Practice Exam Paper
Time: 2 Hours

P1 P2

1. The first term of a geometric series is 20 and the common ratio is $\frac{7}{8}$. The sum to infinity of the series is S_∞.

a. Find the value of S_∞ **(2)**

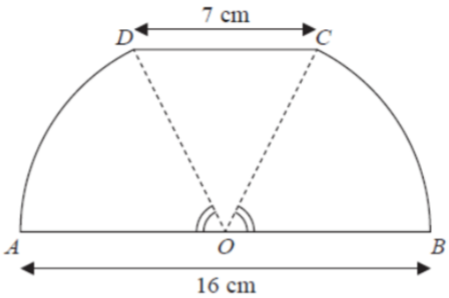
The sum to N terms of the series is S_N

b. Find, to 1 decimal place, the value of S_{12} **(2)**

c. Find the smallest value of N, for which $S_\infty - S_N < 0.5$. **(4)**

(Total Marks: 8)

2.



The figure shows a sketch of a design for a scraper blade. The blade *AOBCDA* consists of an isosceles triangle *COD* joined along its equal sides to sectors *OBC* and *ODA* of a circle with centre *O* and radius 8 cm. Angles *AOD* and *BOC* are equal. *AOB* is a straight line and is parallel to the line *DC*. *DC* has length 7 cm.

a. Show that the angle *COD* is 0.906 radians, correct to 3 significant figures. **(2)**

b. Find the perimeter of *AOBCDA*, giving your answer to 3 significant figures **(3)**

c. Find the area of *AOBCDA*, giving your answer to 3 significant figures **(3)**

(Total Marks: 8)

3a. Express $7 \cos x - 24 \sin x$ in the form $R \cos (x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the value of α to 3 decimal places. **(3)**

b. Hence write down the minimum value of $7 \cos x - 24 \sin x$. **(1)**

c. Solve, for $0 \leq x < 2\pi$, the equation

$$7 \cos x - 24 \sin x = 10,$$

giving your answers to 2 decimal places. **(5)**

(Total Marks: 9)

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4. The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$$m = pe^{-kt},$$

where k and p are positive constants

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

a. Write down the value of p **(1)**

b. Show that $k = \frac{1}{4} \ln 3$ **(4)**

c. Find the value of t when $\frac{dm}{dt} = -\,0.6 \ln 3$ **(6)**

(Total Marks: 11)

5a. Prove that

$$\frac{1}{\sin 2x} - \frac{\cos 2x}{\sin 2x} = \tan x \qquad x \neq 90n$$ **(4)**

b. Hence, or otherwise,

i. Show that tan 15 = 2 - $\sqrt{3}$ **(3)**

ii. Solve, for $0 < x < 360^{\circ}$

$$\text{cosec } 4x - \cot 4x = 1$$ **(5)**

(Total Marks: 12)

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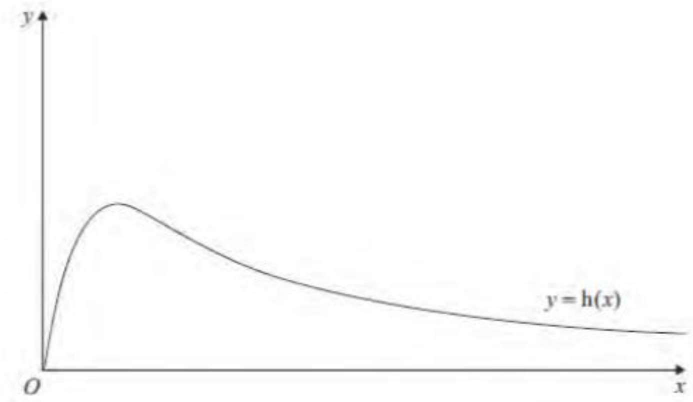
6. $h(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, x \geq 0$

a. Show that $h(x) = \frac{2x}{x^2+5}$ **(4)**

b. Hence, or otherwise, find $h'(x)$ in its simplest form. **(3)**



The figure shows a graph of the curve with equation $y = h(x)$

c. Calculate the range of $h(x)$ **(5)**

(Total Marks: 12)

