# Practice Exam Paper C
# Time: 2 Hours

P1

1. Differentiation with respect to $x$,

$$\frac{6x^2-1}{2\sqrt{x}}$$ **(5)**

**(Total Marks: 5)**

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2a. Solve the inequality,

$x^2 + 3x > 10$ **(4)**

b. Find the ser of values of $x$ which satisfy both of the following inequalities,

$3x - 2 < x + 3$
$x^2 + 3x > 10$ **(3)**

**(Total Marks: 6)**

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3. The points $A$, $B$ and $C$ have coordinates (-3, 0), (5, -2) and (4, 1) respectively.

Find an equation for the straight line which passes through $C$ and is parallel to $AB$. Give your answer in the form $ax + by = c$, where $a$, $b$ and $c$ are integers. **(4)**

**(Total Marks: 4)**

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4. The curve $C$ has the equation $y = x^2 + 2x + 4$.

a. Express $x^2 + 2x + 4$ in the form $a(x + b)^2 + c$ and hence state the coordinates of the minimum point of $C$. **(4)**

The straight line $l$ has the equation $x + y = 8$.

b. Sketch $l$ and $C$ on the same set of axes. **(3)**

c. Find the coordinates of the points where $l$ and $C$ intersect **(4)**

**(Total Marks: 11)**

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5. The curve $C$ has the equation $y = f(x)$. Given that,

$$\frac{dy}{dx} = 3 - \frac{2}{x^2}, x \neq 0$$

And that the point $A$ on $C$ has coordinates (2, 6)

a. Find an equation for $C$ **(5)**

b. Find an equation for the tangent to $C$ at $A$, giving your answer in the form $ax + by + c = 0$ where $a$, $b$ and $c$ are integers. **(4)**

c. Show that the line $y = x + 3$ is also a tangent to $C$. **(3)**

**(Total Marks: 12)**

6. Solve the simultaneous equations,

$$x + y = 2$$

$$3x^2 - 2x + y^2 = 2$$

**(7)**

**(Total Marks: 7)**

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7a. Given that $t = \log_3 x$, find expression in terms of $t$ for,

i. $\log_3 x^2$

ii. $\log_9 x$ **(4)**

b. Hence, or otherwise, find the 3 significant figures the values of $x$ such that,

$\log_3 x^2 - \log_9 x = 4$ **(3)**

**(Total Marks: 7)**

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8. The circle $C$ has centre $(-3, 2)$ and passes through the point $(2, 1)$.

a. Find an equation for $C$. **(4)**

b. Show that the point with coordinates $(-4, 7)$ lies on $C$. **(1)**

c. Find an equation for the tangent to $C$ at the point $(-4, 7)$. Give your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers. **(5)**

**(Total Marks: 10)**

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9. $f(x) = 2 + 6x^2 - x^3$

a. Find the coordinates of the stationary points of the curve $y = f(x)$. **(5)**

b. Determine whether each stationary point is a maximum or minimum point. **(3)**

c. Sketch the curve $y = f(x)$. **(2)**

d. State the set of values of $k$ for which the equation $f(x) = k$ has three solutions. **(1)**

**(Total Marks: 11)**

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10. Solve the equation,

$$\sin^2 x = 4 \cos x$$

For the values of $x$ in the interval $0 \leq x \leq 360^{\circ}$ **(7)**

**(Total Marks: 7)**

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11. Given that for small values of $x$,

$$(1 + ax)^n \approx 1 - 24x + 270x^2$$

Where $n$ is an integer and $n > 1$

a. Show that $n = 16$ and find the value of $a$, **(7)**

b. Use your value of a and a suitable value of $x$ to estimate the value of (0.9985)16, giving your answer to 5 decimal places. **(3)**

**(Total Marks: 10)**

12a. Solve for $0 \leq x < 180^{o}$

$$\sin(2x - 30) + 1 = 0.4$$

Giving your answers to 1 decimal place. **(5)**

b. Find all values of $x$, in the interval $0 \leq x < 360^{o}$, for which,

$$9\cos^2 x - 11\cos x + 3\sin^2 x = 0$$

Giving your answers to 1 decimal place. **(7)**

**(Total Marks: 12)**

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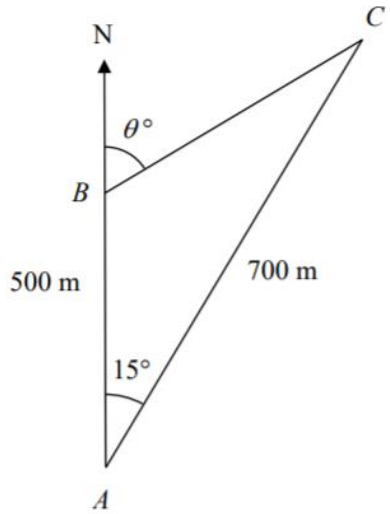
13.




