Practice Exam Paper G
Time: 2 Hours

P1

1a. Evaluate $(5\frac{4}{9})^{-\frac{1}{2}}$ **(2)**

b. Find the value of x such that,

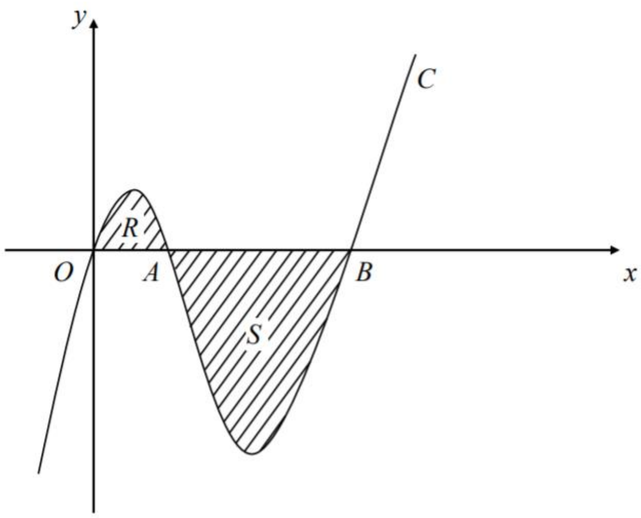
$$\frac{1+x}{x} = \sqrt{3},$$

giving your answer in the form $a + b\sqrt{3}$ where a and b are rational. **(4)**

(Total Marks: 6)

2. The figure shows part of the curve C with equation $y = f(x)$ where,

$$f(x) = x^3 - 6x^2 + 5x$$



The curve crosses the x-axis at the origin O and at the points A and B.

a. Factorise f(x) completely **(3)**

b. Write down the x-coordinates of the points A and B **(1)**

c. Find the gradient of C at A **(3)**

The region R is bounded by C and the line OA, and the region S is bounded by C and the line AB.

d. Use integration to find the area of the combined regions R and S, shown shaded in the figure. **(7)**

(Total Marks: 14)

3. Express $\frac{y+3}{(y+1)(y+2)} - \frac{y+1}{(y+2)(y+3)}$ as a single fraction in its simplest form. **(5)**

(Total Marks: 5)

4. Given that $2 \sin 2x = \cos 2x$

a. Show that $\tan 2x = 0.5$ **(1)**

b. Hence, find the values of x, to one decimal place, in the interval, $0 \leq x < 360$ for which $2 \sin 2x = \cos 2x$. **(5)**

(Total Marks: 6)

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5. $f(x) = x^3 - x^2 - 7x + c$, where c is a constant.

Given that $f(4) = 0$

a. Find the value of c **(2)**

b. Factorise $f(x)$ as the product of a linear factor and a quadratic factor. **(3)**

c. Hence show that, apart from $x = 4$, there are no real values of x for which $f(x) = 0$. **(2)**

(Total Marks: 7)

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6. Find in degrees, the value of x in the interval $0 \leq x \leq 360^\circ$ for which,

$$2 \cos^2 x - \cos x - 1 = \sin^2 x$$

Given your answers to 1 decimal place where appropriate. **(8)**

(Total Marks: 8)

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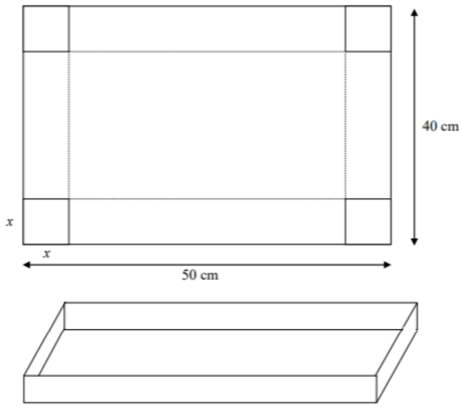
7. A rectangular sheet of metal measures 50 cm by 40 cm. Squares of side x cm are cut from each corner of the sheet and the remainder is folded along the dotted lines to make an open tray, as shown in the figure.



