 0$, where k is a constant, has no real roots.

Find the set of possible values of k **(4)**

(Total Marks: 4)

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5. The curve C has equation $y = f(x)$, $x > 0$, and $f'(x) = 4x - 6\sqrt{x} + \frac{8}{x^2}$
Given that the point $P(4, 1)$ lies on C,

a. Find $f(x)$ and simplify your answer. **(6)**

b. Find an equation of the normal to C at the point $P(4, 1)$. **(4)**

(Total Marks: 10)

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6. $f(x) = x^2 + 4kx + (3 + 11k)$, where k is a constant.

a. Express $f(x)$ in the form $(x + p)^2 + q$, where p and q are constants to be found in terms of k. **(3)**

Given that the equation $f(x) = 0$ has no real roots
b. Find the set of possible values of k. **(4)**

Given that $k = 1$
c. Sketch the graph of $y = f(x)$, showing the coordinates of any point at which the graph crosses a coordinate axis. **(3)**

(Total Marks: 13)

7. The points A and B have coordinates $(5, -1)$ and $(13, 11)$ respectively.

a. Find the coordinates of the mid-point of AB. **(2)**

Given that AB is a diameter of the circle C,

b. Find an equation for C **(4)**

(Total Marks: 6)

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8. Find, giving your answer to 3 significant figures where appropriate, the value of x for which,

a. $3^x = 5$ **(3)**

b. $\log_2 (2x + 1) - \log_2 x = 2$ **(4)**

(Total Marks: 7)

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9. Find all the values of x, to 1 decimal place, in the interval $0^\circ \le x < 360^\circ$ for which,

$5 \sin (x + 30) = 3$ **(4)**

b. Find all the values of x, to 1 decimal place, in the interval $0^\circ \le x < 360^\circ$ for which,

$\tan^2 x = 4$ **(5)**

(Total Marks: 9)

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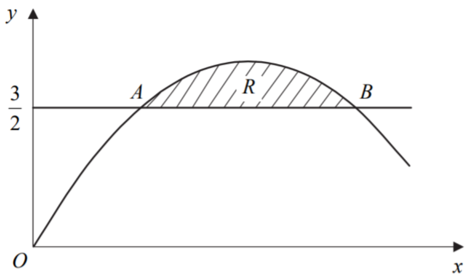
10. The figure shows the shaded region R which is bounded by the curve $y = -2x^2 + 4x$ and the line $y = \frac{3}{2}$. The points A and B are the points of intersection of the line and the curve



Find,

a. The x-coordinates of the points A and B **(4)**

b. The exact area of R. **(6)**

(Total Marks: 10)

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11. $f(x) = 1 - \frac{3}{x+2} + \frac{3}{(x+2)^2}, x \neq 2$

a. Show that $f(x) = \frac{Ax^2+Ax+A}{(x+2)^2}$, $x \neq 2$, where A is an integer to be found. **(4)**

b. Show that $x^2 + x + 1 > 0$, for all values of x **(3)**

c. Show that $f(x) > 0$ for all values of x, $x \neq 2$ **(1)**

(Total Marks: 8)

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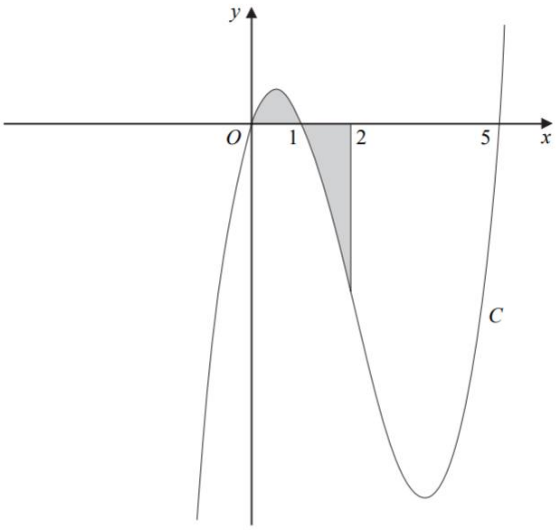
12. The figure shows a sketch of part of the curve C with equation,

$$y = x(x - 1)(x - 5)$$

Use calculus to find the total area of the infinite region, shown shaded in the figure, that is between $x = 0$ and $x = 2$ and is bounded by C, the x-axis and the line $x = 2$. **(9)**