The figure shows 3 yachts $A$, $B$ and $C$ which are assumed to be in the same horizontal plane. Yacht $B$ is 500 m due north of yacht $A$ and yacht $C$ is 700 m from $A$.

The bearing of $C$ from $A$ is 015°.

a. Calculate the distance between yacht $B$ and yacht $C$, in metres to 3 significant figures. **(3)**

The bearing of yacht $C$ from yacht B is $\theta°$, as shown in the figure.

b. Calculate the value of $\theta$. **(4)**

**(Total Marks: 7)**

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14. A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of $75\pi$ cm$^3$ . The cost of polishing the surface area of this glass cylinder is £2 per cm$^2$ for the curved surface area and £3 per cm$^2$ for the circular top and base areas.

Given that the radius of the cylinder is $r$ cm,

a. Show that the cost of the polishing, £$C$, is given by, $C = 6\pi r^2 + \frac{300\pi}{r}$ **(4)**

b. Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound. **(5)**

c. Justify that the answer that you have obtained in part *(b)* is a minimum **(1)**

**(Total Marks: 10)**

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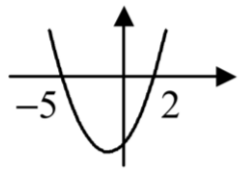
**Total Marks: 120**

# Mark Scheme

| | | |
|---|---|---|
| **1** | $\frac{6x^2-1}{2\sqrt{x}} = 3x^{\frac{3}{2}} - \frac{1}{2}x^{-\frac{1}{2}}$ | **M1**<br>**A1** |
| | $\frac{d}{dx}(3x^{\frac{3}{2}} - \frac{1}{2}x^{-\frac{1}{2}}) = \frac{9}{2}x^{\frac{1}{2}} + \frac{1}{4}x^{-\frac{3}{2}}$ | **M1**<br>**A2** |

| | | |
|---|---|---|
| **2a** | $x^2 + 3x - 10 > 0$<br>$(x + 5)(x - 2) > 0$ | **M1** |
| |  | **M1** |
| | $x < -5$<br>$x > 2$ | **A2** |
| **2b** | $3x - 2 < x + 3$<br>$2x < 5$ | **M1** |
| | $x < \frac{5}{2}$ | **A1** |
| | Both inequalities are satisfied by,<br>$x < -5$ or $2 < x < \frac{5}{2}$ | **A1** |

| | | |
|---|---|---|
| **3** | Gradient $AB = \frac{-2-0}{5-(-3)} = -\frac{1}{4}$ | **M1**<br>**A1** |
| | $y - 1 = -\frac{1}{4}(x - 4)$ | **M1** |
| | $4y - 4 = -x + 4$<br>$x + 4y = 8$ | **A1** |

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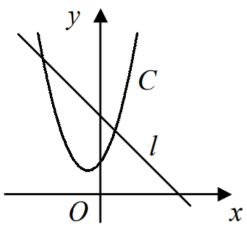
|---|---|---|
| **4a** | $x^2 + 2x + 4 = (x + 1)^2 - 1 + 4$ | **M1** |
| | $= (x + 1)^2 + 3$ | **A1** |
| | Therefore minimum: $(-1, 3)$ | **A2** |
| **4b** |  | **B2**<br>**B1** |
| **4c** | $x^2 + 2x + 4 = 8 - x$<br>$x^2 + 3x - 4 = 0$<br>$(x + 4)(x - 1) = 0$ | **M1** |
| | $x = -4$<br>$x = 1$ | **A1** |
| | Coordinates,<br>$(-4, 12)$<br>$(1, 7)$ | **M1**<br>**A1** |

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|---|---|---|
| **5a** | $y = \int (3 - \frac{2}{x^2})\, dx$<br>$y = 3x + 2x^{-1} + c$ | **M1**<br>**A2** |
| | At the point (2, 6)<br>$6 = 6 + 1 + c$<br>$c = -1$ | **M1** |
| | $y = 3x + 2x^{-1} - 1$ | **A1** |
| **5b** | Gradient $= 3 - \frac{1}{2} = \frac{5}{2}$ | **M1 A1** |