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7. The curve C has equation

$$3^{x-1} + xy - y^2 + 5 = 0$$

Show that $\frac{dy}{dx}$ at the point (1, 3) on the curve C can be written in the form $\frac{1}{\lambda} \ln (\mu e^3)$, where λ and μ are integers to be found. **(7)**

(Total Marks: 7)

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8. The curve C has equation,

$$\cos 2x + \cos 3y = 1, -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}, \quad 0 \leq y \leq \frac{\pi}{6}$$

a. Find $\frac{dy}{dx}$ in terms of x and y **(3)**

The point P lies on C where $x = \frac{\pi}{6}$

b. Find the value of y at P. **(3)**

c. Find the equation of the tangent to C at P, giving your answer in the form $ax + by + c\pi = 0$, where a, b and c are integers. **(3)**

(Total Marks: 9)

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9. Relative to a fixed origin O, the point A has position vector $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and the point B has position vector $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$. The points A and B lie on a straight line l.

a. Find $\overrightarrow{AB}$ **(2)**

b. Find a vector equation of l **(2)**

The point C has position vector $2\mathbf{i} + \mathrm{p}\mathbf{j} - 4\mathbf{k}$ with respect to O, where p is a constant. Given that AC is perpendicular to l, find

c. The value of p **(4)**

d. The distance AC **(2)**

(Total Marks: 10)

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10. A curve C has parametric equations,

$$x = \sin^2 t, y = 2 \tan t \qquad 0 \leq t < \frac{\pi}{2}$$

a. Find $\frac{dy}{dx}$ in terms of t **(4)**

The tangent to C at the point where $t = \frac{\pi}{3}$ cuts the x-axis at the point P.

b. Find the x-coordinate of P **(6)**

(Total Marks: 10)

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11. Using the substitution $x = 2 \cos u$, or otherwise, find the exact value of,

$$\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{4-x^2}}\, dx$$ **(7)**

(Total Marks: 7)

12a.Express $4 \operatorname{cosec}^2 2x - \operatorname{cosec}^2 x$ in terms of $\sin x$ and $\cos x$. **(2)**

b. Hence show that, $4 \operatorname{cosec}^2 2x - \operatorname{cosec}^2 x = \sec^2 x$ **(4)**

c. Hence or otherwise solve, for $0 < x < \pi$, $4 \operatorname{cosec}^2 2x - \operatorname{cosec}^2 x = 4$ **(3)**

giving your answer in terms of π

(Total Marks: 9)

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13. Given that, $f(x) = \ln x$, $x > 0$

Sketch on separate axes the graph of,

a. $y = f(x)$

b. $y = |f(x)|$

c. $y = -f(x - 4)$

Show, on each diagram, the point where the graph meets or crosses the x-axis. In each case, state the equation of the asymptote.

(Total Marks: 8)

Total Marks: 120

Mark Scheme

1a	$S_\infty = \frac{20}{1-\frac{7}{8}} = 160$	**M1** **A1**
1b	$S_{12} = \frac{20\left(1-\left(\frac{7}{8}\right)^{12}\right)}{1-\frac{7}{8}} = 127.733 \ldots..$	**M1** **A1**
1c	$160 - \frac{20\left(1-\left(\frac{7}{8}\right)^{N}\right)}{1-\frac{7}{8}} < 0.5$	**M1**
	$160\ (\left(\frac{7}{8}\right)^{N} < 0.5$	**M1**
	$N \log\left(\frac{7}{8}\right) < \log\left(\frac{0.5}{160}\right)$	**M1**
	$N > \frac{\log\left(\frac{0.5}{160}\right)}{\log\left(\frac{7}{8}\right)} = 43.19823 \ldots$ $N = 44$	**A1**

