Practice Exam Paper B
Time: 2 Hours

P1

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1. $f(x) = (\sqrt{x} + 3)^2 + (1 - 3\sqrt{x})^2$

Show that f(x) can be written in the form $ax + b$ where a and b are integers to be found **(3)**

(Total Marks: 3)

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2. Given that,

$$\frac{dy}{dx} = 2x^3 + 1$$

And that $y = 3$ when $x = 0$, find the value of y when $x = 2$ **(6)**

(Total Marks: 6)

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3. Solve the equation, $x^{\frac{3}{2}} = 27$ **(2)**

b. Express $(2\frac{1}{4})^{-\frac{1}{2}}$ where a and b are rational. **(2)**

(Total Marks: 4)

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4. The straight line l_1 has the equation $3x – y = 0$.
The straight line l_2 has the equation $x + 2y – 4 = 0$

a. Sketch l_1 and l_2 on the same diagram, showing the coordinates of any points where each line meets the coordinate axes. **(3)**

b. Find in exact fractions, the coordinates of the point where l_1 and l_2 intersect. **(3)**

(Total Marks: 6)

5a. Sketch on the same diagram the graphs of $y = (x – 1)^2(x – 5)$ and $y = 8 – 2x$.
Label on your diagram the coordinates of any points where each graph meets the coordinate axes. **(5)**

b. Explain how your diagram shows that there is only one solution, α, to the equation $(x – 1)^2(x – 5) = 8 – 2x$ **(1)**

c. State the integer, n, such that, $n < \alpha < n + 1$ **(1)**

(Total Marks: 7)

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6. Given that, $y = \sqrt{x} - \frac{4}{\sqrt{x}}$

a. Find $\frac{dy}{dx}$ **(3)**

b. Find $\frac{d^2y}{dx^2}$ **(2)**

c. Show that, $4x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} – y = 0$ **(3)**

(Total Marks: 8)

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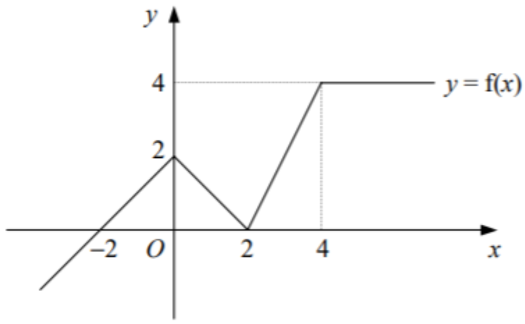
7. The figure below shows the graph of $y = f(x)$



a. Write down the number of solutions that exist for the equation,

i. $f(x) = 1$

ii. $f(x) = -x$ **(2)**

b. Labelling the axes in a similar way, sketch on separate diagrams the graphs of

i. $y = f(x - 2)$

ii. $y = f(2x)$ **(6)**

(Total Marks: 8)

8a. Expand $(1 + 3x)^8$ in ascending powers of x up to and including the term in x^3 . You should simplify each coefficient in your expansion. **(4)**

b. Use your series, together with a suitable values of x, which you should state to estimate the value of $(1.003)^8$, giving your answers to 8 significant figures. **(3)**

(Total Marks: 7)

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9. Evaluate, $\log_3 27 – \log_8 4$ **(4)**

b. Solve the equation $4^x – 3(2^{x+1}) = 0$ **(5)**

(Total Marks: 9)

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10. The circle C has the equation,

$$x^2 + y^2 + 10x – 8y + k = 0$$

where k is constant.

Given that the point with coordinates (-6, 5) lies on C.

a. Find the value of k **(2)**

b. Find the coordinates of the centre and the radius of C **(3)**

A straight line passes through the point A (2, 3) is a tangent to C at the point B.

c. Find the length AB in the form $k\sqrt{3}$ **(5)**

(Total Marks: 10)

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11. The polynomial f(x) is given by,

$$f(x) = x^3 + kx^2 - 7x - 15$$

where k is a constant

When f(x) is divided by (x + 1) the remainder r
When f(x) is divided by (x - 3) the remainder is 3r.

a. Find the value of k **(5)**

b. Find the value of r **(1)**

c. Show that (x – 5) is a factor of f(x) **(2)**

d. Show that there is only one real solution to the equation f(x) = 0 **(4)**

(Total Marks: 12)

12a. Given that,

$$5 \cos x - 2 \sin x = 0,$$

Show that, tan x = 2.5 **(2)**

b. Solve, for $0 \leq x \leq 180$, the equation

$$5 \cos 2x - 2 \sin 2x = 0$$

Giving your answer to 1 decimal place. **(4)**

(Total Marks: 6)

