Practice Exam Paper D
Time: 2 Hours

P1

1. Express $\sqrt{22.5}$ in the form $k\sqrt{10}$ **(4)**

(Total Marks: 4)

2a. Solve the inequality, $4(x – 2) < 2x + 5$ **(2)**

b. Find the value of y such that, $4^{y+1} = 8^{2y-1}$ **(4)**

(Total Marks: 6)

3. The curve with equation $y = \sqrt{8x}$ passes through the point A with x-coordinates 2. Find an equation for the tangent to the curve at A. **(7)**

(Total Marks: 7)

4a. Simplify, $\frac{x^2+7x+12}{2x^2+9x+4}$ **(3)**

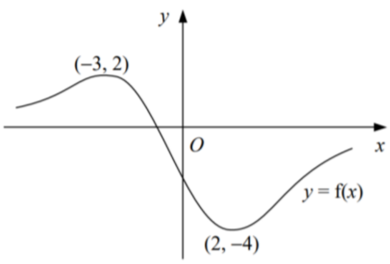
4b. Solve the equation

$$\ln(x^2 + 7x + 12) - 1 = \ln(2x^2 + 9x + 4)$$

giving your answer in terms of e **(4)**

(Total Marks: 7)

5. The figure shows the curve $y = f(x)$ which has a maximum points at (-3, 2) and a minimum point at (2, -4).



a. Showing the coordinates of any stationary points, sketch on a separate diagram the graph of $y = 3f(2x)$. **(4)**

b. Write down the values of the constants a and b such that the curve with equations $y = a + f(x + b)$ has a minimum point at the origin O. **(2)**

(Total Marks: 6)

6. Show that, $\int_2^3 (6\sqrt{x} - \frac{4}{\sqrt{x}})\, dx = k\sqrt{3}$

Where k is an integer to be found. **(6)**

(Total Marks: 6)

7. The circle C has centre (5, 2) and passes through the point (7, 3)

a. Find the length of the diameter of C. **(2)**

b. Find an equation for C. **(2)**

c. Show that the line $y = 2x - 3$ is a tangent to C and find the coordinates of the point of contact. **(5)**

(Total Marks: 9)

8a. Sketch on the same diagram the graphs of $y = \sin 2x$ and $y = \tan \frac{x}{2}$ for x in the interval $0 \leq x \leq 360°$. **(4)**

b. Hence state how many solutions exist to the equation

$$\sin 2x = \tan \frac{x}{2}$$

for x in the interval $0 \leq x \leq 360°$ and give a reason for your answer. **(2)**

(Total Marks: 6)

9a. Expand $(2 + x)^4$ in ascending powers of x, simplifying each coefficient. **(4)**

b. Find the integers A, B and C, such that, $(2 + x)^4 + (2 - x)^4 \equiv A + Bx^2 + Cx^4$ **(2)**

c. Find the real values of x for which, $(2 + x)^4 + (2 - x)^4 = 126$ **(3)**

(Total Marks: 9)

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10. $f(x) = 2x^3 - 5x^2 + x + 2$.

a. Show that $(x - 2)$ is a factor of $f(x)$. **(2)**

b. Fully factorise $f(x)$. **(4)**

c. Solve the equation $f(x) = 0$. **(1)**

d. Find the values of θ in the interval $0 \leq \theta \leq 360$ for which $2 \sin^3 \theta - 5 \sin^2 \theta + \sin \theta + 2 = 0$. **(4)**

(Total Marks: 12)

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11. The curve C has the equation

$$y = 3 - x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}, x > 0$$

a. Find the coordinates of the points where C crosses the x-axis. **(4)**

b. Find the exact coordinates of the stationary point of C **(5)**

c. Determine the nature of the stationary point. **(2)**

d. Sketch the curve C. **(2)**

(Total Marks: 13)

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12. Solve the equation,

$$\log_5 (4x + 3) - \log_5 (x - 1) = 2$$

(4)

(Total Marks: 4)

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13. The figure shows the curve C with equation $y = 3x - 4\sqrt{x} + 2$, and the tangent to C at the point A.

