Practice Exam Paper A
Time: 2 Hours

P1

1a. Express $\frac{18}{\sqrt{3}}$ in the form $k\sqrt{3}$ **(2)**

b. Express $(1 - \sqrt{3})(4 - 2\sqrt{3})$ in the form $a + b\sqrt{3}$, where a and b are integers. **(2)**

(Total Marks: 4)

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2. $f(x) = 2x^2 - 4x + 1$

a. Find the values of constants a, b and c such that, $f(x) = a(x + b)^2 + c$ **(4)**

b. State the equation of the line of symmetry of the curve, $y = f(x)$ **(1)**

c. Solve the equation $f(x) = 3$, giving your answers in exact form **(3)**

(Total Marks: 8)

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3. The curve C with equation $y = f(x)$ is such that, $\frac{dy}{dx} = 3x^2 + 4x + k$

Where k is a constant.

Given that C passes through the points (0, -2) and (2, 18).

a. Show that $k = 2$ and find an equation for C **(7)**

b. Show that the line with equation $y = x - 2$ is a tangent to C and find the coordinates of the point of contact. **(5)**

(Total Marks: 12)

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4. Solve, for $0 \leq x < 360$, the equation,

$$3\cos^2 x + \sin^2 x + 5 \sin x = 0$$

(7)

(Total Marks: 7)

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5. A circle has the equation $x^2 + y^2 - 6y - 7 = 0$

a. Find the coordinates of the centre of the circle **(2)**

b. Find the radius of the circle **(2)**

(Total Marks: 4)

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6. Expand $(1 + x)^4$ in ascending powers of x. **(2)**

b. Using your expansion, express each of the following in the form $a + b\sqrt{2}$, where a and b are integers.
(i) $(1 + \sqrt{2})^4$

(ii) $(1 - \sqrt{2})^8$ **(7)**

(Total Marks: 9)

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7. $f(x) = x^3 - 4x^2 - 3x + 18$

a. Show that $(x - 3)$ is a factor of $f(x)$ **(2)**

b. Fully factorise $f(x)$ **(4)**

c. Using your answer to part *b*, write down the coordinates of one of the turning points of the curve $y = f(x)$ and give a reason for your answer. **(2)**

d. Using differentiation, find the x-coordinate of the other turning point of the curve $y = f(x)$ **(5)**

(Total Marks: 13)

8a. Sketch on the same diagram the graphs of $y = \sin 2x$ and $y = \tan \frac{x}{2}$ for x in the interval, $0 \leq x \leq 360^{\circ}$ **(4)**

b. Hence state how many solutions exist to the equation,

$$\sin 2x = \tan \frac{x}{2}$$

for x in the interval $0 \leq x \leq 360^{\circ}$ and give a reason for your answer. **(2)**

(Total Marks: 6)

9a. Find the value of a such that,

$$\log_a 27 = 3 + \log_a 8$$

(3)

b. Solve the equation

$$2^{x + 3} = 6^{x - 1}$$

giving your answer to 3 significant figures. **(4)**

(Total Marks: 7)

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10. The curve C has the equation,

$$y = 3 - x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}, x > 0$$

a. Find the coordinates of the points where C crosses the x-axis. **(4)**

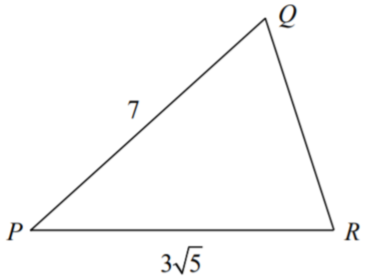
b. Find the exact coordinates of the stationary point of C. **(5)**

c. Determine the nature of the stationary point **(2)**

d. Sketch the curve C. **(2)**

(Total Marks: 13)

11. The figure shows triangle PQR in which $PQ = 7$ and $PR = 3\sqrt{5}$.



Given that $\sin(Q\hat{P}R) = \frac{2}{3}$ and that $Q\hat{P}R$ is acute,

a. Find the exact value of $\cos(Q\hat{P}R)$ in its simplest form. **(2)**

b. Show that $QR = 2\sqrt{6}$ **(4)**

c. Find PQR in degrees to 1 decimal place. **(3)**

(Total Marks: 9)

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12. Solve the simultaneous equations,

$$x - 3y + 7 = 0$$
$$x^2 + 2xy - y^2 = 7$$

(7)

(Total Marks: 7)

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13. Given that,

$$\frac{dy}{dx} = \frac{x^3 - 4}{x^3}, x \neq 0$$

a. Find $\frac{d^2y}{dx^2}$

Given also that $y = 0$ when $x = -1$ **(3)**

b. Find the value of y when $x = 2$ **(6)**

(Total Marks: 9)

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14. Find the set of values of x for which,

a. $6x - 11 > x + 4$ **(2)**

b. $x^2 - 6x - 16 < 0$ **(3)**

c. Both $6x - 11 > x + 4$ and $x^2 - 6x - 16 > 0$ **(1)**

(Total Marks: 6)

