Practice Exam Paper E
Time: 2 Hours

P1

1a. Solve the inequality, $x^2 - 13x + 30 < 0$ **(3)**

b. Hence find the set of values of y such that, $2^{2y} - 13(2^y) + 30 < 0$ **(3)**

(Total Marks: 6)

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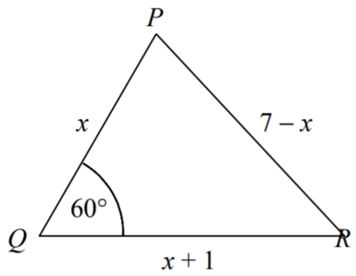
2. The figure shows the triangle PQR in which PQ = x, PR = $7 - x$, QR = $x + 1$ and $P\hat{Q}R = 60^\circ$.



Find the values of x. **(4)**

(Total Marks: 4)

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3. The coefficient of x^2 in the binomial expansion of $(1 + kx)^7$, where k is a positive constant, is 525.

a. Find the value of k **(3)**

Using this value of k,

b. Show that the coefficient of x^3 in the expansion is 4375 **(2)**

c. Find the first three terms in the expansion in ascending powers of x of $(2 - x)(1 + kx)^7$ **(3)**

(Total Marks: 8)

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4. Given that,

$$8 \tan x - 3 \cos x = 0$$

Show that

$$3 \sin^2 x + 8 \sin x - 3 = 0$$

(3)

b. Find to 2 decimal places, the values of x in the interval $0 \leq x \leq 360$ such that, $8 \tan x - 3 \cos x = 0$ **(5)**

(Total Marks: 8)

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5. Given that,

$$\frac{dy}{dx} = \frac{x^3 - 4}{x^3}, x \neq 0$$

And that $y = 0$ when $x = -1$, find the value of y when $x = 2$ **(8)**

(Total Marks: 8)

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6. The circle C has centre (-1, 6) and radius $2\sqrt{5}$

a. Find an equation for C **(2)**

The line $y = 3x - 1$ intersects C at the points A and B.

b. Find the x-coordinates of A and B **(4)**

c. Show that $AB = 2\sqrt{10}$ **(3)**

(Total Marks: 9)

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7. $f(x) = 3 - \frac{x-1}{x-3} + \frac{x+11}{2x^2-5x-3}$, $x < -1$

a. Show that, $f(x) = \frac{4x-1}{2x+1}$ **(5)**

Given that $f'(x) = \frac{6}{(2x+1)^2}$,

b. Find an equation for the tangent to the curve $y = f(x)$ at the point where $x = -2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**

(Total Marks: 8)

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8. $f(x) = 2 - x + 3x^{\frac{2}{3}}$, $x > 0$

a. Find $f'(x)$ and $f''(x)$ **(3)**

b. Find the coordinates of the turning point of the curve $y = f(x)$. **(4)**

c. Determine whether the turning point is a maximum or minimum point **(2)**

(Total Marks: 9)

9a. Sketch the curve $y = 5^{x-1}$, showing the coordinates of any points of intersection with the coordinate axes. **(2)**

b. Find, to 3 significant figures, the x-coordinates of the points where the curve $y = 5^{x-1}$ intersects
i. The straight line $y = 10$,
ii. The curve $y = 2x$. **(6)**

(Total Marks: 8)

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10. $f(x) = 2x^3 + 3x^2 - 6x + 1$

a. Find the remainder when $f(x)$ is divided by $(2x - 1)$. **(2)**

bi. Find the remainder when $f(x)$ is divided by $(x + 2)$.
ii. Hence, or otherwise, solve the equation

$$2x^3 + 3x^2 - 8x - 8 = 0$$

Giving your answers to 2 decimal places where appropriate. **(7)**

(Total Marks: 9)

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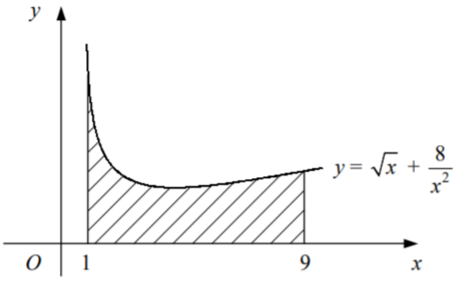
11. The figure shows the curve with equation $y = \sqrt{x} + \frac{8}{x^2}$, $x > 0$



a. Find the coordinates of the minimum point of the curve **(7)**
b. Show that the area of the shaded region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 9$ is $24\frac{4}{9}$. **(5)**