a. Show that the volume, V cm^3 , of the tray is given by

$$V = 4x(x^2 - 45x + 500)$$ **(3)**

b. State the range of possible values of x. **(1)**

c. Find the value of x for which V is a maximum. **(4)**

d. Hence find the maximum value of V. **(2)**

e. Justify that the value of V you found in part (d) is a maximum **(2)**

(Total Marks: 12)

8a. Using the substitution $u = 2^x$, show that the equation $4^x - 2^{(x+1)} - 15 = 0$ can be written in the form $u^2 - 2u - 15 = 0$ **(2)**

b. Hence solve the equation $4^x - 2^{(x+1)} - 15 = 0$, giving your answers to 2 d.p. **(4)**

(Total Marks: 6)

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9. A circle has centre (3, 4) and radius $3\sqrt{2}$. A straight line l has equation $y = x + 3$.

a. Write down an equation of the circle C. **(2)**

b. Calculate the exact coordinates of the two points where the line l intersects C, giving your answers in surds. **(5)**

c. Find the distance between these two points. **(2)**

(Total Marks: 9)

10a. Write down the first four terms of the binomial expansion, in ascending powers of x, of $(1 + 3x)^n$, where $n > 2$ **(2)**

Given that the coefficient of x^3 in this expansion is ten times the coefficient of x^2 ,

b. Find the value of n **(2)**

c. Find the coefficient of x^4 in the expansion. **(2)**

(Total Marks: 6)

11a. $f(x) = 5 \sin 3x$, $0 \leq x \leq 180$
a. Sketch the graph of $f(x)$, indicating the value of x at each point where the graph intersects the x-axis (3)

b. Write down the coordinates of all the maximum and minimum points of $f(x)$ (3)

c. Calculate the values of x for which $f(x) = 2.5$ (4)

(Total Marks: 10)

12a. Given that $3 + 2 \log_2 x = \log_2 y$, show that $y = 8x$ **(3)**

b. Hence, or otherwise, find the roots α or β, where $\alpha < \beta$, of the equation,

$$3 + 2 \log_2 x = \log_2 (14x - 3)$$ **(3)**

c. Show that $\log_2 \alpha = -2$. **(1)**

d. Calculate $\log_2 \beta$, giving your answer to 3 significant figures **(3)**

(Total Marks: 10)

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13. Given that $f(x) = (2x^{\frac{3}{2}} - 3x^{-\frac{3}{2}})^2 + 5$, $x > 0$
a. Find to 3 significant figures, the value of x for which $f(x) = 5$. **(3)**

b. Show that $f(x)$ may be written in the form $Ax^3 + \frac{B}{x^3} + C$, where A, B and C are constants to be found. **(3)**

c. Hence evaluate $\int_{1}^{-2} f(x)\, dx$ **(5)**

Total Marks: 11)

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14. For the curve C with equation $y = x^4 - 8x^2 + 3$,

a. Find $\frac{dy}{dx}$ **(2)**

b. Find the coordinates of each of the stationary points **(5)**

c. Determine the nature of each stationary point **(3)**

Total Marks: 10)

Total Marks: 120

Mark Scheme

1a	$= (\frac{49}{9})^{-\frac{1}{2}} = \sqrt{\frac{9}{49}} = \frac{3}{7}$	**M1** **A1**
1b	$1 + x = \sqrt{3}x$ $1 = x(\sqrt{3} - 1)$	**M1**
	$x = \frac{1}{\sqrt{3}-1}$	**A1**
	$x = \frac{1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$ $= \frac{\sqrt{3}+1}{2-1}$ $= \frac{1}{2} + \frac{1}{2}\sqrt{3}$	**M1** **A1**