Given that A has the x-coordinate 4,

a. Show that the tangent to C at A has the equation $y = 2x - 2$. **(6)**

The shaded region is bounded by C, the tangent to C at A and the positive coordinate axes.

b. Find the area of the shaded region. **(8)**

(Total Marks: 14)

14a. Solve the equation

$$2 \sin^2 x - 2 \cos x - \cos^2 x = 1,$$

For the values of x in the interval $0 \leq x \leq 360°$. Give your answers to 1 decimal place. **(8)**

b. Sketch the curve $y = \sin (x - 30)°$ for x in the interval $-180 < x < 180$ **(2)**

(Total Marks: 10)

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15. The sides of a triangle have lengths of 7 *cm*, 8 *cm* and 10 *cm*.

Find the area of the triangle correct to 3 significant figures. **(8)**

(Total Marks: 8)

Total Marks: 120

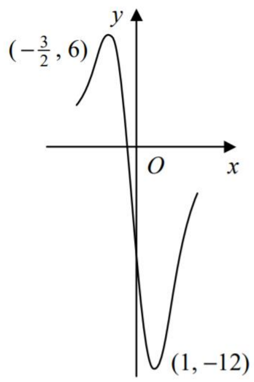
Mark Scheme

1	$= \sqrt{\frac{45}{2}} = \frac{3\sqrt{5}}{\sqrt{2}}$	**M1** **A1**
	$\frac{3\sqrt{5}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3}{2}\sqrt{10}$	**M1** **A1**

2a	$4x - 8 < 2x + 5$ $2x < 13$	**M1**
	$x < 6.5$	**A1**
2b	$(2^2)^{y+1} = (2^3)^{2y-1}$	**M1**
	$2^{2y+2} = 2^{6y-3}$	**A1**
	$2y + 2 = 6y - 3$	**M1**
	$y = \frac{5}{4}$	**A1**

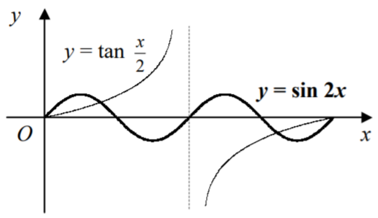
3	$x = 2,\ y = \sqrt{16} = 4$	**B1**
	$y = \sqrt{8}\,\sqrt{x} = 2\sqrt{2}x^{\frac{1}{2}}$	**B1**
	$\frac{dy}{dx} = \sqrt{2}x^{-\frac{1}{2}}$	**M1** **A1**
	gradient $= \frac{\sqrt{2}}{\sqrt{2}} = 1$	**M1**
	$y - 4 = 1\,(x - 2)$ $y = x + 2$	**M1** **A1**

4a	$\frac{(x+3)(x+4)}{(2x+1)(x+4)} = \frac{x+3}{2x+1}$	**M1** **A2**
4b	$\ln(x^2 + 7x + 12) - \ln(2x^2 + 9x + 4) = 1$	**M1**
	$\ln\frac{x+3}{2x+1} = 1$	**A1**
	$x + 3 = e(2x + 1)$ $3 - e = x(2e - 1)$	**M1**
	$x = \frac{3-e}{2e-1}$	**A1**

5a	 	**B2** **M2**
5b	$a = 5$ $b = 2$	**B2**

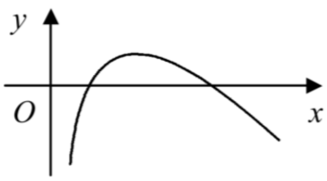
6	$\int_2^3 (6\sqrt{x} - \frac{4}{\sqrt{x}})\,dx = [4x^{\frac{3}{2}} - 8x^{\frac{1}{2}}]_2^3$	**M1** **A2**
	$= [4(3\sqrt{3}) - 8\sqrt{3}] - [4(2\sqrt{2} - 8\sqrt{2}]$	**M1** **B1**
	$= (12\sqrt{3} - 8\sqrt{3}) - (8\sqrt{2} - 8\sqrt{2})$ $= 4\sqrt{3}$	**A1**