(Total Marks: 12)

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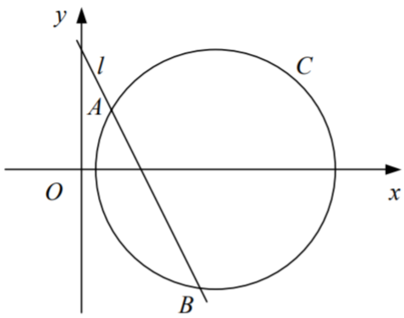
12. The figure shows the circle C and the straight line l. The centre of C lies on the x-axis and l intersects C at the points A (2, 4) and B (8, −8).



a. Find the gradient of l **(2)**

b. Find the coordinates of the mid-point of AB **(2)**

c. Find the coordinates of the centre of C **(5)**

d. Show that C has the equation $x^2 + y^2 - 18x + 16 = 0$ **(3)**

(Total Marks: 12)

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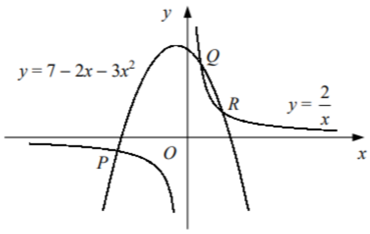
13. The figure shows the curves with equations $y = 7 - 2x - 3x^2$ and $y = \frac{2}{x}$



The two curves intersect at the points P, Q and R.
a. Show that the x-coordinates of P, Q and R satisfy the equation
$3x^3 + 2x^2 - 7x + 2 = 0$ **(2)**

Given that P has coordinates (−2, −1)
b. Find the coordinates of Q and R **(6)**

(Total Marks: 9)

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14. The straight line l_1 passes through the point A (−2, 5) and the point B (4, 1).
a. Find an equation for l_1 in the form $ax + by = c$, where a, b and c are integers **(4)**

The straight line l_2 passes through B and is perpendicular to l_1.
b. Find an equation for l_2. **(3)**

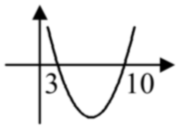
Given that l_2 meets the y-axis at the point C,

c. Show that triangle ABC is isosceles. **(4)**

Total Marks: 11)

Total Marks: 120

Mark Scheme

1a	$(x - 3)(x - 10) < 0$	**M1**
		**M1**
	$3 < x < 10$	**A1**
1b	Let $x = 2^y$ $3 < 2^y < 10$ $\lg 3 < y \lg 2 < \lg 10$	**M1**
	$\frac{\lg 3}{\lg 2} < y < \frac{\lg 10}{\lg 2}$	**M1**
	$1.58 < y < 3.32$	**A1**

2	Using the cosine rule, $(7 - x)^2 = x^2 + (x + 1)^2 - [2\times x \times (x + 1) \times \cos 60]$	**M1** **A1**
	$49 - 14x + x^2 = x^2 + x^2 + 2x + 1 - x^2 - x$ $15x = 48$ $x = \frac{16}{5}$	**M1** **A1**

3a	$(1 + kx)^7 = \ldots. + \binom{7}{2}(kx)^2 + \ldots$	**B1**
	$\frac{7\times 6}{2} \times k^2 = 525$ $k^2 = \frac{525}{21} = 25$	**M1**
	$k > 0$ Therefore, $k = 5$	**A1**
3b	$(1 + 5x)^7 = \ldots. + \binom{7}{3}(kx)^2 + \ldots$ Therefore, coefficient of $x^3 = \frac{7\times 6\times 5}{3\times 2} \times 125 = 4375$	**M1** **A1**
3c	$(1 + 5x)^7 = 1 + 35x + 525x^2 + \ldots$	**B1**
	$(2 - x)(1 + 5x)^7 = (2 - x)(1 + 35x + 525x^2 + \ldots)$ $= 2 + 70x + 1050x^2 - x - 35x^2$	**M1**
	$= 2 + 69x + 1015x^2$	**A1**

