Practice Exam Paper F
Time: 2 Hours

P1

1a. By completing the square, find in terms of the constant k the roots of the equation $x^2 + 4kx - k = 0$. **(4)**

b. Hence find the set of values of k for which the equation has no real roots. **(4)**

(Total Marks: 8)

2a. Describe fully a single transformation that maps the graph of $y = \frac{1}{x}$ onto the graph of $y = \frac{3}{x}$. **(2)**

b. Sketch the graph of $y = \frac{3}{x}$ and write down the equations of any asymptotes. **(3)**

c. Find the values of the constant c for which the straight line $y = c - 3x$ is a tangent to the curve $y = \frac{3}{x}$ **(4)**

(Total Marks: 9)

3a. Given that $y = 2^x$, fin expression in terms of y for,
i. 2^{x+2}
ii. 2^{3-x} **(4)**

b. Show that using the substitution $y = 2^x$, the equation

$$2^{x+2} + 2^{3-x} = 33$$

can be rewritten as

$$4y^2 - 33y + 8 = 0.$$ **(2)**

c. Hence solve the equation, $2x + 2 + 23 – x = 33$ **(4)**

(Total Marks: 10)

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4. The straight line l_1 has gradient 2 and passes through the point with coordinates $(4, -5)$.

a. Find an equation for l_1 in the form $y = mx + c$. **(2)**

The straight line l_2 is perpendicular to the line with equation $3x - y = 4$ and passes through the point with coordinates $(3, 0)$.

b. Find an equation for l_2. **(3)**

c. Find the coordinates of the point where l_1 and l_2 intersect. **(3)**

(Total Marks: 8)

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5. The curve C has the equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = (x + 2)^3$$

a. Sketch the curve C, showing the coordinates of any points of intersection with the coordinate axes. **(3)**

b. Find $\mathrm{f}'(x)$. **(4)**

The straight line l is the tangent to C at the point $P\ (-1, 1)$.

c. Find an equation for l. **(3)**

The straight line m is parallel to l and is also a tangent to C.

d. Show that m has the equation $y = 3x + 8$ **(4)**

(Total Marks: 14)

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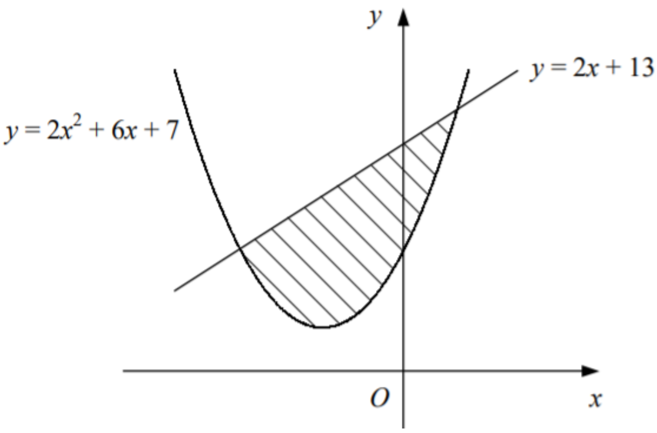
6. The figure shows the curve $y = 2x^2 + 6x + 7$ and the straight line $y = 2x + 13$



a. Find the coordinates of the points where the curve and the line intersect. **(4)**

b. Find the area of the shaded region bounded by the curve and line **(7)**

(Total Marks: 11)

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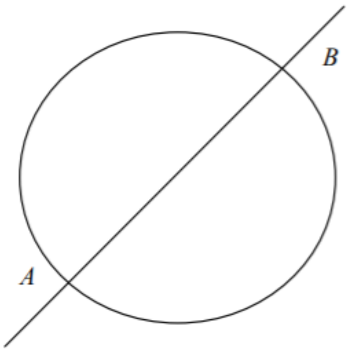
7. The points $A(-3, -2)$ and $B(8 , 4)$ are at the ends of a diameter of the circle shown in the figure.



a. Find the coordinates of the centre of the circle **(2)**

b. Find an equation of the diameter AB, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

c. Find an equation of tangent to the circle at B **(3)**

The line l passes through A and the origin.

d. Find the coordinates of the point at which l intersects the tangent to the circle at B, giving your answer as exact fractions **(4)**

(Total Marks: 13)

8. The equation $x^2 + 5kx + 2k = 0$, where k is a constant, has real roots

a. Prove that $k(25k - 8) \geq 0$. **(2)**

b. Hence find the set of possible values of k. **(4)**

c. Write down the values of k for which the equation $x^2 + 5kx + 2k = 0$ has equal roots. **(1)**

(Total Marks: 7)

9a. Expand $(2\sqrt{x} + 3)^2$ **(2)**

b. Hence evaluate $\int_1^2 (2\sqrt{x} + 3)^2 \, dx$, giving your answer in the form $a + b\sqrt{2}$, where a and b are integers. **(5)**

(Total Marks: 7)