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|---|---|---|
| | $y - 6 = \frac{5}{2}(x - 2)$ | **M1** |
| | $2y - 12 = 5x - 10$<br>$5x - 2y + 2 = 0$ | **A1** |
| **5c** | $3x + 2x^{-1} - 1 = x + 3$<br>$3x^2 + 2 - x = x^2 + 3x$ | **M1** |
| | $x^2 - 2x + 1 = 0$<br>$(x - 1)^2 = 0$<br>Repeated roots, therefore tangent. | **M1**<br>**A1** |

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| **6** | $x + y = 2$<br>$y = 2 - x$ | **M1** |
| | Sub into $3x^2 - 2x + y^2 = 2$<br>$3x^2 - 2x + (2 - x)^2 = 2$ | **M1** |
| | $2x^2 - 3x + 1 = 0$ | **A1** |
| | $(2x - 1)(x - 1) = 0$ | **M1** |
| | $x = \frac{1}{2}$<br>$x = 1$ | **A1** |
| | $x = \frac{1}{2}$, $y = \frac{3}{2}$<br>$x = 1$, $y = 1$ | **M1**<br>**A1** |

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|---|---|---|
| **7ai** | $= 2\log_3 x = 2t$ | **M1**<br>**A1** |
| **7aii** | $\frac{log_3 x}{log_3 x} = \frac{log_3 x}{2} = \frac{1}{2}t$ | **M1**<br>**A1** |
| **7b** | $2t - \frac{1}{2}t = 4$<br>$t = \frac{8}{3}$ | **M1** |
| | $\log_3 x = \frac{8}{3}$<br>$x = 3^{\frac{8}{3}} = 18.7$ | **M1**<br>**A1** |

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|---|---|---|
| **8a** | Radius = $\sqrt{25 + 1} = \sqrt{26}$ | **M1**<br>**A1** |
| | $(x + 3)^2 + (y - 2)^2 = (\sqrt{26})^2$ | **M1** |
| | $(x + 3)^2 + (y - 2)^2 = 26$ | **A1** |
| **8b** | At the point (-4, 7)<br>L.H.S = _$(-4 + 3)^2 + (7 - 2)^2 = 1 + 25 = 26$<br>Therefore, lies on circle. | **B1** |
| **8c** | Gradient of radius = $\frac{7-2}{-4-(-3)} = -5$ | **M1** |
| | Gradient on tangent = $\frac{-1}{-5} = \frac{1}{5}$ | **M1**<br>**A1** |
| | $y - 7 = \frac{1}{5}(x + 4)$ | **M1** |
| | $5y - 35 = x + 4$<br>$x - 5y + 39 = 0$ | **A1** |

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|---|---|---|
| **9a** | $f'(x) = 12x - 3x^2$ | **M1**<br>**A1** |
| | For stationary points, $12x - 3x^2 = 0$<br>$3x(4 - x)$<br>$x = 0$<br>$x = 4$ | **M1** |
| | When $x = 0$, $y = 2$<br>When $x = 4$, $y = 34$<br>Coordinates: (0, 2) and (4, 34) | **A2** |
| **9b** | $f''(x) = 12 - 6x$ | **M1** |

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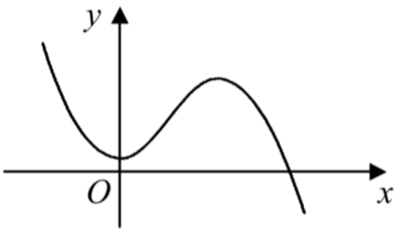
|---|---|---|
| | f''(0) = 12<br>f''(0) > 0<br>Therefore, (0, 2) minimum | **A1** |
| | f''(4) = -12<br>f''($x$) < 0<br>Therefore, (4, 34) maximum | **A1** |
| **9c** |  | **B2** |
| **9d** | $2 < k < 34$ | **B1** |

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|---|---|---|
| **10** | $1 - \cos^2 x = 4 \cos x$ | **M1** |
| | $\cos^2 x + 4 \cos x - 1 = 0$ | **A1** |
| | $\cos x = -2 - \sqrt{5}$ (no solutions)<br>$\cos x = -2 + \sqrt{5}$ | **M1**<br>**A1** |
| | $x = 76.3$,<br>$x = 360 - 76.3$ | **B1**<br>**M1** |
| | $x = 76.3^o$<br>$x = 283.6^o$ | **A1** |