2a	$\cos COD = \frac{8^2+8^2-7^2}{2\times8\times8}$	**M1**
	Angle COD = 0.95056… = 0.906	**A1**
2b	$s = 8\theta$	**M1**
	$\theta = \frac{\pi - COD}{2} = 1.12$	**M1**
	23 + (16 x θ) = 40.9 cm	**A1**
2c	Area of a triangle = $\frac{1}{2} \times 8 \times 8 \times \sin 0.906$	**M1**
	Area of sector = $\frac{1}{2} \times 8^2 \times 1.11797932$	**M1**
	Total area = area of two sectors + area of triangle = 96.7 cm^2	**A1**

3a	7 cos x – 24 sin x = R cos (x + α) 7 cos x – 24 sin x = R cos x cos α – R sin x sin α Equate cos x: 7 = R cos α Equate sin x: 24 = R sin α $R = \sqrt{7^2 + 24^2} = 25$	**B1**
	tan α = $\frac{24}{7}$ α = 1.28700….	**M1**
	α = 1.287 Hence, 7 cos x – 24 sin x = 25 cos (x + 1.287)	**A1**
3b	Minimum value = -25	**B1**
3c	7 cos x – 24 sin x = 10 25 cos (x + 1.287) = 10 cos (x + 1.287) = $\frac{10}{25}$	**M1**
	PV = 1.159279481	**M1**
	So, x + 1.287 = 1.15929…., 5.123906…, 7.442465…	**M1**
	x = 3.836906, 6.155465	**A1** **A1**

4a	p = 7.5	**B1**
4b	$2.5 = 7.5e^{-4k}$	**M1**
	$e^{-4k} = \frac{1}{3}$	**M1**
	$-4k = \ln\left(\frac{1}{3}\right)$	**M1**
	$-4k = -\ln(3)$	**A1**

	$k = \frac{1}{4}\ln(3)$	
4c	$\frac{dm}{dt} = -kpe^{-kt}$	**M1** **A1**
	$-\frac{1}{4}\ln 3 \times 7.5e^{-\frac{1}{4}(\ln 3)t} = -0.6\ln 3$ $e^{-\frac{1}{4}(\ln 3)t} = \frac{2.4}{7.5} = 0.32$	**M1** **A1**
	$-\frac{1}{4}(\ln 3)\ t = \ln(0.32)$	**M1**
	$t = 4.1486$	**A1**

5a	$\frac{1}{\sin 2x} - \frac{\cos 2x}{\sin 2x} = \frac{1-\cos 2x}{\sin 2x}$	**M1**
	$= \frac{2\sin^2 x}{2\sin x\cos x}$	**M1** **A1**
	$= \frac{\sin x}{\cos x} = \tan x$	**A1**
5bi	$\tan 15 = \frac{1}{\sin 30} - \frac{\cos 30}{\sin 30}$	**M1**
	$\tan 15 = \frac{1}{\frac{1}{2}} - \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2 - \sqrt{3}$	**M1** **A1**
5bii	$\tan 2x = 1$	**M1**
	$2x = 45$	**A1**
	$2x = 45 + 180$	**M1**
	$x = 22.5^{\circ}, 112.5^{\circ}, 202.5^{\circ}, 292.5^{\circ}$	**A1** **A1**

6a	$\frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x+2)(x^2+5)} = \frac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$	**M1** **A1**
	$= \frac{2x(x+2)}{(x+2)(x^2+5)}$	**M1**
	$= \frac{2x}{(x^2+5)}$	**A1**
6b	$h'(x) = \frac{(x^2+5)\times 2 - 2x\times 2x}{(x^2+5)^2}$	**M1** **A1**
	$h'(x) = \frac{10-2x^2}{(x^2+5)^2}$	**A1**
6c	Maximum occurs when $h'(x) = 0$ $10 - 2x^2 = 0$	**M1**
	$x = \sqrt{5}$	**A1**
	When $x = \sqrt{5}$ $h(x) = \frac{\sqrt{5}}{5}$	**M1** **A1**
	Range of $h(x)$ is $0 \le h(x) \le \frac{\sqrt{5}}{5}$	**A1**

