



# Practice Exam Paper D

Time: 2 Hours

P1

1. Express  $\sqrt{22.5}$  in the form  $k\sqrt{10}$  (4)

(Total Marks: 4)

2a. Solve the inequality,  $4(x - 2) < 2x + 5$  (2)

b. Find the value of  $y$  such that,  $4^{y+1} = 8^{2y-1}$  (4)

(Total Marks: 6)

3. The curve with equation  $y = \sqrt{8x}$  passes through the point  $A$  with  $x$ -coordinates 2. Find an equation for the tangent to the curve at  $A$ . (7)

(Total Marks: 7)

4a. Simplify,  $\frac{x^2+7x+12}{2x^2+9x+4}$  (3)

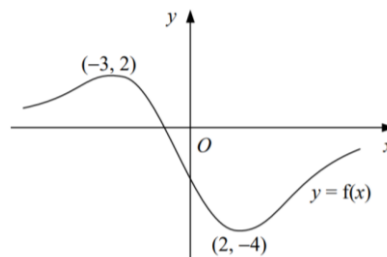
4b. Solve the equation

$$\ln(x^2 + 7x + 12) - 1 = \ln(2x^2 + 9x + 4)$$

giving your answer in terms of  $e$  (4)

(Total Marks: 7)

5. The figure shows the curve  $y = f(x)$  which has a maximum points at  $(-3, 2)$  and a minimum point at  $(2, -4)$ .



a. Showing the coordinates of any stationary points, sketch on a separate diagram the graph of  $y = 3f(2x)$ . (4)

b. Write down the values of the constants  $a$  and  $b$  such that the curve with equations  $y = a + f(x + b)$  has a minimum point at the origin  $O$ . (2)

(Total Marks: 6)

6. Show that,  $\int_2^3 (6\sqrt{x} - \frac{4}{\sqrt{x}}) dx = k\sqrt{3}$   
Where  $k$  is an integer to be found. (6)

(Total Marks: 6)

7. The circle  $C$  has centre  $(5, 2)$  and passes through the point  $(7, 3)$

a. Find the length of the diameter of  $C$ . (2)

b. Find an equation for  $C$ . (2)

c. Show that the line  $y = 2x - 3$  is a tangent to  $C$  and find the coordinates of the point of contact. (5)

(Total Marks: 9)

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8a. Sketch on the same diagram the graphs of  $y = \sin 2x$  and  $y = \tan \frac{x}{2}$  for  $x$  in the interval  $0 \leq x \leq 360^\circ$ . (4)

b. Hence state how many solutions exist to the equation

$$\sin 2x = \tan \frac{x}{2}$$

for  $x$  in the interval  $0 \leq x \leq 360^\circ$  and give a reason for your answer. (2)

(Total Marks: 6)

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9a. Expand  $(2 + x)^4$  in ascending powers of  $x$ , simplifying each coefficient. (4)

b. Find the integers  $A$ ,  $B$  and  $C$ , such that,  $(2 + x)^4 + (2 - x)^4 \equiv A + Bx^2 + Cx^4$  (2)

c. Find the real values of  $x$  for which,  $(2 + x)^4 + (2 - x)^4 = 126$  (3)

(Total Marks: 9)

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10.  $f(x) = 2x^3 - 5x^2 + x + 2$ .

a. Show that  $(x - 2)$  is a factor of  $f(x)$ . (2)

b. Fully factorise  $f(x)$ . (4)

c. Solve the equation  $f(x) = 0$ . (1)

d. Find the values of  $\theta$  in the interval  $0 \leq \theta \leq 360$  for which  $2 \sin^3 \theta - 5 \sin^2 \theta + \sin \theta + 2 = 0$ . (4)

(Total Marks: 12)

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11. The curve  $C$  has the equation

$$y = 3 - x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}, x > 0$$

a. Find the coordinates of the points where  $C$  crosses the  $x$ -axis. (4)

b. Find the exact coordinates of the stationary point of  $C$  (5)

c. Determine the nature of the stationary point. (2)

d. Sketch the curve  $C$ . (2)

(Total Marks: 13)

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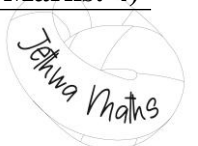
12. Solve the equation,

$$\log_5(4x + 3) - \log_5(x - 1) = 2$$

(4)

(Total Marks: 4)

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13. The figure shows the curve  $C$  with equation  $y = 3x - 4\sqrt{x} + 2$ , and the tangent to  $C$  at the point  $A$ .