(Total Marks: 9)

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13. A diesel lorry is driven from Birmingham to Bury at a steady speed of v kilometres per hour. The total cost of the journey, £C, is given by

$$C = \frac{1400}{v} + \frac{2v}{7}$$

a. Find the value of v for which C is a minimum **(5)**

b. Find $\frac{d^2C}{dv^2}$ and hence verify that C is a minimum for this value of v **(2)**

c. Calculate the minimum total cost of the journey **(2)**

(Total Marks: 9)

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14. $f(x) = x^4 - x^3 + 3x^2 + ax + b$

Where a and b are constants.

When $f(x)$ is divided by $(x - 1)$ the remainder is 4

When $f(x)$ is divided by $(x + 2)$ the remainder is 22.

Find the value of a and the value of b **(5)**

(Total Marks: 5)

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15. The radioactive decay of a substance is given by,

$$R = 1000e^{-ct}, t \geq 0$$

Where R is the number of atoms at time t years and c is a positive constant

a. Find the number of atoms when the substance started to decay **(1)**

It takes 5730 years for half of the substance to decay

b. Find the value of c to 3 significant figures. **(4)**

c. Calculate the number of atoms that will be left when $t = 22\,920$. **(2)**

(Total Marks: 7)

16a. Prove that

$$\frac{\cos x}{1+\sin\ x}+\frac{1+\sin x}{\cos x}\equiv\frac{2}{\cos x}$$

(4)

b. Hence or otherwise, find for 0 < x < 360, all the solutions of,

$$\frac{\cos x}{1+\sin\ x}+\frac{1+\sin x}{\cos x}=4$$

(2)

(Total Marks: 6)

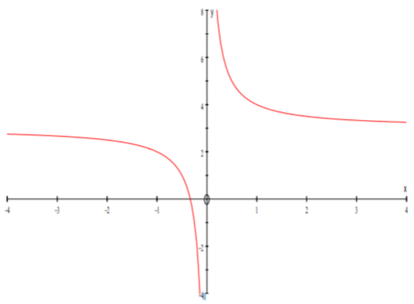
Total Marks: 120

Mark Scheme

1a	$y = 5 - (2 \times 3) = -1$	**B1**
1b	Gradient of $L = \frac{1}{2}$	**B1**
	$y - (-1) = \frac{1}{2}(x - 3)$	**M1** **A1**
	$x - 2y - 5 = 0$	**A1**

2a	$3\sqrt{5}$	**B1**
2b	$\frac{2(3+\sqrt{5})}{(3-\sqrt{5})} \times \frac{(3+\sqrt{5})}{(3+\sqrt{5})}$	**M1**
	$\frac{2(14+6\sqrt{5})}{4}$	**B1** **M1** **M1**
	$= 7 + 3\sqrt{5}$	**a1**

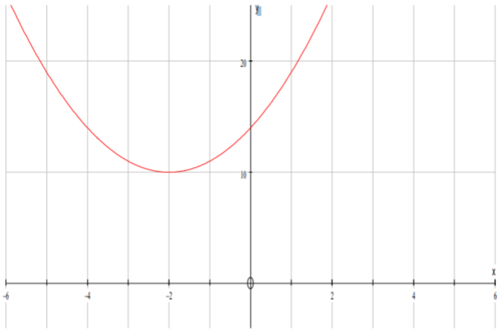
3a	Shape of $f(x)$	**B1**
	Moved up	**M1**
	Asymptotes $y = 3$	**B1**
	$x = 0$	**B1**



3b	$\frac{1}{x} + 3 = 0$	**M1**
	$x = -\frac{1}{3}$	**A1**

4	Use of $b^2 - 4ac$	**M1**
	$(-3)^2 - 4 \times 2 \times -(k + 1) < 0$	**A1**
	$8k < -17$	**M1**
	$k < -\frac{17}{8}$	**A1**