7a	Attempt to differentiate implicitly.	**B1**
	$3^{x-1}\ln 3 + (y + x\frac{dy}{dx}) - 2y\frac{dy}{dx} = 0$	**M1**
	$xy \rightarrow \ldots + y + x\frac{dy}{dx}$	**B1**
	$\ldots + y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$	**A1**
	Sub (1, 3) $3^{(1-1)}\ln 3 + 3 + (1)\frac{dy}{dx} - 2(3)\frac{dy}{dx} = 0$	**M1**
	$\ln 3 + 3 + \frac{dy}{dx} - 6\frac{dy}{dx} = 0$ $3 + \ln 3 = 5\frac{dy}{dx}$ $\frac{dy}{dx} = \frac{3+\ln 3}{5}$	**M1**

	$\frac{dy}{dx} = \frac{1}{5}(\ln e^3 + \ln 3) = \frac{1}{5}(3e^3)$	**A1**

8a	$-2\sin 2x - 3\sin 3y\frac{dy}{dx} = 0$	**M1** **A1**
	$\frac{dy}{dx} = -\frac{2\sin 2x}{3\sin 2y}$	**A1**
8b	At $x = \frac{\pi}{6}$ $\text{Cos}\left(\frac{2\pi}{6}\right) + \cos 3y = 1$	**M1**
	$\cos 3y = \frac{1}{2}$	**A1**
	$3y = \frac{\pi}{3}$ $y = \frac{\pi}{9}$	**A1**
8c	At $\left(\frac{\pi}{6}, \frac{\pi}{9}\right)$ $\frac{dy}{dx} = -\frac{2\sin 2\left(\frac{\pi}{6}\right)}{3\sin 3\left(\frac{\pi}{9}\right)} = -\frac{2\sin\frac{\pi}{3}}{3\sin\frac{\pi}{3}} = -\frac{2}{3}$	**M1**
	$y - \frac{\pi}{9} = -\frac{2}{3}\left(x - \frac{\pi}{6}\right)$	**M1**
	Leading to $6x + 9y - 2\pi = 0$	**A1**

9a	$AB = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} - (\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) = -3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$	**M1** **A1**
9b	$r = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k} + \lambda(-3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$	**M1** **A1**
9c	$AC = 2\mathbf{i} + p\mathbf{j} - 4\mathbf{k} - (\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$ $= \mathbf{i} + (p + 3)\mathbf{j} - 6\mathbf{k}$	**B1**
	$AC.\,AB = \begin{pmatrix} 1 \\ p+3 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 5 \\ -3 \end{pmatrix} = 0$	**M1**
	$-3 + 5p + 15 + 18 = 0$ Leading to $p = -6$	**M1** **A1**
9d	$AC^2 = (2 - 1)^2 + (-6 + 3)^2 + (-4-2)^2 = 46$	**M1**
	$AC = \sqrt{46}$	**A1**

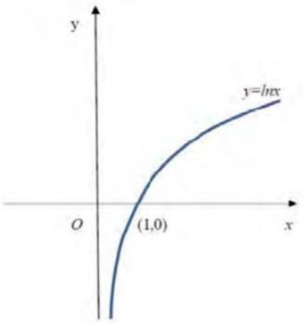
10a	$\frac{dx}{dt} = 2\sin t\cos t$ $\frac{dy}{dx} = 2\sec^2 t$	**B1** **B1**
	$\frac{dy}{dx} = \frac{\sec^2 t}{\sin t\cos t}$	**M1** **A1**
10b	At $t = \frac{\pi}{3}$, $x = \frac{3}{4}$ $y = 2\sqrt{3}$	**B1**
	$\frac{dy}{dx} = \frac{\sec^2\frac{\pi}{3}}{\sin\frac{\pi}{3}\cos\frac{\pi}{3}} = \frac{16}{\sqrt{3}}$	**M1** **A1**
	$y - 2\sqrt{3} = \frac{16}{\sqrt{3}}\left(x - \frac{3}{4}\right)$	**M1**
	$y = 0$ $x = \frac{3}{8}$	**M1** **A1**