Given that  $A$  has the  $x$ -coordinate 4,

a. Show that the tangent to  $C$  at  $A$  has the equation  $y = 2x - 2$ . (6)

The shaded region is bounded by  $C$ , the tangent to  $C$  at  $A$  and the positive coordinate axes.

b. Find the area of the shaded region. (8)

**(Total Marks: 14)**

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14a. Solve the equation

$$2 \sin^2 x - 2 \cos x - \cos^2 x = 1,$$

For the values of  $x$  in the interval  $0 \leq x \leq 360^\circ$ . Give your answers to 1 decimal place. (8)

b. Sketch the curve  $y = \sin(x - 30)^\circ$  for  $x$  in the interval  $-180 < x < 180$  (2)

**(Total Marks: 10)**

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15. The sides of a triangle have lengths of 7 cm, 8 cm and 10 cm.

Find the area of the triangle correct to 3 significant figures. (8)

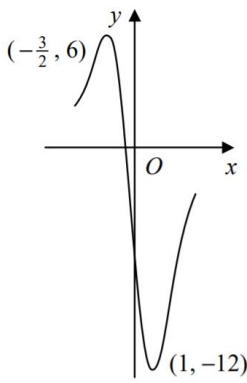
**(Total Marks: 8)**

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**Total Marks: 120**



## Mark Scheme

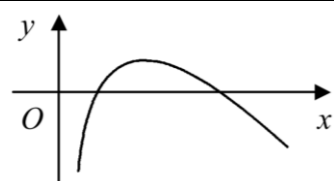
<b>1</b>	$= \sqrt{\frac{45}{2}} = \frac{3\sqrt{5}}{\sqrt{2}}$	<b>M1</b> <b>A1</b>
	$\frac{3\sqrt{5}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3}{2}\sqrt{10}$	<b>M1</b> <b>A1</b>
<b>2a</b>	$4x - 8 < 2x + 5$	<b>M1</b>
	$2x < 13$	<b>A1</b>
	$x < 6.5$	<b>A1</b>
<b>2b</b>	$(2^2)^{y+1} = (2^3)^{2y-1}$	<b>M1</b>
	$2^{2y+2} = 2^{6y-3}$	<b>A1</b>
	$2y + 2 = 6y - 3$	<b>M1</b>
	$y = \frac{5}{4}$	<b>A1</b>
<b>3</b>	$x = 2, y = \sqrt{16} = 4$	<b>B1</b>
	$y = \sqrt{8} \sqrt{x} = 2\sqrt{2}x^{\frac{1}{2}}$	<b>B1</b>
	$\frac{dy}{dx} = \sqrt{2}x^{-\frac{1}{2}}$	<b>M1</b> <b>A1</b>
	gradient = $\frac{\sqrt{2}}{\sqrt{2}} = 1$	<b>M1</b>
	$y - 4 = 1(x - 2)$	<b>M1</b>
	$y = x + 2$	<b>A1</b>
<b>4a</b>	$\frac{(x+3)(x+4)}{(2x+1)(x+4)} = \frac{x+3}{2x+1}$	<b>M1</b> <b>A2</b>
<b>4b</b>	$\ln(x^2 + 7x + 12) - \ln(2x^2 + 9x + 4) = 1$	<b>M1</b>
	$\ln \frac{x+3}{2x+1} = 1$	<b>A1</b>
	$x + 3 = e(2x + 1)$	<b>M1</b>
	$3 - e = x(2e - 1)$	<b>M1</b>
	$x = \frac{3-e}{2e-1}$	<b>A1</b>
<b>5a</b>		<b>B2</b> <b>M2</b>
<b>5b</b>	$a = 5$ $b = 2$	<b>B2</b>
<b>6</b>	$\int_2^3 (6\sqrt{x} - \frac{4}{\sqrt{x}}) dx = [4x^{\frac{3}{2}} - 8x^{\frac{1}{2}}]_2^3$	<b>M1</b> <b>A2</b>
	$= [4(3\sqrt{3}) - 8\sqrt{3}] - [4(2\sqrt{2}) - 8\sqrt{2}]$	<b>M1</b> <b>B1</b>
	$= (12\sqrt{3} - 8\sqrt{3}) - (8\sqrt{2} - 8\sqrt{2})$	<b>M1</b>
	$= 4\sqrt{3}$	<b>A1</b>