4a	$\frac{8 \sin x}{\cos x} - 3 \cos x = 0$	**M1**
	$8 \sin x - 3 \cos^2 x = 0$ $8 \sin x - 3(1 - \sin^2 x) = 0$	**M1**
	$3 \sin^2 x + 8 \sin x - 3 = 0$	**A1**
4b	$(3 \sin x - 1)(\sin x + 3) = 0$	**M1**
	$\sin x = -3$ (no solutions) $\sin x = \frac{1}{3}$	**A1**
	$x = 19.47$ $x = 180 - 19.47$	**B1** **M1**
	$x = 19.47^\circ$ $x = 160.53^\circ$	**A1**

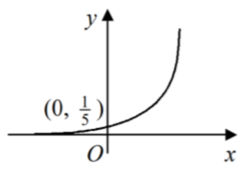
5	$\int(1 - 4x^{-3})\, dx$	**M1**
	$y = x + 2x^{-2} + c$	**M1** **A2**
	$x = -1$ $y = 0$ $0 = -1 + 2 + c$	**M1**

	$c = -1$	**A1**
	$y = x + 2x^{-2} - 1$ When $x = 2$, $y = 2 + \frac{1}{2} - 1 = \frac{3}{2}$	**M1** **A1**

6a	$(x + 1)^2 + (y - 6)^2 = (2\sqrt{5})^2$	**M1**
	$(x + 1)^2 + (y - 6)^2 = 20$	**A1**
6b	sub. $y = 3x - 1$ into equation C $(x + 1)^2 + [(3x - 1) - 6]^2 = 20$	**M1**
	$(x + 1)^2 + (3x - 7)^2 = 20$	**A1**
	$x^2 - 4x + 3 = 0$ $(x - 1)(x - 3) = 0$	**M1**
	$x = 1$ $x = 3$	**A1**
6c	$x = 1$, $y = 2$ $x = 3$, $y = 8$	**B1**
	$AB = \sqrt{(3-1)^2 + (8-2)^2}$ $= \sqrt{40}$ $= 2\sqrt{10}$	**M1** **A1**

7a	$f(x) = 3 - \frac{x-1}{x-3} + \frac{x+11}{(2x+1)(x-3)}$	**B1**
	$= \frac{3(2x^2-5x-3)-(x-1)(2x+1)+(x+11)}{(2x+1)(x-3)}$	**M1** **A1**
	$= \frac{4x^2-13x+3}{(2x+1)(x-3)}$ $= \frac{(4x-1)(x-3)}{(2x+1)(x-3)}$ $= \frac{4x-1}{2x+1}$	**M1** **A1**
7b	$x = -2$ $y = 3$ Gradient = $\frac{2}{3}$	**A1**
	$y - 3 = \frac{2}{3}(x + 2)$	**M1**
	$3y - 9 = 2x + 4$ $2x - 3y + 13 = 0$	**A1**

8a	$f'(x) = -1 + 2x^{-\frac{1}{3}}$	**M1** **A1**
	$f''(x) = -\frac{2}{3}x^{-\frac{4}{3}}$	**A1**
8b	For turning point, $-1 + 2x^{-\frac{1}{3}} = 0$	**M1**
	$x^{\frac{1}{3}} = 2$	**M1**
	$x = 8$	**A1**
	Therefore, (8, 6)	**A1**
8c	$f''(8) = -\frac{1}{24}$ $f''(x) < 0$ Therefore maximum	**M1** **A1**

9a		**B2**

9bi	$5^{x-1} = 10$ $(x - 1) \lg 5 = \log 10 = 1$	**M1**
	$x = \frac{1}{\lg 5} + 1 = 2.43$	**M1** **A1**
9bii	$5^{x-1} = 2^{x}$ $(x - 1) \lg 5 = x \lg 2$	**M1**
	$x (\lg 5 - \lg 2) = \lg 5$	**M1**
	$x = \frac{\lg 5}{\lg 5 - \lg 2} = 1.76$	**A1**

10a	$f(\frac{1}{2}) = \frac{1}{4} + \frac{3}{4} - 3 + 1 = -1$	**M1** **A1**
10bi	$f(-2) = -16 + 12 + 12 + 1 = 9$	**B1**
10bii	$x = -2$ is a soltion to $f(x) = 9$	**M1** **A1**
	$x + 2 \overline{)\,2x^3 + 3x^2 - 6x - 8}$ quotient $2x^2 - x - 4$ $2x^3 + 4x^2$ $-x^2 - 6x$ $-x^2 - 2x$ $-4x - 8$ $-4x - 8$	**M1** **A1**
	$(x + 2)(2x^2 - x - 4) = 0$ $x = -2$ $x = -1.19$ $x = -1.69$	**M1** **A1**