2a	$x(x^2 - 6x + 5)$	**M1**
	$x(x - 1)(x - 5)$	**M1** **A1**
2b	1 and 5	**B1**
2c	$\frac{dy}{dx} = 3x^2 - 12x + 5 = -4$	**M1** **A1**
	At $x = 1$, $\frac{dy}{dx} = 3 - 12 + 5 = -4$	**A1**
2d	$\int (x^3 - 6x^2 + 5x)\, dx = \frac{x^4}{2} - \frac{6x^3}{3} + \frac{5x^2}{2}$	**M1** **A1**
	$[\frac{x^4}{2} - \frac{6x^3}{3} + \frac{5x^2}{2}]_0^1 = \frac{1}{4} - 2 + \frac{5}{2} = \frac{3}{4}$	**M1** **A1**
	At 5, $\frac{625}{4} - 250 + \frac{125}{2} = -31\frac{1}{4}$	**A1**
	To find S: $-31\frac{1}{4} - \frac{3}{4} = -32$	**M1**
	Total area $= 32 + \frac{3}{4} = 32\frac{3}{4}$	**A1**

3	$\frac{y+3}{(y+1)(y+2)} - \frac{y+1}{(y+2)(y+3)} = \frac{(y+3)^2-(y+1)^2}{(y+1)(y+2)(y+3)}$	**M1**
	$= \frac{(y^2+6y+9)-(y^2+2y+1)}{(y+1)(y+2)(y+3)}$ $= \frac{4y+8}{(y+1)(y+2)(y+3)}$	**M1** **A1**
	$= \frac{4(y+2)}{(y+1)(y+2)(y+3)}$ $= \frac{4}{(y+1)(y+3)}$	**M1** **A1**

4a	$\frac{\sin 2x}{\cos 2x} = \tan 2x$ $\tan 2x = 0.5$	**M1**
4b	$\tan 2x = 0.5$ $2x = 26.6^o$	**B1**
	$2x = 206.6$	**B1**
	$2x = 386.6, 566.6$	**B1**
	$x = 13.3, 103.3, 193.3, 283.3$	**M1** **A1**

5a	$64 - 16 - 28 + c = 0$	**M1**
	$c = -20$	**A1**
5b	$(x - 4)(x^2 + 3x + 5)$	**B1** **M1**

		A1
5c	For $x^2 + 3x + 5$, $b^2 - 4ac = -11 < 0$	**M1**
	Therefore, no real roots.	**A1**

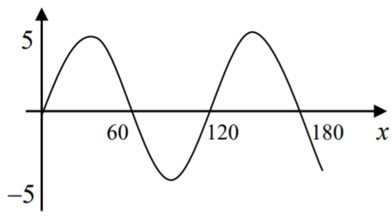
6	$2\cos^2 x - \cos x - 1 = 1 - \cos^2 x$	**M1**
	$3\cos^2 x - \cos x - 2 = 0$	**A1**
	$(3\cos x + 2)(\cos x - 1) = 0$ $\cos x = -\frac{2}{3}$ $\cos x = 1$	**M1** **A1**
	$x = 0^o$ $x = 131.8^o$	**B1** **A1**
	$x = (360 - 131.8) = 228.2^o$	**M1** **A1**

7a	$l = (50 - 2x)$ $W = (40 - 2x)$	**B1**
	$V = x(50 - 2x)(40 - 2x)$	**M1**
	$V = x(2000 - 80x - 100x + 4x^2)$ $= 4x(x^2 - 45x + 500)$	**A1**
7b	$0 < x < 20$	**B1**
7c	$\frac{dV}{dx} = 12x^2 - 360x + 2000$	**M1** **A1**
	$\frac{dV}{dx} = 0$ $3x^2 - 90x + 500 = 0$	**M1**
	$x = 22.6$	**A1**
7d	$V_{max} = 4 \text{ x } 7.36\ (7.36)^2$ $= 6564$	**M1** **A1**
7e	$V'' = 24x - 360$ When $x = 6564$ $V'' = -183$	**M1**
	$-183 < 0$, therefore maximum	**A1**

8a	$4^x = (2^x)^2 = u^2$	**M1**
	$u^2 - 2u - 15 = 0$	**A1**
8b	$u^2 - 2u - 15 = (u - 5)(u + 3)$	**M1** **A1**
	$u = 5$ $2^x = 5$	**M1**
	$x = \frac{\log 5}{\log 2} = 2.32$	**A1**