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10. Find all the values of x in the interval $0 \leq x \leq 360$ for which,

a. $\cos(x + 75) = 0$ **(3)**

b. $\sin 2x = 0.7$, giving your answers to one decimal place. **(5)**

(Total Marks: 8)

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11. Given that $\log_2 x = a$, find, in terms of a, the simplest form of,

a. $\log_2 (16x)$ **(2)**

b. $\log_2 \left(\frac{x^4}{2}\right)$ **(3)**

c. Hence or otherwise, solve,

$$\log_2 (16x) = \log_2 \left(\frac{x^4}{2}\right)$$

giving your answer in its simplest surd form. **(4)**

(Total Marks: 9)

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12. Express,

$$\frac{2x^3 + x^2}{x^2 - 4} \times \frac{x - 2}{2x^2 - 5x - 3}$$

As a single fraction in its simplest form. **(5)**

(Total Marks: 5)

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13. Given that,

$$(2 + x)^5 + (2 - x)^5 = A + Bx^2 + Cx^4$$

Find the values of the constants A, B and C **(6)**

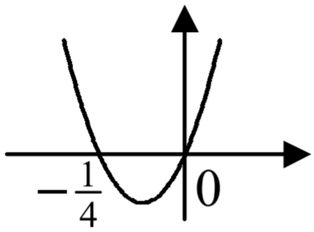
b. Using the substitution $y = x^2$ and your answers to part a, solve,

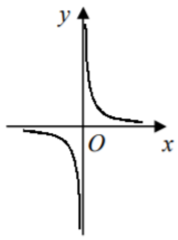
$$(2 + x)^5 + (2 - x)^5 = 349$$

(5)

(Total Marks: 11)

Mark Scheme

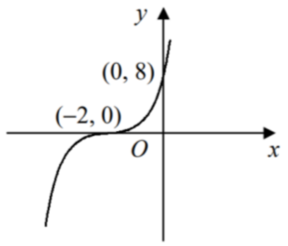
1a	$(x + 2k)^2 - (2k)^2 - k = 0$	**M1**
	$(x + 2k)^2 = 4k^2 + k$	**A1**
	$x + 2k = \pm\sqrt{4k^2 + k}$	**M1**
	$x = -2k \pm \sqrt{4k^2 + k}$	**A1**
1b	No real roots if, $4k^2 + k < 0$	**M1**
	$k(4k + 1) < 0$, critical values, $-\frac{1}{4}, 0$	**A1**
		
	$-\frac{1}{4} < k < 0$	**M1** **A1**

2a	Stretch by factor of 3 in y-direction about x-axis	**B2**
2b	Asymptotes: x = 0 and y = 0 	**B2** **B1**
2c	$\frac{3}{x} = c - 3x$ $3 = cx - 3x^2$	**M1**
	$3x^2 - cx + 3 = 0$ tangent, therefore equal roots, $b^2 - 4ac = 0$ $(-c)^2 - (4 \times 3 \times 3) = 0$	**M1** **A1**
	$c^2 = 36$ $c = \pm 6$	**A1**

3ai	$2^{x+2} = 2^2 \times 2^x = 4y$	**M1** **A1**
3aii	$2^{3-x} = \frac{2^3}{2^x} = \frac{8}{y}$	**M1** **A1**
3b	$2^{x+2} + 2^{3-x} = 33$ $4y + \frac{8}{y} = 33$ $4y^2 + 8 = 33y$	**M1**
	$4y^2 - 33y + 8 = 0$	**A1**
3c	$(4y - 1)(y - 8) = 0$	**M1**
	$y = \frac{1}{4}$ $y = 8$	**A1**
	$x = -2$ $x = 3$	**A2**

4a	$y + 5 = 2(x - 4)$	**M1**
	$y = 2x - 13$	**A1**
4b	$3x - y = 4$ $y = 3x - 4$	**M1** **A1**

	Therefore, gradient = 3 gradient of $l_2 = -\frac{1}{3}$	
	$y - 0 = -\frac{1}{3}(x - 3)$	**A1**
4c	$2x - 13 = -\frac{1}{3} + 1$ $x = 6$	**M1** **A1**
	Therefore, point of intersection is (6, -1)	**A1**

5a		**B3**
5b	$f(x) = (x + 2)(x^2 + 4x + 4)$ $f(x) = x^3 + 4x^2 + 4x + 2x^2 + 8x + 9$	**M1**
	$f(x) = x^2 + 6x^2 + 12x + 8$	**A1**
	$f'(x) = 3x^2 + 12x + 12$	**M1** **A1**
5c	gradient = 3 – 12 + 12 = 3	**B1**
	$y - 1 = 3(x + 1)$ $y = 3x + 4$	**M1** **A1**
5d	Gradient $m = 3$ $3x^2 + 12x + 12 = 3$ $x^2 + 4x + 3 = 0$ $(x + 1)(x + 3) = 0$	**M1**
	$x = -1$ (at P) and -3	**A1**
	$x = -3$, therefore $y = -1$ $y + 1 = 3(x + 3)$	**M1**
	$y = 3x + 8$	**A1**