5a	$4x \rightarrow kx^2$	**M1**
	$f(x) = 2x^2 - 4x^{\frac{3}{2}} - 8x^{-1} + C$	**A1** **A1** **A1**
	At $x = 4$, $y = 1$ $1 = (2 \times 16) - \left(4 \times 4^{\frac{3}{2}}\right) - (8 \times 4^{-1}) + c$	**M1**
	$c = 3$	**A1**
5b	$f'(4) = 16 - (6 \times 2) + \frac{8}{16} = \frac{9}{2}$	**M1**
	Gradient of normal is $-\frac{2}{9} = -\frac{1}{m}$	**M1**
	Equation of normal: $y - 1 = -\frac{2}{9}(x - 4)$ $2x + 9y - 17 = 0$	**M1** **A1**

6a	$(x + 2k)^2$	**M1**
	$(x + 2k)^2 - 4k^2 + (3 + 11k)$	**M1** **A1**
6b	$4k^2 - 11k - 3 = 0$ $(4k + 1)(k - 3) = 0$	**M1**
	$k = -\frac{1}{4}$ $k = 3$	**A1**
	Using $b^2 - 4ac < 0$ $4k^2 - 11k - 3 < 0$	**M1**
	$-\frac{1}{4} < k < 3$	**A1**
6c	Shape	**B1**
	Minimum in correct quadrant, no touching x-axis, not on the y-axis, no other maximum or minimum.	**B1**
	(0, 14) marked	**B1**
		

7a	$\left(\frac{5+13}{2}, \frac{-1+11}{2}\right) = (9, 5)$	**M1** **A1**
7b	$r^2 = (9 - 5)^2 + (5 - - 1)^2 = 52$	**M1**
	Equation of circle, $(x - 9)^2 + (y - 5)^2 = 52$	**M1** **A1** **A1**

8a	$\log 3^x = \log 5$	**M1**
	$x = \frac{\log 5}{\log 3}$	**A1**
	$x = 1.46$	**A1**
8b	$\log_2 \left(\frac{2x+1}{x}\right) = 2$	**M1**
	$\frac{2x+1}{x} = 2^2$	**M1**
	$2x + 1 = 4x$	**M1**
	$x = \frac{1}{2}$	**A1**

9a	$\sin(x + 30) = \frac{3}{5}$	**B1**
	$x + 30 = 36.9$	**B1**
	$x + 30 = 143.1$	**M1**
	$x = 6.9$ $x = 113.1$	**A1**
9b	$\tan x = \pm 2$	**B1**
	$\tan x = 2$ $x = 63.4$ $x = 243.4$	**B1** **M1**
	$\tan x = -2$	**M1**

	$x = 116.6^{O}$	
	$x = 296.6^{O}$	**M1**

10a	$\frac{3}{2} = -2x^2 + 4x$	**M1**
	$4x^2 - 8x + 3 = 0$	**A1**
	$(2x - 1)(2x - 3) = 0$	**M1**
	$x = \frac{1}{2}$ $x = \frac{3}{2}$	**A1**
10b	Area of $R = \int_{\frac{1}{2}}^{\frac{3}{2}} (-2x^2 + 4x)dx - \frac{3}{2}$	**B1**
	$\int(-2x^2 + 4x)dx = [-\frac{2}{3}x^2 + 2x^2]$	**M1** **A1**
	$\int_{\frac{1}{2}}^{\frac{3}{2}}(-2x^2 + 4x)dx = \left(-\frac{2}{3} \times \frac{3^3}{2^3} + 2 \times \frac{3^2}{2^2}\right) - \left(-\frac{2}{3} \times \frac{1}{2^3} + 2 \times \frac{1}{2^2}\right)$	**M1** **M1**
	$= \frac{11}{6}$ Area of R $= \frac{11}{6} - \frac{3}{2} = \frac{1}{3}$	**A1**