9a	$(x - 3)^2 + (y - 4)^2 = 18$	**M1** **A1**
9b	$y = x + 3$ $(x - 3)^2 + (x - 1)^2 = 18$	**M1**
	$2x^2 - 8x = 8$	**A1**
	$x = 2 \pm \sqrt{8}$ $y = 5 \pm \sqrt{8}$	**M1** **A1** **A1**
9c	Distance $= \sqrt{((2\sqrt{8})^2 + (2\sqrt{8})^2}$	**M1**
	$= 8$	**A1**

10a	$1 + n(3x) + \frac{n(n-1)}{2}(3x)^2 + \frac{n(n-1)(n-2)}{3!}(3x)^3$	**B1** **B1**
10b	$\frac{n(n-1)(n-2)}{6} \times 27 = 10 \times \frac{n(n-1)}{2} \times 9$	**M1**
	$n = 12$	**A1**
10c	$\frac{n(n-1)(n-2)(n-3)}{4!} \times (3x)^4$	**M1**
	Therefore, when $n = 12$, coefficient = 40095	**A1**

11a		
	Shape	**B1**
	60, 120, 180 on x-axis	**B1**
	5, -5 on y-axis	**B1**
11b	(30, 5) (150, 5) (90, -5)	**B1** **B1** **B1**
11c	$f(x) = 2.5$ $\sin 3x = \frac{1}{2}$ $3x = 30$	**B1**
	$3x = 30, 160, 390, ,510$	**M1** **M1**
	$x = 10, 50, 130, 170$	**A1**

12a	$2 \log x = \log x^2$	**B1**
	$\log_2 (\frac{y}{x^2}) = 3$	**M1**
	$\frac{y}{x^2} = 2^3$ $y = 8x^2$	**A1**
12b	$14x - 3 = 8x^2$	**M1**
	$(4x - 1)(2x - 3) = 0$ Roots = $\frac{1}{4}, \frac{3}{2}$	**M1** **A1**
12c	$\log_2 \alpha = \log_2 \frac{1}{4} = \log_2 (2^{-2}) = -2$	**B1**
	$\log_2 1.5 = k$ $2^k = 1.5$	**M1**
	$k = \frac{\log 1.5}{\log 2} = 0.585$	**M1** **A1**

13a	$2x^{\frac{3}{2}} - 3x^{-\frac{3}{2}} = 0$	**M1**
	$x^3 = \frac{3}{2}$ $x = 1.447\ldots. = 1.14$ (3 s.f)	**M1**
13b	$f(x) = 4x^3 + 9x^{-3} - 12 + 5$	**B1**
	$= 4x^3 + \frac{9}{x^3} - 7$	**B1** **B1**
13c	$\int_1^2 f(x)dx = [x^4 - \frac{9}{2}x^{-2} - 7x]_1^2$	**M1** **A2**
	$= (2^4 - \frac{9}{2} \times 2^{-2} - 14) - (1 - \frac{9}{2} - 7)$	**M1**
	$= 11\frac{3}{8}$	**A1**

14a	$\frac{dy}{dx} = 4x^3 - 16x$	**M1** **A1**
14b	$4x^3 - 16x = 0$	**M1**
	$4x(x^2 - 4) = 0$ $x = 0, 2, -2$	**A2**
	$y = 3, -13, -13$	**M1** **A1**
14c	$\frac{d^2y}{dx^2} = 12x^2 - 16$	**M1**
	$x = 0$ (maximum) $x = 2$ (minimum) $x = -2$ (minimum)	**A1** **A1**

Topic List

Q1	Surds
Q2	Shaded regions
Q3	Algebraic fractions
Q4	Solving trig. equations
Q5	Factor theorem
Q6	Solving trig
Q7	Maxima and minima problem
Q8	Indices laws
Q9	Equations of straight lines
Q10	Binomial expansion
Q11	Sketching trig functions
Q12	Logarithms
Q13	Integrals
Q14	Stationary points