Practice Exam Paper C
Time: 2 Hours

P1

1. Differentiation with respect to x,

$$\frac{6x^2-1}{2\sqrt{x}}$$

(5)

(Total Marks: 5)

2a. Solve the inequality,

$x^2 + 3x > 10$ **(4)**

b. Find the ser of values of x which satisfy both of the following inequalities,

$3x - 2 < x + 3$
$x^2 + 3x > 10$ **(3)**

(Total Marks: 6)

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3. The points A, B and C have coordinates (-3, 0), (5, -2) and (4, 1) respectively.

Find an equation for the straight line which passes through C and is parallel to AB. Give your answer in the form $ax + by = c$, where a, b and c are integers. **(4)**

(Total Marks: 4)

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4. The curve C has the equation $y = x^2 + 2x + 4$.

a. Express $x^2 + 2x + 4$ in the form $a(x + b)^2 + c$ and hence state the coordinates of the minimum point of C. **(4)**

The straight line l has the equation $x + y = 8$.

b. Sketch l and C on the same set of axes. **(3)**

c. Find the coordinates of the points where l and C intersect **(4)**

(Total Marks: 11)

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5. The curve C has the equation $y = f(x)$. Given that,

$$\frac{dy}{dx} = 3 - \frac{2}{x^2}, x \neq 0$$

And that the point A on C has coordinates (2, 6)

a. Find an equation for C **(5)**

b. Find an equation for the tangent to C at A, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers. **(4)**

c. Show that the line $y = x + 3$ is also a tangent to C. **(3)**

(Total Marks: 12)

6. Solve the simultaneous equations,

$$x + y = 2$$

$$3x^2 - 2x + y^2 = 2$$

(7)

(Total Marks: 7)

7a. Given that $t = \log_3 x$, find expression in terms of t for,

i. $\log_3 x^2$

ii. $\log_9 x$ **(4)**

b. Hence, or otherwise, find the 3 significant figures the values of x such that,

$\log_3 x^2 - \log_9 x = 4$ **(3)**

(Total Marks: 7)

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8. The circle C has centre $(-3, 2)$ and passes through the point $(2, 1)$.

a. Find an equation for C. **(4)**

b. Show that the point with coordinates $(-4, 7)$ lies on C. **(1)**

c. Find an equation for the tangent to C at the point $(-4, 7)$. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(5)**

(Total Marks: 10)

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9. $f(x) = 2 + 6x^2 - x^3$

a. Find the coordinates of the stationary points of the curve $y = f(x)$. **(5)**

b. Determine whether each stationary point is a maximum or minimum point. **(3)**

c. Sketch the curve $y = f(x)$. **(2)**

d. State the set of values of k for which the equation $f(x) = k$ has three solutions. **(1)**

(Total Marks: 11)

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10. Solve the equation,

$$\sin^2 x = 4 \cos x$$

For the values of x in the interval $0 \leq x \leq 360^{\circ}$ **(7)**

(Total Marks: 7)

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11. Given that for small values of x,

$$(1 + ax)^n \approx 1 - 24x + 270x^2$$

Where n is an integer and $n > 1$

a. Show that $n = 16$ and find the value of a, **(7)**

b. Use your value of a and a suitable value of x to estimate the value of (0.9985)16, giving your answer to 5 decimal places. **(3)**

(Total Marks: 10)

12a. Solve for $0 \le x < 180^{o}$

$$\sin(2x - 30) + 1 = 0.4$$

Giving your answers to 1 decimal place. **(5)**

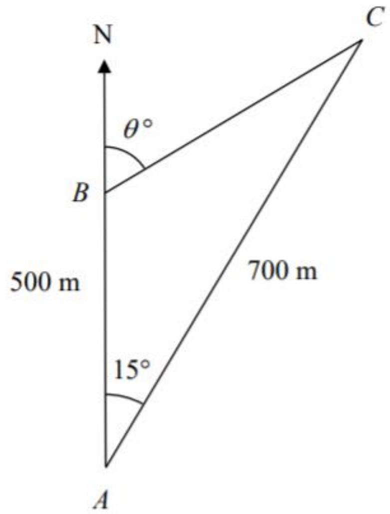
b. Find all values of x, in the interval $0 \le x < 360^{o}$, for which,

$$9\cos^2 x - 11\cos x + 3\sin^2 x = 0$$

Giving your answers to 1 decimal place. **(7)**

(Total Marks: 12)

13.



The figure shows 3 yachts *A*, *B* and *C* which are assumed to be in the same horizontal plane. Yacht *B* is 500 m due north of yacht *A* and yacht *C* is 700 m from *A*.

The bearing of *C* from *A* is 015°.

a. Calculate the distance between yacht *B* and yacht *C*, in metres to 3 significant figures. **(3)**

The bearing of yacht *C* from yacht B is $\theta°$, as shown in the figure.

b. Calculate the value of θ. **(4)**

(Total Marks: 7)

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14. A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of 75π cm^3 . The cost of polishing the surface area of this glass cylinder is £2 per cm^2 for the curved surface area and £3 per cm^2 for the circular top and base areas.