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15. Evaluate, $(36^{\frac{1}{2}} + 16^{\frac{1}{4}})^{\frac{1}{3}}$ **(3)**

b. Solve the equation $3x^{-\frac{1}{2}} - 4 = 0$ **(3)**

(Total Marks: 6)

Total Marks: 120

Mark Scheme

1a	$\frac{18}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$	**M1**
	$= 6\sqrt{3}$	**A1**
1b	$= 4 - 2\sqrt{3} - 4\sqrt{3} + 6$	**M1**
	$= 10 - 6\sqrt{3}$	**A1**

2a	$f(x) = 2[x^2 - 2x] + 1$	**M1**
	$= 2[(x-1)^2 - 1] + 1$	**M1**
	$= 2(x-1)^2 - 1$	**A2**
2b	$x = 1$	**B1**
2c	$2(x-1)^2 - 1 = 3$ $(x-1)^2 = 2$	**M1**
	$x = 1 \pm \sqrt{2}$	**M1** **A1**

3a	$y = \int (3x^2 + 4x + k)dx$ $y = x^3 + 2x^2 + kx + c$	**M1** **A2**
	At the point (0, -2) $c = -2$	**B1**
	At the point (2, 18) $18 = 8 + 8 + 2k - 2$	**M1**
	$k = 2$	**A1**
	$y = x^3 + 2x^2 + 2x - 2$	**A1**
3b	$x^3 + 2x^2 + 2x - 2 = x - 2$ $x^3 + 2x^2 + x = 0$ $x(x^2 + 2x + 1) = 0$	**M1**
	$x(x+1)^2 = 0$	**M1**
	As there is a repeated root, there is a tangent	**A1**
	Point of contact is at $x = -1$	**M1**
	When $x = -1$ $y = -3$ Point of contact: (-1, -3)	**A1**

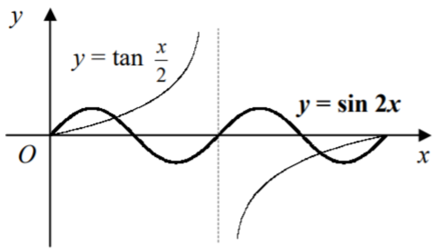
4	$3(1 - \sin^2 x) + \sin^2 x + 5\sin x = 0$	**M1**
	$2\sin^2 x - 5\sin x - 3 = 0$	**A1**
	$(2\sin x + 1)(\sin x - 3) = 0$	**M1**
	$\sin x = 3$ No solutions $\sin x = -\frac{1}{2}$	**A1**
	$x = 180 + 30, 360 = 30$	**B1** **M1**
	$x = 210^o, 330^o$	**A1**

5a	$x^2 + (y - 3)^2 - 9 - 7 = 0$	**M1**
	Therefore centre (0, 3)	**A1**
5b	$x^2 + (y - 3)^2 = 16$	**M1**
	Therefore, radius = 4	**A1**

6a	$= 1 + 4x + 6x^2 + 4x^3 + x^4$	**M1** **A1**
6bi	$1 + 4(\sqrt{2}) + 6(\sqrt{2})^2 + 4(\sqrt{2})^3 + (\sqrt{2})^4$	**M1**
	$= 1 + 4\sqrt{2} + 6(2) + 4(2\sqrt{2}) + 4$	**M1**

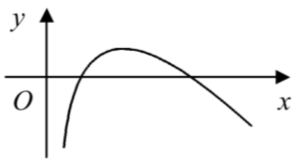
	$= 17 + 12\sqrt{2}$	**A1**
6bii	$(1 - \sqrt{2})^4 = 17 + 12\sqrt{2}$	**B1**
	$(1 - \sqrt{2})^8 = [(1 - \sqrt{2})^4]^2 = (17 - 12\sqrt{2})^2$	**M1**
	$= 289 - 408\sqrt{2} + 288$	**M1**
	$= 577 - 408\sqrt{2}$	**A1**

7a	$f(3) = 27 – 36 – 9 + 18 = 0$ Therefore $(x – 3)$ is a factor	**M1** **A1**
7b	` $x - 3 \overline{)\, x^3 - 4x^2 - 3x + 18}$, quotient $x^2 - x - 6$ $x^3 - 3x^2$ $-x^2 - 3x$ $-x^2 + 3x$ $-6x + 18$ $-6x + 18$	**M1** **A1**
	$f(x) = (x – 3)(x^2 – x – 6)$ $f(x) = (x – 3)(x + 2)(x – 3)$	**M1**
	$= (x + 2)(x – 3)^2$	**A1**
7c	(3, 0)	**B1**
	$(x – 3)$ is a repeated factor of $f(x)$ therefore, x-axis is tangent where $x = 3$	**B1**
7d	$f'(x) = 3x^2 – 8x – 3$	**M1** **A1**
	For stationary point, $3x^2 – 8x – 3 = 0$	**M1**
	$(3x + 1)(x – 3) = 0$	**M1**
	$x = -\frac{1}{3}, 3$ Therefore, $x = -\frac{1}{3}$	**A1**

8a		**B2** **B2**
8b	4 solutions	**B1**
	As the graph intersects at 4 points.	**B1**