7a	$= 2 \times \sqrt{4+1} = 2\sqrt{5}$	**M1** **A1**
7b	$(x-5)^2 + (y-2)^2 = (\sqrt{5})^2$ $(x-5)^2 + (y-2)^2 = 5$	**M1** **A1**
7c	Substituting, $y = 2x - 3$ into equation C $(x-5)^2 + [(2x-3)-2]^2 = 5$	**M1**
	$(x-5)^2 + (2x-5)^2 = 5$	**A1**
	$x^2 - 6x + 9 = 0$ $(x-3)^2 = 0$	**M1**
	Repeated root, therefore, a tangent	**A1**
	Therefore, point on contact is (3, 3)	**A1**

8a		**B2** **B2**
8b	There are 4 solutions	**B1**
	As the graphs intersect at 4 times	**B1**

9a	$= 2^4 + 4(2^3)(x) + 6(2^2)(x^2) + 4(2)(x^3) + x^4$	**M1** **A1**
	$= 16 + 32x + 24x^2 + 8x^3 + x^4$	**B1** **A1**
9b	$(2-x)^4 = 16 - 32x + 24x^2 - 8x^3 + x^4$	**M1**
	$(2+x)^4 + (2-x)^4 = 32 + 48x^2 + 2x^4$ $A = 32$ $B = 48$ $C = 2$	**A1**
9c	$32 + 48x^2 + 2x^4 = 136$ $x^4 + 24x^2 - 52 = 0$ $(x^2 + 26)(x^2 - 2) = 0$	**M1**
	$x^2 = -26$ (no real solutions) or, $x^2 = 2$	**A1**
	$x = \pm\sqrt{2}$	**A1**

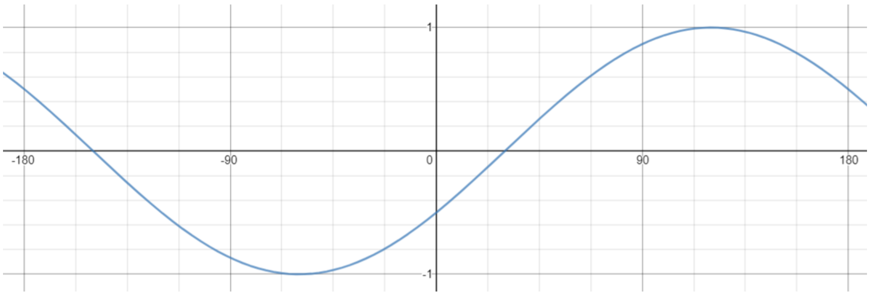
10a	$f(2) = 16 - 20 + 2 + 2 = 0$ Therefore, $(x-2)$ is a factor	**M1** **A1**
10b	$2x^2 - x - 1$ $x - 2 \,)\, 2x^3 - 5x^2 + x + 2$ $2x^3 - 4x^2$ $-x^2 + x$ $-x^2 + 2x$ $-x + 2$ $-x + 2$	**M1** **A1**
	$f(x) = (x-2)(2x^2 - x - 1)$ $= (x-2)(2x+1)(x-1)$	**M1** **A1**
10c	$x = -\frac{1}{2}$ $x = 2$	**B1**
10d	$\sin\theta = 2$ (no solutions) $\sin\theta = -\frac{1}{2}$ $\sin\theta = 1$	**M1** **B1**
	$\theta = 90^\circ, 210^\circ, 330^\circ$	**A2**