Given that the radius of the cylinder is r cm,

a. Show that the cost of the polishing, £C, is given by, $C = 6\pi r^2 + \frac{300\pi}{r}$ **(4)**

b. Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound. **(5)**

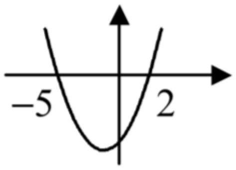
c. Justify that the answer that you have obtained in part *(b)* is a minimum **(1)**

(Total Marks: 10)

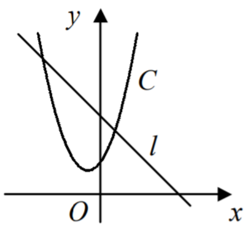
Total Marks: 120

Mark Scheme

1	$\frac{6x^2-1}{2\sqrt{x}} = 3x^{\frac{3}{2}} - \frac{1}{2}x^{-\frac{1}{2}}$	**M1** **A1**
	$\frac{d}{dx}(3x^{\frac{3}{2}} - \frac{1}{2}x^{-\frac{1}{2}}) = \frac{9}{2}x^{\frac{1}{2}} + \frac{1}{4}x^{-\frac{3}{2}}$	**M1** **A2**

2a	$x^2 + 3x - 10 > 0$ $(x + 5)(x - 2) > 0$	**M1**
		**M1**
	$x < -5$ $x > 2$	**A2**
2b	$3x - 2 < x + 3$ $2x < 5$	**M1**
	$x < \frac{5}{2}$	**A1**
	Both inequalities are satisfied by, $x < -5$ or $2 < x < \frac{5}{2}$	**A1**

3	Gradient $AB = \frac{-2-0}{5-(-3)} = -\frac{1}{4}$	**M1** **A1**
	$y - 1 = -\frac{1}{4}(x - 4)$	**M1**
	$4y - 4 = -x + 4$ $x + 4y = 8$	**A1**

4a	$x^2 + 2x + 4 = (x + 1)^2 - 1 + 4$	**M1**
	$= (x + 1)^2 + 3$	**A1**
	Therefore minimum: (-1, 3)	**A2**
4b		**B2** **B1**
4c	$x^2 + 2x + 4 = 8 - x$ $x^2 + 3x - 4 = 0$ $(x + 4)(x - 1) = 0$	**M1**
	$x = -4$ $x = 1$	**A1**
	Coordinates, (-4, 12) (1, 7)	**M1** **A1**

5a	$y = \int (3 - \frac{2}{x^2})\, dx$ $y = 3x + 2x^{-1} + c$	**M1** **A2**
	At the point (2, 6) $6 = 6 + 1 + c$ $c = -1$	**M1**
	$y = 3x + 2x^{-1} - 1$	**A1**
5b	Gradient $= 3 - \frac{1}{2} = \frac{5}{2}$	**M1 A1**

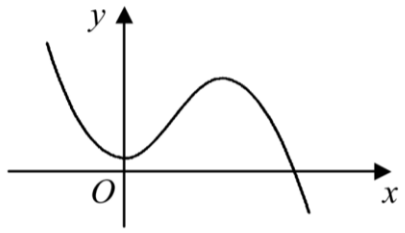
	$y - 6 = \frac{5}{2}(x - 2)$	**M1**
	$2y - 12 = 5x - 10$ $5x - 2y + 2 = 0$	**A1**
5c	$3x + 2x^{-1} - 1 = x + 3$ $3x^2 + 2 - x = x^2 + 3x$	**M1**
	$x^2 - 2x + 1 = 0$ $(x - 1)^2 = 0$ Repeated roots, therefore tangent.	**M1** **A1**

6	$x + y = 2$ $y = 2 - x$	**M1**
	Sub into $3x^2 - 2x + y^2 = 2$ $3x^2 - 2x + (2 - x)^2 = 2$	**M1**
	$2x^2 - 3x + 1 = 0$	**A1**
	$(2x - 1)(x - 1) = 0$	**M1**
	$x = \frac{1}{2}$ $x = 1$	**A1**
	$x = \frac{1}{2}$, $y = \frac{3}{2}$ $x = 1$, $y = 1$	**M1** **A1**

7ai	$= 2 \log_3 x = 2t$	**M1** **A1**
7aii	$\frac{log_3 x}{log_3 x} = \frac{log_3 x}{2} = \frac{1}{2}t$	**M1** **A1**
7b	$2t - \frac{1}{2}t = 4$ $t = \frac{8}{3}$	**M1**
	$\log_3 x = \frac{8}{3}$ $x = 3^{\frac{8}{3}} = 18.7$	**M1** **A1**