9a	$\log_a 27 – \log_a 8 = 3$ $\log_a \frac{27}{8} = 3$	**M1**
	$a^3 = \frac{27}{8}$ $a = \sqrt[3]{\frac{27}{8}}$ $a = \frac{3}{2}$	**M1** **A1**
9b	$(x + 3)\log 2 = (x – 1)\log 6$	**M1**
	$x(\log 6 – \log 2) = 3 \log 2 + \log 6$	**M1**
	$x = \frac{3\log 2+\log 6}{\log 6-\log 2}$ $x = 3.52$	**M1** **A1**

10a	$3 – x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} = 0$ $3x^{\frac{1}{2}} - x - 2 = 0$	**M1**
	$x - 3x^{\frac{1}{2}} + 2 = 0$	**M1**

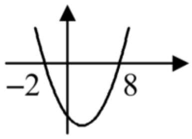
	x = 1 and 4	**A1**
	Therefore, (1, 0) and (4, 0)	**A1**
10b	$\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{1}{2}} + x^{-\frac{3}{2}}$	**M1** **A1**
	For minimum, -$-\frac{1}{2}x^{-\frac{1}{2}} + x^{-\frac{3}{2}}$ = 0 -$\frac{1}{2}x^{-\frac{3}{2}}$(x – 2) = 0	**M1**
	x = 2 $y = 3 - \sqrt{2} - \frac{2}{\sqrt{2}}$	**A1**
	$(2, 3 - 2\sqrt{2})$	**A1**
10c	$\frac{d^2y}{dx^2} = \frac{1}{4}x^{-\frac{3}{2}} - \frac{3}{2}x^{-\frac{5}{2}}$	**M1**
	When x = 2, $\frac{d^2y}{dx^2} = \frac{1}{8\sqrt{2}} - \frac{3}{8\sqrt{2}} = -\frac{1}{4\sqrt{2}}$ $\frac{d^2y}{dx^2} < 0$, therefore maximum	**A1**
10d		**B2**

11a	$\cos^2 P = 1 - (\frac{2}{3})^2 = \frac{5}{9}$	**M1**
	As the angle is acute, $Q\hat{P}R = \sqrt{\frac{5}{9}} = \frac{1}{3}\sqrt{5}$	**A1**
11b	$QR^2 = 7^2 + (3\sqrt{5})^2 - (2 \text{ x } 7 \text{ x } 3\sqrt{5} \text{ x } \frac{1}{3}\sqrt{5})$	**M1** **A1**
	$QR^2 = 49 + 45 - 70 = 24$	**M1**
	$QR = \sqrt{24} = 2\sqrt{6}$	**A1**
11c	$\frac{\sin Q}{3\sqrt{5}} = \frac{\frac{2}{3}}{2\sqrt{6}}$	**M1**
	$\sin Q = \frac{\sqrt{5}}{\sqrt{6}}$	**M1**
	$P\hat{Q}R = 65.9^{O}$	**A1**

12	$x - 3y + 7 = 0$ $x = 3y - 7$	**M1**
	$x^2 + 2xy - y^2 = 7$ $(3y - 7)^2 + 2y(3y - 7) - y^2 = 8$	**M1**
	$y^2 - 4y + 3 = 0$	**A1**
	$(y - 1)(y - 3) = 0$	**M1**
	$y = 1, y = 3$	**A1**
	When y = 1 x = -4	**M1**
	When y = 3, x = 2	**A1**

13a	$\frac{dy}{dx} = 1 - 4x^{-3}$	**B1**
	$\frac{d^2y}{dx^2} = 12x^{-4}$	**M1 A1**
13b	$y = \int 1 - 4x^{-3}\, dx$ $y = x + 2x^{-2} + c$	**M1 A2**
	x = -1	**M1**

	$y = 0$ Therefore, $c = -1$	
	$y = x + 2x^{-2} - 1$ When x = 2, $y = 2 + \frac{1}{2} - 1 = \frac{3}{2}$	**M1 A1**

14a	$5x > 15$	**M1**
	$x > 3$	**A1**
14b	$(x + 2)(x - 8) < 0$	**M1**
		**M1**
	$2 < x < 8$	**A1**
14c	$3 < x < 8$	**B1**

15a	$= (6 + \sqrt[4]{16})^{\frac{1}{3}}$ $= (6 + 2)^{\frac{1}{3}}$	**B1** **M1**
	$= \sqrt[3]{8}$ $= 2$	**A1**
16c	$\frac{3}{\sqrt{x}} = 4$	**M1**
	$\sqrt{x} = \frac{3}{4}$	**M1**
	$x = \frac{9}{16}$	**A1**

<u>Topic List</u>

Q1	Surds
Q2	Completing the square
Q3	Integration with repeated roots
Q4	Solving trigonometric equations
Q5	Circle equations
Q6	Binomial expansion
Q7	Factor theorem and stationary points
Q8	Drawing trig graphs
Q9	Logarithms and exponentials
Q10	Stationary points and drawing graphs
Q11	Cosine rule and sine rule
Q12	Simultaneous equations
Q13	Differentiation and Integration
Q14	Inequalities
Q15	Solving equations