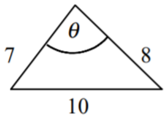
11a	$3 - x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} = 0$ $3x^{\frac{1}{2}} - x - 2 = 0$	**M1**
	$x - 3x^{\frac{1}{2}} + 2 = 0$ $(x^{\frac{1}{2}} - 1)(x^{\frac{1}{2}} - 2) = 0$	**M1**
	$x^{\frac{1}{2}} = 1$ $x^{\frac{1}{2}} = 2$	**A1**
	$x = 1, y = 0$ $x = 4, y = 0$ Therefore coordiantes are (1, 0) and (4, 0)	**A1**
11b	$\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{1}{2}} + x^{-\frac{3}{2}}$	**M1** **A1**
	For stationary point, $-\frac{1}{2}x^{-\frac{1}{2}} + x^{-\frac{3}{2}} = 0$	**M1**
	$-\frac{1}{2}x^{-\frac{3}{2}}(x - 2) = 0$ $x = 2$ $y = 3 - \sqrt{2} - \frac{2}{\sqrt{2}}$	**A2**
11c	$\frac{d^2y}{dx^2} = \frac{1}{4}x^{-\frac{3}{2}} - \frac{3}{2}x^{-\frac{5}{2}}$	**M1**
	when $x = 2$, $\frac{d^2y}{dx^2} = \frac{1}{8\sqrt{2}} - \frac{3}{8\sqrt{2}} = -\frac{1}{4\sqrt{2}}$ $\frac{d^2y}{dx^2} < 0$, therefore maximum	**A1**
11d		**B2**

12	$\log_5 \frac{4x+3}{x-1} = 2$	**M1**
	$\frac{4x+3}{x-1} = 5^2 = 25$	**M1**
	$4x + 3 = 25(x - 1)$	**M1**
	$21x = 28$ $x = \frac{4}{3}$	**A1**

13a	$x = 4, y = 12 - 8 + 2 = 6$	**B1**
	$\frac{dy}{dx} = 3 - 2x^{-\frac{1}{2}}$	**M1** **A1**
	Gradient = $3 - 1 = 2$	**M1**
	$y - 6 = 2(x - 4)$	**M1**
	$y = 2x - 2$	**A1**
13b	Area under curve = $\int_0^4 (3x - 4\sqrt{x} + 2)dx$ $= [\frac{3}{2}x^2 - \frac{8}{3}x^{\frac{3}{2}} + 2x]_0^4$	**M1** **A2**
	$= (24 - \frac{64}{3} + 8) - (0) = 10\frac{2}{3}$	**M1**
	Tangent meets x-axis when $y = 0$, $x = 1$	**M1**
	Area of triangle = $\frac{1}{2} \times 3 \times 6 = 9$	**A1**
	Shaded area = $10\frac{2}{3} - 9 = \frac{5}{3}$	**M1** **A1**

14a	$2(1 - \cos^2 x) - 2\cos x - \cos^2 x = 1$	**M1**
	$3\cos^2 x + 2\cos x - 1 = 0$	**A1**

	$(3\cos x - 1)(\cos x + 1) = 0$	**M1**
	$\cos x = -1$ $\cos x = \frac{1}{3}$	**A1**
	$x = 180$ $x = 360 - 70.5$	**B2 M1**
	$x = 70.5^{o}, 180^{o}, 289.5^{o}$	**A1**
14b		**B2**

15		
	$10^2 = 7^2 + 8^2 - (2 \times 7 \times 8 \times \cos x)$	**M1**
	$\cos x = \frac{49+63-100}{112} = \frac{13}{112}$	**M1**
	$x = 83.335$	**A1**
	Area $= \frac{1}{2} \times 7 \times 8 \times \sin 83.335$	**M1**
	$= 27.8$ cm^2 (3 s.f)	**A1**

Topic List

Q1	Surds
Q2	Inequalities
Q3	Equation of tangents
Q4	Algebraic fractions and natural logarithms
Q5	Graph sketching
Q6	Definite integrals
Q7	Circles
Q8	Drawing trig. graphs
Q9	Binomial expansion
Q10	Factor theorem, trig. equations
Q11	Stationary points, maxima and minima
Q12	Solving logarithms
Q13	Differentiation and integration
Q14	Solving trig. equations
Q15	Cosine rule and area of a triangle