8a	Radius = $\sqrt{25+1} = \sqrt{26}$	**M1** **A1**
	$(x + 3)^2 + (y - 2)^2 = (\sqrt{26})^2$	**M1**
	$(x + 3)^2 + (y - 2)^2 = 26$	**A1**
8b	At the point (-4, 7) L.H.S = _$(-4 + 3)^2 + (7 - 2)^2 = 1 + 25 = 26$ Therefore, lies on circle.	**B1**
8c	Gradient of radius = $\frac{7-2}{-4-(-3)} = -5$	**M1**
	Gradient on tangent = $\frac{-1}{-5} = \frac{1}{5}$	**M1** **A1**
	$y - 7 = \frac{1}{5}(x + 4)$	**M1**
	$5y - 35 = x + 4$ $x - 5y + 39 = 0$	**A1**

9a	$f'(x) = 12x - 3x^2$	**M1** **A1**
	For stationary points, $12x - 3x^2 = 0$ $3x(4 - x)$ $x = 0$ $x = 4$	**M1**
	When $x = 0$, $y = 2$ When $x = 4$, $y = 34$ Coordinates: (0, 2) and (4, 34)	**A2**
9b	$f''(x) = 12 - 6x$	**M1**

	$f''(0) = 12$ $f''(0) > 0$ Therefore, (0, 2) minimum	**A1**
	$f''(4) = -12$ $f''(x) < 0$ Therefore, (4, 34) maximum	**A1**
9c		**B2**
9d	$2 < k < 34$	**B1**

10	$1 - \cos^2 x = 4 \cos x$	**M1**
	$\cos^2 x + 4 \cos x - 1 = 0$	**A1**
	$\cos x = -2 - \sqrt{5}$ (no solutions) $\cos x = -2 + \sqrt{5}$	**M1** **A1**
	$x = 76.3$, $x = 360 - 76.3$	**B1** **M1**
	$x = 76.3^o$ $x = 283.6^o$	**A1**

11a	$(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$	**B2**
	$an = -24$ $\frac{1}{2}a^2n(n - 1) = 270$	**M1**
	$a = -\frac{24}{n}$ $\frac{288}{n}(n-1) = 270$	**M1**
	$288n - 288 = 270n$	**M1**
	$18n = 288$ $n = \frac{288}{18} = 16$ $a = -\frac{3}{2}$	**A2**
11b	$1 - \frac{3}{2}x = 0.9985$ $x = 0.001$	**B1**
	$(0.9985)^{16} \approx 1 - 0.024 + 0.00270$	**M1**
	= 0.97627 (5 d.p)	**A1**

12a	$\sin(2x - 30) = -0.6$	**B1**
	$2x - 30 = 216.87$	**M1**
	$x = \frac{216.98+30}{2} = 123.4^o$	**A1**
	$2x - 30 = 360 - 36.9$	**M1**
	$x = \frac{323.1+30}{2} = 176.6^o$	**A1**
12b	$9 \cos^2 x - 11 \cos x + 3(1 - \cos^2 x) = 0$	**M1**
	$6 \cos^2 x - 11 \cos x + 3(\sin^2 x + \cos^2 x) = 0$ $6 \cos^2 x - 11 \cos x + 3 = 0$	**A1**
	$(3 \cos x - 1)(2 \cos x - 3) = 0$	**M1**
	$\cos x = \frac{3}{2}$ (no solutions) $\cos x = \frac{1}{3}$	**A1**
	$x = 70.5^o$	**B1**

	$x = 360 - 70.5$ $x = 289.5^o$	**M1** **A1**

13a	$BC^2 = 700^2 + 500^2 - 2 \times 500 \times 700 \cos 15$	**M1** **A1**
	$BC = 253$	**A1**
13b	$\frac{\sin B}{700} = \frac{\sin 15}{253}$	**M1**
	$\sin B = \sin 15 \times \frac{700}{253} = 0.716$ $B = 134.2^o$	**M1**
	$\theta = 180 - 134.2$	**M1**
	$\theta = 045.8$	**A1**

14a	Cost of polishing top and bottom = $3 \times 2\pi r^2$	**B1**
	Volume = $\frac{75\pi}{\pi r^2}$	**B1**
	$C = 6\pi r^2 + \frac{300\pi}{r}$	**M1** **A1**
14b	$\frac{dC}{dr} = 12\pi r - \frac{300\pi}{r^2}$	**M1** **A1**
	$12\pi r - \frac{300\pi}{r^2} = 0$	**M1**
	Solving, for $r = 3$	**M1**
	$C = 483$	**A1**

15	$\sin x \left(\frac{\sin x}{\cos x}\right) = 3\cos x + 2$	**M1**
	$\left(\frac{1-\cos^2 x}{\cos x}\right) = 3\cos x + 2$	**M1**
	$1 - \cos^2 x = 3\cos^2 x + 2\cos x$ Therefore, $0 = 4\cos^2 x + 2\cos x - 1$	**A1**

Topic List

Q1	Differentiation
Q2	Inequalities
Q3	Equations of straight lines
Q4	Completing the square and curve sketching
Q5	Integration and tangents
Q6	Solving simultaneous equations
Q7	Logarithms
Q8	Circles
Q9	Stationary points
Q10	Solving trig. equations
Q11	Binomial expansion
Q12	Solving trig. equations
Q13	Cosine and sine rule
Q14	Modelling with calculus, maxima and minima