<b>7a</b>	$= 2 \times \sqrt{4+1} = 2\sqrt{5}$	<b>M1</b> <b>A1</b>
<b>7b</b>	$(x-5)^2 + (y-2)^2 = (\sqrt{5})^2$ $(x-5)^2 + (y-2)^2 = 5$	<b>M1</b> <b>A1</b>
<b>7c</b>	Substituting, $y = 2x - 3$ into equation C $(x-5)^2 + [(2x-3)-2]^2 = 5$	<b>M1</b>
	$(x-5)^2 + (2x-5)^2 = 5$	<b>A1</b>
	$x^2 - 6x + 9 = 0$ $(x-3)^2 = 0$	<b>M1</b>
	Repeated root, therefore, a tangent	<b>A1</b>
	Therefore, point on contact is (3, 3)	<b>A1</b>

<b>8a</b>		<b>B2</b> <b>B2</b>
<b>8b</b>	There are 4 solutions As the graphs intersect at 4 times	<b>B1</b> <b>B1</b>

<b>9a</b>	$= 2^4 + 4(2^3)(x) + 6(2^2)(x^2) + 4(2)(x^3) + x^4$	<b>M1</b> <b>A1</b>
	$= 16 + 32x + 24x^2 + 8x^3 + x^4$	<b>B1</b> <b>A1</b>
<b>9b</b>	$(2-x)^4 = 16 - 32x + 24x^2 - 8x^3 + x^4$	<b>M1</b>
	$(2+x)^4 + (2-x)^4 = 32 + 48x^2 + 2x^4$ $A = 32$ $B = 48$ $C = 2$	<b>A1</b>
	$32 + 48x^2 + 2x^4 = 136$ $x^4 + 24x^2 - 52 = 0$ $(x^2 + 26)(x^2 - 2) = 0$	<b>M1</b>
<b>9c</b>	$x^2 = -26$ (no real solutions) or, $x^2 = 2$	<b>A1</b>
	$x = \pm \sqrt{2}$	<b>A1</b>

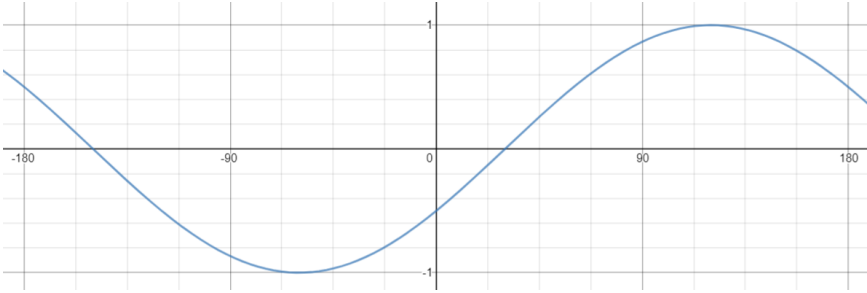
<b>10a</b>	$f(2) = 16 - 20 + 2 + 2 = 0$ Therefore, $(x-2)$ is a factor	<b>M1</b> <b>A1</b>
<b>10b</b>	$\begin{array}{r} 2x^2 - x - 1 \\ x-2 \overline{) 2x^3 - 5x^2 + x + 2} \\ \underline{2x^3 - 4x^2} \phantom{+ x + 2} \\ -x^2 + x \phantom{+ 2} \\ \underline{-x^2 + 2x} \phantom{+ 2} \\ -x + 2 \\ \underline{-x + 2} \\ 0 \end{array}$	<b>M1</b> <b>A1</b>
	$f(x) = (x-2)(2x^2 - x - 1)$ $= (x-2)(2x+1)(x-1)$	<b>M1</b> <b>A1</b>
<b>10c</b>	$x = -\frac{1}{2}$ $x = 2$	<b>B1</b>
<b>10d</b>	$\sin \theta = 2$ (no solutions) $\sin \theta = -\frac{1}{2}$ $\sin \theta = 1$	<b>M1</b> <b>B1</b>
	$\theta = 90^\circ, 210^\circ, 330^\circ$	<b>A2</b>