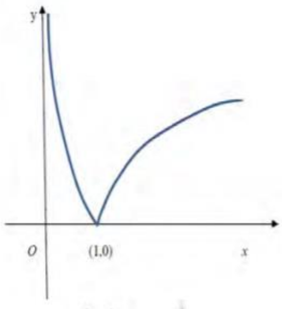
11	$\frac{dx}{du} = -2\sin u$	**B1**
	$\int \frac{1}{x^2\sqrt{4-x^2}}\,dx = \int \frac{1}{(2\cos u)^2\sqrt{4-(2\cos u)^2}} \times -2\sin u\,du$	**M1**
	$= \int \frac{-2\sin u}{4\cos^2 u\sqrt{4\sin^2 u}}\,du$	**M1**

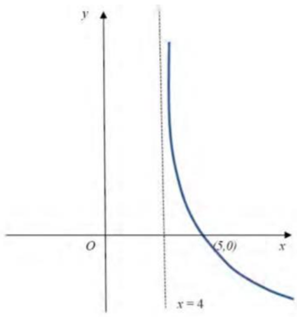
	$= -\frac{1}{4}\int \frac{1}{\cos^2 u}\, du$	**M1**
	$= -\frac{1}{4}\tan u\ (+C)$	**M1**
	$x = \sqrt{2}$ $\sqrt{2} = 2\cos u$ $u = \frac{\pi}{4}$ $x = 1$ $1 = 2\cos u$ $u = \frac{\pi}{3}$	**M1**
	$[-\frac{1}{4}\tan u]_{\frac{\pi}{3}}^{\frac{\pi}{4}} = -\frac{1}{4}(\tan\frac{\pi}{4} - \tan\frac{\pi}{3})$ $= -\frac{1}{4}(1-\sqrt{3})$ $= \frac{\sqrt{3}-1}{4}$	**A1**

12a	$4\operatorname{cosec}^2 2x - \operatorname{cosec}^2 x = \frac{4}{\sin^2 2x} - \frac{1}{\sin^2 x}$ $= \frac{4}{(2\sin x\cos x)^2} - \frac{1}{\sin^2 x}$	**B1** **B1**
12b	$\frac{4}{(2\sin x\cos x)^2} - \frac{1}{\sin^2 x} = \frac{4}{4\sin^2 x\cos^2 x} - \frac{1}{\sin^2 x}$ $= \frac{1}{\sin^2 x\cos^2 x} - \frac{\cos^2 x}{\sin^2 x\cos^2 x}$	**M1**
	$= \frac{\sin^2 x}{\sin^2 x\cos^2 x}$	**M1**
	$= \frac{1}{\cos^2 x} = \sec^2 x$	**M1** **A1**
12c	$\sec^2 x = 4$ $\sec x = \pm 2$ $\cos x = \pm\frac{1}{2}$	**M1**
	$x = \frac{\pi}{3}, \frac{2\pi}{3}$	**A1** **A1**

13

a) 

b) 

c) 

13a	ln graph crossing x axis at (1,0)	**B1**
	Asymptote at $x = 0$	**B1**
13b	Shape including cusp	**B1**
	Touches or crosses the x axis at (1,0)	**B1**
	Asymptote given as $x = 0$	**B1**
13c	Shape	**B1**
	Crosses at (5, 0)	**B1**
	Asymptote given as $x = 4$	**B1**

Topic List

Q1	Sequences and series
Q2	Radians
Q3	Trig.
Q4	Exponential modelling
Q5	Trig
Q6	Functions
Q7	Implicit differentiation
Q8	Differentiation
Q9	Vectors
Q10	Parametric differentiation
Q11	Integration using substitution
Q12	Trig proof and solving equations
Q13	Sketching and transforming graphs