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| **11a** | $(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$ | **B2** |
| | $an = -24$<br>$\frac{1}{2}a^2n(n-1) = 270$ | **M1** |
| | $a = -\frac{24}{n}$<br>$\frac{288}{n}(n-1) = 270$ | **M1** |
| | $288n - 288 = 270n$ | **M1** |
| | $18n = 288$<br>$n = \frac{288}{18} = 16$<br>$a = -\frac{3}{2}$ | **A2** |
| **11b** | $1 - \frac{3}{2}x = 0.9985$<br>$x = 0.001$ | **B1** |
| | $(0.9985)^{16} \approx 1 - 0.024 + 0.00270$ | **M1** |
| | = 0.97627 (5 d.p) | **A1** |

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|---|---|---|
| **12a** | $\sin(2x - 30) = -0.6$ | **B1** |
| | $2x - 30 = 216.87$ | **M1** |
| | $x = \frac{216.98+30}{2} = 123.4^o$ | **A1** |
| | $2x - 30 = 360 - 36.9$ | **M1** |
| | $x = \frac{323.1+30}{2} = 176.6^o$ | **A1** |
| **12b** | $9 \cos^2 x - 11 \cos x + 3(1 - \cos^2 x) = 0$ | **M1** |
| | $6 \cos^2 x - 11 \cos x + 3(\sin^2 x + \cos^2 x) = 0$<br>$6 \cos^2 x - 11 \cos x + 3 = 0$ | **A1** |
| | $(3 \cos x - 1)(2 \cos x - 3) = 0$ | **M1** |
| | $\cos x = \frac{3}{2}$ (no solutions)<br>$\cos x = \frac{1}{3}$ | **A1** |
| | $x = 70.5^o$ | **B1** |

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|---|---|---|
| | $x = 360 – 70.5$<br>$x = 289.5^o$ | **M1**<br>**A1** |

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|---|---|---|
| **13a** | $BC^2 = 700^2 + 500^2 – 2 \text{ x } 500 \text{ x } 700 \cos 15$ | **M1**<br>**A1** |
| | $BC = 253$ | **A1** |
| **13b** | $\frac{\sin B}{700} = \frac{\sin 15}{253}$ | **M1** |
| | $\sin B = \sin 15 \text{ x } \frac{700}{253} = 0.716$<br>$B = 134.2^o$ | **M1** |
| | $\theta = 180 – 134.2$ | **M1** |
| | $\theta = 045.8$ | **A1** |

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|---|---|---|
| **14a** | Cost of polishing top and bottom = $3 \text{ x } 2\pi r^2$ | **B1** |
| | Volume = $\frac{75\pi}{\pi r^2}$ | **B1** |
| | $C = 6\pi r^2 + \frac{300\pi}{r}$ | **M1**<br>**A1** |
| **14b** | $\frac{dC}{dr} = 12\pi r - \frac{300\pi}{r^2}$ | **M1**<br>**A1** |
| | $12\pi r - \frac{300\pi}{r^2} = 0$ | **M1** |
| | Solving, for $r = 3$ | **M1** |
| | $C = 483$ | **A1** |

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|---|---|---|
| **15** | $\sin x \left(\frac{\sin x}{\cos x}\right) = 3 \cos x + 2$ | **M1** |
| | $\left(\frac{1-cos^2 x}{\cos x}\right) = 3 \cos x + 2$ | **M1** |
| | $1 – \cos^2 x = 3 \cos^2 x + 2 \cos x$<br>Therefore, $0 = 4 \cos^2 x + 2 \cos x - 1$ | **A1** |

**Topic List**

| | |
|---|---|
| **Q1** | Differentiation |
| **Q2** | Inequalities |
| **Q3** | Equations of straight lines |
| **Q4** | Completing the square and curve sketching |
| **Q5** | Integration and tangents |
| **Q6** | Solving simultaneous equations |
| **Q7** | Logarithms |
| **Q8** | Circles |
| **Q9** | Stationary points |
| **Q10** | Solving trig. equations |
| **Q11** | Binomial expansion |
| **Q12** | Solving trig. equations |
| **Q13** | Cosine and sine rule |
| **Q14** | Modelling with calculus, maxima and minima |