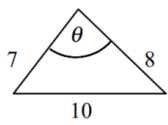
<b>11a</b>	$3 - x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} = 0$ $3x^{\frac{1}{2}} - x - 2 = 0$	<b>M1</b>
	$x - 3x^{\frac{1}{2}} + 2 = 0$ $(x^{\frac{1}{2}} - 1)(x^{\frac{1}{2}} - 2) = 0$	<b>M1</b>
	$x^{\frac{1}{2}} = 1$ $x^{\frac{1}{2}} = 2$	<b>A1</b>
	$x = 1, y = 0$ $x = 4, y = 0$ Therefore coordinates are (1, 0) and (4, 0)	<b>A1</b>
<b>11b</b>	$\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{1}{2}} + x^{-\frac{3}{2}}$	<b>M1</b> <b>A1</b>
	For stationary point, $-\frac{1}{2}x^{-\frac{1}{2}} + x^{-\frac{3}{2}} = 0$	<b>M1</b>
	$\frac{1}{2}x^{-\frac{3}{2}}(x - 2) = 0$ $x = 2$ $y = 3 - \sqrt{2} - \frac{2}{\sqrt{2}}$	<b>A2</b>
<b>11c</b>	$\frac{d^2y}{dx^2} = \frac{1}{4}x^{-\frac{3}{2}} - \frac{3}{2}x^{-\frac{5}{2}}$ when $x = 2$ , $\frac{d^2y}{dx^2} = \frac{1}{8\sqrt{2}} - \frac{3}{8\sqrt{2}} = -\frac{1}{4\sqrt{2}}$ $\frac{d^2y}{dx^2} < 0$ , therefore maximum	<b>M1</b> <b>A1</b>
	<b>11d</b> 	<b>B2</b>

<b>12</b>	$\log_5 \frac{4x+3}{x-1} = 2$	<b>M1</b>
	$\frac{4x+3}{x-1} = 5^2 = 25$	<b>M1</b>
	$4x + 3 = 25(x - 1)$	<b>M1</b>
	$21x = 28$ $x = \frac{4}{3}$	<b>A1</b>

<b>13a</b>	$x = 4, y = 12 - 8 + 2 = 6$	<b>B1</b>
	$\frac{dy}{dx} = 3 - 2x^{-\frac{1}{2}}$	<b>M1</b> <b>A1</b>
	Gradient = $3 - 1 = 2$	<b>M1</b>
	$y - 6 = 2(x - 4)$	<b>M1</b>
	$y = 2x - 2$	<b>A1</b>
<b>13b</b>	Area under curve = $\int_0^4 (3x - 4\sqrt{x} + 2) dx$ $= [\frac{3}{2}x^2 - \frac{8}{3}x^{\frac{3}{2}} + 2x]_0^4$ $= (24 - \frac{64}{3} + 8) - (0) = 10\frac{2}{3}$	<b>M1</b> <b>A2</b> <b>M1</b>
	Tangent meets x-axis when $y = 0, x = 1$	<b>M1</b>
	Area of triangle = $\frac{1}{2} \times 3 \times 6 = 9$	<b>A1</b>
	Shaded area = $10\frac{2}{3} - 9 = \frac{5}{3}$	<b>M1</b> <b>A1</b>

<b>14a</b>	$2(1 - \cos^2 x) - 2 \cos x - \cos^2 x = 1$	<b>M1</b>
	$3 \cos^2 x + 2 \cos x - 1 = 0$	<b>A1</b>

	$(3 \cos x - 1)(\cos x + 1) = 0$	<b>M1</b>
	$\cos x = -1$ $\cos x = \frac{1}{3}$	<b>A1</b>
	$x = 180$ $x = 360 - 70.5$	<b>B2 M1</b>
	$x = 70.5^\circ, 180^\circ, 289.5^\circ$	<b>A1</b>
<b>14b</b>		<b>B2</b>

<b>15</b>		
	$10^2 = 7^2 + 8^2 - (2 \times 7 \times 8 \times \cos x)$	<b>M1</b>
	$\cos x = \frac{49+63-100}{112} = \frac{13}{112}$	<b>M1</b>
	$x = 83.335$	<b>A1</b>
	Area = $\frac{1}{2} \times 7 \times 8 \times \sin 83.335$	<b>M1</b>
	= $27.8 \text{ cm}^2$ (3 s.f)	<b>A1</b>

## Topic List

<b>Q1</b>	Surds
<b>Q2</b>	Inequalities
<b>Q3</b>	Equation of tangents
<b>Q4</b>	Algebraic fractions and natural logarithms
<b>Q5</b>	Graph sketching
<b>Q6</b>	Definite integrals
<b>Q7</b>	Circles
<b>Q8</b>	Drawing trig. graphs
<b>Q9</b>	Binomial expansion
<b>Q10</b>	Factor theorem, trig. equations
<b>Q11</b>	Stationary points, maxima and minima
<b>Q12</b>	Solving logarithms
<b>Q13</b>	Differentiation and integration
<b>Q14</b>	Solving trig. equations
<b>Q15</b>	Cosine rule and area of a triangle