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13. $f(x) = 2 + 6x^2 - x^3$
a. Find the coordinates of the stationary points of the curve y = f(x) **(5)**

b. Determine whether each stationary point is a maximum or minimum point **(3)**

c. Sketch the curve y = f(x). **(2)**

d. State the set of values of k for which the equation f(x) = k has three solutions. **(1)**

(Total Marks: 11)

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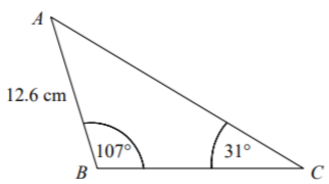
14. The figure shows triangle ABC in which AB = 12.6m, $A\widehat{B}C = 107^{\circ}$ and $A\widehat{C}B = 31^{\circ}$. Find, to 3 significant figures,



a. The length BC **(3)**

b. The area of triangle *ABC* **(2)**

(Total Marks: 5)

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15. Find the area of the finite region enclosed by the curve $y = 5x - x^2$ and the x-axis **(6)**

(Total Marks: 6)

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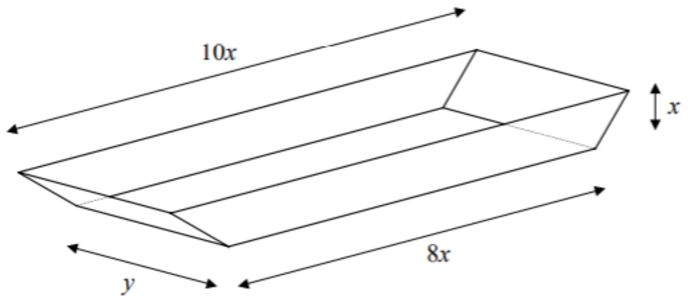
16. The figure shows a tray made from sheet metal.



The horizontal base is a rectangle measuring $8x$ cm by y cm and the two vertical sides are trapezia of height x cm with parallel edges of length $8x$ cm and $10x$ cm. The remaining two sides are rectangles inclined at 45° to the horizontal.

Given that the capacity of the tray is 900 cm^3

a. Find an expression for y in terms of x **(4)**

b. Show that the area of metal used to make the tray, A cm^2, is given by,

$$A = 18x^2 + \frac{200(4+\sqrt{2})}{x}$$

(4)

c. Find the 3 significant figures, the value of x for which A is stationary **(4)**

(Total Marks: 12)

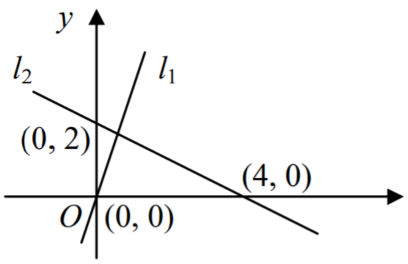
Total Marks: 120

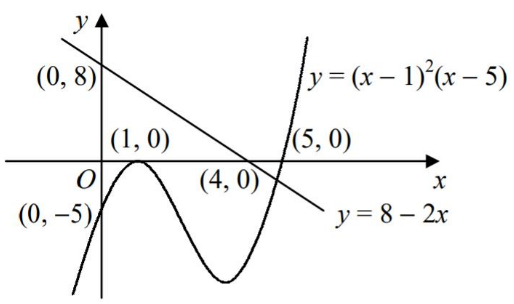
Mark Scheme

1	f(x) = $x + 6\sqrt{x} + 9 - 1 - 6\sqrt{x} + 9x$	**M1** **A1**
	$= 10x + 10$ $a = 10$ $b = 10$	**A1**

2	$y = \int (2x^3 + 1)dx$ $y = \frac{1}{2}x^4 + x + c$	**M1** **A2**
	When $x = 0$, $y = 3$ $c = 3$	**B1**
	$y = \frac{1}{2}x^4 + x + 3$ When $x = 2$, $y = 8 + 2 + 3 = 13$	**M1** **A1**