11a	$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - 16x^{-3}$	**M1** **A2**
	For minimum, $\frac{1}{2}x^{-\frac{1}{2}} - 16x^{-3} = 0$	**M1**
	$\frac{1}{2}x^{-3}\left(x^{\frac{5}{2}} - 32\right) = 0$ $x^{\frac{5}{2}} = 32$	**A1** **M1**
	Therefore coordinate: $(4, \frac{5}{2})$	**A1**
11b	$\int_1^9 (\sqrt{x} + \frac{8}{x^2})\, dx$ $= [\frac{2}{3}x^{\frac{3}{2}} - 8x^{-1}]_1^9$	**M1** **A2**
	$= (18 - \frac{8}{9}) - (\frac{2}{3} - 8)$	**M1**
	$= 24\frac{4}{9}$	**A1**

12a	$\frac{-8-4}{8-2} = -2$	**M1** **A1**
12b	$=(\frac{2+8}{2}, \frac{4-8}{2}) = (5, -2)$	**M1** **A1**
12c	Perpedicular gradient = $\frac{-1}{-2} = \frac{1}{2}$	**M1**
	Perpendicular bisector: $y + 2 = \frac{1}{2}(x - 5)$	**M1** **A1**
	Centre where $y = 0$, $x = 9$ $\rightarrow (9, 0)$	**M1** **A1**
12d	Radius = distance (2, 4) to (9, 0) $= \sqrt{49 + 16} = \sqrt{65}$	**B1**
	$(x - 9)^2 + (y - 0)^2 = (\sqrt{65})^2$	**M1**
	$x^2 - 18x + 81 + y^2 = 65$ $x^2 + y^2 - 18x + 16 = 0$	**A1**

13a	$7 - 2x - 3x^2 = \frac{2}{x}$ $7x - 2x^2 - 3x^3 = 2$	**M1**
	$3x^3 + 2x^2 - 7x + 2 = 0$	**A1**
13b	$x = -2$ is a solution, therefore $(x + 2)$ is a factor	**B1**
	$\begin{array}{r} 3x^2 - 4x + 1 \\ x+2 \,\big)\, 3x^3 + 2x^2 - 7x + 2 \\ 3x^3 + 6x^2 \qquad\qquad \\ -4x^2 - 7x \qquad \\ -4x^2 - 8x \qquad \\ x + 2 \\ x + 2 \end{array}$	**M1** **A1**
	$(x + 2)(3x^2 - 4x + 1) = 0$ $(x + 2)(3x - 1)(x - 1) = 0$	**M1**
	$x = -2$ (at P) $x = \frac{1}{3}$, $y = 6$ $x = 1$, $y = 2$ Coordinates: $(\frac{1}{3}, 6)$ and $(1, 2)$	**A2**

14a	Gradient $= \frac{1-5}{4-(-2)} = -\frac{2}{3}$	**M1** **A1**
	$y - 5 = -\frac{2}{3}(x + 2)$	**M1**
	$3y - 15 = -2x - 4$ $2x + 2y = 11$	**A1**
14b	Gradient $l_2 = \frac{-1}{-\frac{2}{3}} = \frac{3}{2}$	**M1** **A1**
	$y - 1 = \frac{3}{2}(x - 4)$ $3x - 2y = 10$	**A1**
14c	at C, $x = 0$, $y = -5$	**B1**
	$AB = \sqrt{(4+2)^2 + (1-5)^2} = \sqrt{52}$	**M1** **A1**
	$BC = \sqrt{(0-4)^2 + (-5-1)^2} = \sqrt{52}$ $AB = BC$, therefore triangle is isosceles	**A1**

Topic List

Q1	Solving inequalities, logarithms
Q2	Cosine rule
Q3	Binomial expansion
Q4	Trig proof and solving equations
Q5	Integration
Q6	Circles
Q7	Algebraic fractions and equations of lines
Q8	Turning point
Q9	Sketching and solving logs
Q10	Factor and remainder theorem
Q11	Differentiation and integration
Q12	Circles and lines
Q13	Solving equations
Q14	Straight lines