11a	$f(x) = \frac{(x+2)^2 - 3(x+2) + 3}{(x+2)^2}$	**M1** **A1** **A1**
	$= \frac{x^2 + 4x + 4 - 3x - 6 + 3}{(x+2)^2}$ $= \frac{x^2 + x + 1}{(x+2)^2}$ Where $A = 1$	**A1**
11b	$x_2 + x + 1 = (x + \frac{1}{2})^2 + \frac{3}{4}$	**M1** **A1**
	Any squared value will be positive, therefore, expression is positive.	**A1**
11c	$f(x) = \frac{(x + \frac{1}{2})2 + \frac{3}{4}}{(x+2)^2}$ Numerator is positive and as $x \neq -2$, $(x + 2)^2 > 0$, therefore denominator is positive. Hence, $f(x) > 0$	**B1**

12	$y = x(x^2 - 6x + 5)$ $y = x^3 - 6x^2 + 5x$	**M1** **A1**
	$\int(x^3 - 6x^2 + 5x)\,dx = \frac{x^4}{4} - \frac{6x^3}{3} + \frac{5x^2}{2}$	**M1** **A1**
	$[\frac{x^4}{4} - \frac{6x^3}{3} + \frac{5x^2}{2}]_0^1 = (\frac{1}{4} - 2 + \frac{5}{2}) - 0 = \frac{3}{4}$	**M1**
	$[\frac{x^4}{4} - \frac{6x^3}{3} + \frac{5x^2}{2}]_1^2 = (4 - 16 + 10) - \frac{3}{4} = -\frac{11}{4}$	**M1** **A1**
	Total area $= \frac{3}{4} + \frac{11}{4} = \frac{7}{2}$	**M1** **A1**

13a	$\frac{dC}{dv} = -1400v^{-2} + \frac{2}{7}$	**M1** **A1**
	$-1400v^{-2} + \frac{2}{7} = 0$	**M1**
	$v^2 = 4900$	**M1**
	$v = 70$	**A1**
13b	$\frac{d^2c}{dv^2} = 2800v^{-3}$	**M1**
	$v = 70$, $\frac{d^2c}{dv^2} > 0$	**A1**

	Therefore minimum	
13c	$v = 70$ $C = \frac{1400}{70} + \frac{2\times 70}{7}$	**M1**
	$C = 40$	**A1**

14	$f(x) = x^4 - x^3 + 3x^2 + ax + b$ $f(1) = 4$ $f(2) = 22$	**M1**
	$3 + a + b = 4$	**A1**
	$16 + 8 + 12 - 2a + b = 22$	**A1**
	Solving simultaneously, $a = 5$ $b = -4$	**M1** **A1**

15a	1000	**M1**
15b	$1000e^{-5730c} = 500$	**M1**
	$e^{-5730c} = \frac{1}{2}$	**A1**
	$-5730c = \ln\frac{1}{2}$	**M1**
	$c = 0.000121$	**A1**
15c	$R = 1000e^{-2290c} = 62.5$	**M1** **A1**

16a	$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = \frac{cos^2 x+(1+\sin x)^2}{(1+\sin x)\cos x}$	**M1**
	$\frac{cos^2 x+1+2\sin x+sin^2 x}{(1+\sin x)(\cos x)}$	**A1**
	$= \frac{2(1+\sin x)}{(1+\sin x)\cos x}$	**M1**
	$= \frac{2}{\cos x}$	**A1**
16b	$\cos x = \frac{1}{2}$	**M1**
	$x = 60^o, 300^o$	**A1**

Topic List

Q1	Equations of lines
Q2	Simplifying surds
Q3	Reciprocal graph sketching
Q4	Roots
Q5	Integration and normal
Q6	Completing the square and graph sketching
Q7	Circles
Q8	Solving logarithms
Q9	Solving trig. equations
Q10	Area under curves
Q11	Algebraic fractions
Q12	Area of a curve
Q13	Maxima and minima
Q14	Factor theorem
Q15	Exponential modelling
Q16	Trig proof and solving trig