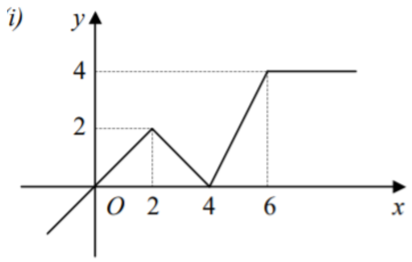
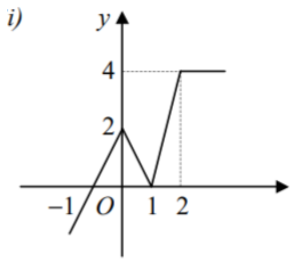
3a	$x = (\sqrt[3]{27})^2 = 3^2 = 9$	**M1** **A1**
3b	$= (\frac{9}{4})^{-\frac{1}{2}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$	**M1** **A1**

4a		**B2** **B1**
4b	$l_1 \rightarrow 6x - 2y = 0$ $l_2 \rightarrow x + 2y - 4 = 0$ Adding, $7x - 4 = 0$ $x = \frac{4}{7}$	**M1** **A1**
	Therefore, point of intersection, $(\frac{4}{7}, \frac{12}{7})$	**A1**

5a		**B3** **B2**

5b	The graphs intersect at exactly one point, therefore there is only one solution.	**B1**
5c	$n = 4$	**B1**

6a	$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} + 2x^{-\frac{3}{2}}$	**M1** **A2**
6b	$\frac{d^2y}{dx^2} = -\frac{1}{4}x^{-\frac{3}{2}} - 3x^{-\frac{5}{2}}$	**M1** **A1**
6c	LHS = $4x^2\left(-\frac{1}{4}x^{-\frac{3}{2}} - 3x^{-\frac{5}{2}}\right) + 4x\left(\frac{1}{2}x^{-\frac{1}{2}} + 2x^{-\frac{3}{2}}\right) - x^{-\frac{1}{2}} - 4x^{-\frac{1}{2}}$ $= -\frac{1}{2}x^{-\frac{1}{2}} - 12x^{-\frac{1}{2}} + 2x^{\frac{1}{2}} + 8x^{-\frac{1}{2}} - x^{-\frac{1}{2}} + 4x^{-\frac{1}{2}}$	**M1** **A1**
	$= 0$	**A1**

7ai	3	**B1**
7aii	1	**B1**
7bi		**B3**
7bii		**B3**

8a	$1 + 8(3x) + \binom{8}{2}(3x)^2 + \binom{8}{3}(3x)^3 + \ldots$	**M1** **A1**
	$= 1 + 24x + 252x^2 + 1512x^3 + \ldots.$	**M1** **A1**
8b.	$x = 0.001$	**B1**
	$(1.003)^8 = 1 + 0.024 + 0.000\,252 + 0.00001512$	**M1**
	= 1.0242535 (to 8 s.f)	**A1**

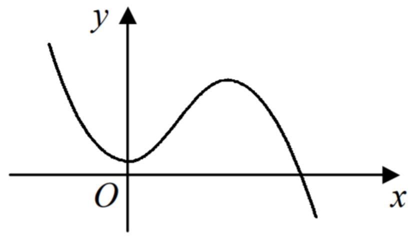
9a	$3 - \log_8 8^{\frac{2}{3}}$	**B1** **M1** **A1**
	$= 3 - \frac{2}{3}$ $= \frac{7}{3}$	**A1**
9b	$(2^2)^x - 3(2 \times 2^x) = 0$	**M1**
	$(2^x)^2 - 6(2^x) = 0$ $2^x(2^x - 6) = 0$	**M1**
	$2^x = 0$ (no solution) $2^x = 6$	**A1**

	$x = \frac{\log 6}{\log 2} = 2.58$ (3 s.f)	**M1** **A1**

10a	(-6, 5) Therefore, 36 + 25 -60 + k = 0	**M1**
	$k = 49$	**A1**
10b	$(x + 5)^2 – 25 + (y – 4)^2 – 16 + 39 = 0$	**M1**
	$(x + 5)^2 + (y – 4)^2 = 2$ Therefore, centre: (-5, 4) Radius = $\sqrt{2}$	**A2**
10c	Distance (2, 3) to centre = $\sqrt{49+1} = \sqrt{50}$	**B1**
	$AB^2 = (\sqrt{50})^2 – (\sqrt{2})^2 = 48$	**M1** **A1**
	$AB = \sqrt{48} = 4\sqrt{3}$	**M1** **A1**