6a	$2x^2 + 6x + 7 = 2x + 13$ $x^2 + 3x - 3 = 0$	**M1**
	$(x + 3)(x - 1) = 0$	**M1**
	$x = -3$ $x = 1$	**A1**
	Therefore coordinates (-3, 7), (1, 15)	**A1**
6b	Area under curve = $\int_{-3}^{1}(2x^2 + 6x + 7)\,dx$ $= [\frac{2}{3}x^3 + 3x^2 + 7x]_{-3}^{1}$	**M1** **A2**
	$= (\frac{2}{3} + 3 + 7) - (-18 + 27 - 21) = 22\frac{2}{3}$	**M1**
	Area of trapezium = $\frac{1}{2} \times (7 + 15) \times 4 = 44$	**B1**
	Shaded area = $44 - 22\frac{2}{3} = 21\frac{1}{3}$	**M1** **A1**

7a	Mid point of $AB = [\frac{1}{2}(-3 + 8), \frac{1}{2}(-2 + 4)] = (\frac{5}{2}, 1)$	**M1** **A1**
7b	$M_{AB} = \frac{4-(-2)}{8-(-3)} = \frac{6}{11}$	**M1** **A1**
	Equation of AB = $y - 4 = \frac{6}{11}(x - 8)$	**M1**
	$11y - 44 = 6x - 48$ $6x - 11y - 4$	**A1**

7c	Gradient of tangent = $-\frac{11}{6}$	**B1**
	Equation: $y - 4 = -\frac{11}{6}(x - 8)$	**M1** **A1**
	Equation of l: $y = \frac{2}{3}x$	**B1**
	Substitute into (c): $\frac{2}{3}x - 4 = -\frac{11}{6}x + \frac{88}{6}$	**M1**
	$x = 7\frac{7}{15}$ $y = 4\frac{44}{45}$	**A1** **A1**

8a	$b^2 - 4ac \geq 0$ $(5k)^2 - 8k \geq 0$ $k(25k - 8) \geq 0$	**M1** **A1**
8b	$k = 0$ $k = \frac{8}{25}$	**B1** **B1**
	$k \leq 0$ $k \leq \frac{8}{25}$	**M1** **M1**
8c	$k = 0$ $k = \frac{8}{25}$	**B1**

9a	$4x + 9 + 12\sqrt{x}$	**B1** **B1**
9b	$\int \left(4x + 12x^{\frac{1}{2}} + 9\right) dx = 2x^2 + 8x^{\frac{3}{2}} + 9x$	**M1** **A1**
	$= (8 + (8 \text{ x } 2^{\frac{3}{2}}) + 18) - (2 + 8 + 9)$	**M1**
	$= 9 + 16\sqrt{2}$	**M1** **A1**

10a	$x = -15^o$ $x = 345^o$	**B1**
	$x + 75 = 360 - 60$	**M1**
	$x = 225, 345$	**A1**
10b	$(2x) = 44.4$	**B1**
	$2x = 135.6$	**B1**
	$2x = 404.4$ $2x = 495.6$	**B1**
	$x = 22.2, 67.8, 202.2, 247.8$	**M1** **A1**

11a	$\log_2 (16x) = \log_2 16 + \log_2 x$	**M1**
	$= 4 + a$	**A1**
11b	$\log_2 \left(\frac{x^4}{2}\right) = \log_2 x^4 - \log_2 2$	**M1**
	$= 4 \log_2 x - \log_2 2$	**M1**
	$= 4a - 1$	**A1**
11c	$\frac{1}{2} = 4 + a - (4a - 1)$	**M1**
	$a = \frac{3}{2}$	**A1**
	$\log_2 x = \frac{3}{2}$ $x = 2^{\frac{3}{2}}$	**M1**
	$x = \sqrt{8}$ $x = 2\sqrt{2}$	**A1**

12	$\frac{x^2(2x+1)}{(x+2)(x-2)} \times \frac{x-2}{(2x+1)(x-3)}$	**M1** **A2**
	$= \frac{x^2}{(x+2)(x-3)}$	**M1** **A1**

13a	Using coefficients 1, 5, 10, 10, 5, 1 as necessary	**M1**
	Using powers x^5 $2x^4$ 2^2x^3	**M1**
	Completing the method	**M1**
	$A = 64$	**B1**
	$B = 160$ $C = 20$	**A2**
13b	$20x^4 + 160x^2 + 64 = 349$	**M1**
	$4y^2 + 32y - 57 = 0$	**A1**
	Solving for y	**M1**
	Replacing by x^2 and completing to obtain all relevant values of x	**M1**
	$\pm\sqrt{\frac{3}{2}}$	**A1**

Topic List

Q1	Completing the square and roots
Q2	Sketching and transforming graphs
Q3	Index laws
Q4	Straight lines
Q5	Curve sketching, differentiation and tangents
Q6	Simultaneous equations and areas
Q7	Equations of lines
Q8	Real roots
Q9	Integrations
Q10	Solving trig.
Q11	Solving logarithms
Q12	Simplifying algebraic fractions
Q13	Binomial expansion