11a	f(-1) = r Therefore -1 + k + 7 – 15 = r	**M1**
	$k = r + 9$	**A1**
	f(3) = 3r Therefore, 27 + 9k – 21 – 15 = 3r 3k = r + 3	**M1**
	Solving simultaneously by subtracting, 2k = -6	**M1**
	$k = -3$	**A1**
11b	$r = -3 – 9 = -12$	**B1**
11c	$f(x) = x^3 – 3x^2 – 7x – 15$ f(5) = 125 – 75 – 35 – 15 = 0 Therefore (x – 5) is a factor	**M1** **A1**
11d	$$\begin{array}{r l} & x^2 + 2x + 3 \\ x-5 \,\big)\!\! & x^3 - 3x^2 - 7x - 15 \\ & x^3 - 5x^2 \\ & \quad\; 2x^2 - 7x \\ & \quad\; 2x^2 - 10x \\ & \qquad\quad 3x - 15 \\ & \qquad\quad 3x - 15 \end{array}$$	**M1** **A1**
	$(x – 5)(x^2 + 2x + 3) = 0$ x = 5 or $x^2 + 2x + 3 = 0$ $b^2 – 4ac = 2^2 – (4 \times 1 \times 3) = -8$	**M1**
	$b^2 – 4ac < 0$, therefore no real soltions to quadratic, therefore only one real solution.	**A1**

12a	5 cos x = 2 sin x $\frac{5}{2} = \frac{\sin x}{\cos x}$	**M1**
	tan x = 2.5	**A1**
12b	tan 2x = 2.5 2x = 68.199 2x = 180 + 68.199	**B1** **M1**
	2x = 68.199 2x = 248.199	**M1**
	x = 34.1 x = 124.1	**A1**

13a	f'(x) = 12x – 3x^2	**M1** **A1**
	To find the stationary point, $12x - 3x^2 = 0$ $3x(4 - x) = 0$	**M1**
	$x = 0, y = 2$ $x = 4, y = 34$ Therefore coordinates: (0, 2) and (4, 34)	**A2**
13b	f''(x) = 12 – 6x	**M1**
	f''(0) = 12 f''(x) > 0, therefore, (0, 2) is a minimum	**A1**
	f''(4) = -12 f''(x) < 0, therefore, (4, 34) is a maximum	**A1**
13c	 	**B2**
13d	$2 < k < 34$	**B1**

14a	$B\hat{A}C = 180 - (107 + 31) = 42$	**B1**
	$\frac{BC}{\sin 42} = \frac{12.6}{\sin 31}$	**M1**
	BC = $\frac{12.6 \sin 42}{\sin 31}$ = 16.4 cm (3 s.f)	**A1**
14b	$= \frac{1}{2} \times 12.6 \times 16.37 \times sin107 = 98.6\ cm^2$	**M1** **A1**

15	$5x - x^2 = 0$ $x(5 - x) = 0$ Crosses x-axis at (0,0) and (5, 0)  	**B1**
	Area = $\int_0^5 (5x - x^2)dx$ $= [\frac{5}{2}x^2 - \frac{1}{3}x^3]_0^5$	**M1** **A2**
	$= (\frac{125}{2} - \frac{125}{3}) - (0) = 20\frac{5}{6}$	**M1** **A1**

16a	Area of XS	**M1**
	$= \frac{1}{2} \times (8x + 10x) \times x = 9x^2$	**M1**
	Volume = $9x^2y = 900$	**M1**
	$y = \frac{100}{x^2}$	**A1**
16b	Width of sloping sides = $\sqrt{2}x$	**B1**
	$A = 8xy + 2(9x^2) + 2(\sqrt{2}\ xy)$	**M1**
	$A = 18x^2 + 2xy(4 + \sqrt{2})$ $A = 18x^2 + 2x(4 + \sqrt{2}) \times \frac{100}{x^2}$	**M1**
	$A = 18x^2 + \frac{200(4+\sqrt{2})}{x}$	**A1**
16c	$\frac{dA}{dx} = 36x - 200(4 + \sqrt{2})\ x^{-2}$	**M1** **A1**
	For stationary point,	**M1**

	$36x - 200(4 + \sqrt{2})\, x^{-2} = 0$	
	$x^3 = \frac{200(4+\sqrt{2})}{36}$ $x = 3.11$	**A1**

Topic List

Q1	Surds
Q2	Integration
Q3	Solving equations with indices
Q4	Straight lines
Q5	Drawing graphs
Q6	Differentiation
Q7	Graph transformations
Q8	Binomial expansion
Q9	Logarithms
Q10	Circles
Q11	Remainder factor theorem, algebraic division
Q12	Solving trigonometric equations
Q13	Differentiation and stationary points
Q14	Sine and cosine rule
Q15	Area under a curve
Q16	Maximum/minimum point