Practice Exam Paper H
Time: 2 Hours

P1 P2

1. $y = \sqrt{(10x - x^2)}$

a. Copy and complete the table below, giving the values of y to 2 decimal places. **(2)**

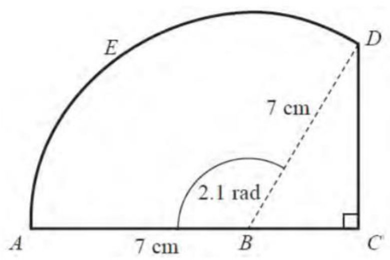
x	1	1.4	1.8	2.2	2.6	3
y	3	3.47			4.39	

b. Use the trapezium rule, with all the values of y from your table, to find an approximation for the value of,

$\int_1^3 \sqrt{(10x - x^2)}\ dx$ **(4)**

(Total Marks: 6)

2.



The figure shows the shape *ABCDEA* which consists of a right-angled triangle *BCD* joined to a sector *ABDEA* of a circle with radius 7 cm and centre *B*.

A, *B* and *C* lie on a straight line with *AB* = 7 cm.

Given that the size of angle *ABD* is exactly 2.1 radians,

a. Find, in cm, the length of the arc *DEA*, **(2)**

b. Find, in cm, the perimeter of the shape *ABCDEA*, giving your answer to 1 decimal place. **(4)**

(Total Marks: 6)

3. Rabbits were introduced onto an island. The number of rabbits, *P*, *t* years after they were introduced is modelled by the equation,

$$P = 80e^{\frac{1}{5}t}, t \geq 0$$

a. Write down the number of rabbits that were introduced to the island **(1)**

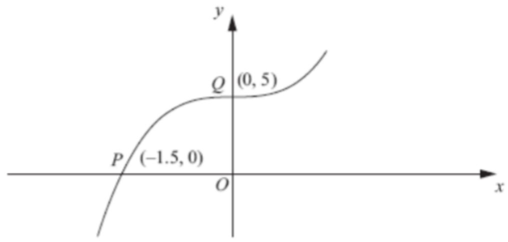
b. Find the number of years it would take for the number of rabbits to first exceed 1000. **(2)**

c. Find $\frac{dP}{dt}$ **(2)**

d. Find *P* when $\frac{dP}{dt} = 50$ **(3)**

(Total Marks: 8)

4. The figure shows part of the curve with equation $y = f(x)$,



The curve passes through the points $Q(0, 2)$ and $P(-3, 0)$ as shown.

a. Find the value of $ff(-3)$. **(2)**

On separate diagrams, sketch the curve with equation

b. $y = f^{-1}(x)$ **(2)**

c. $y = f(|x|) - 2$ **(2)**

d. $y = 2f(\frac{1}{2}x)$ **(3)**

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

(Total Marks: 9)

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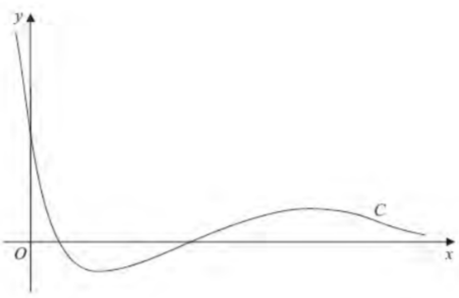
5. The figure shows a sketch of the curve C with the equation $y = (2x^2 - 5x + 2)e^{-x}$



a. Find the coordinates of the point where C crosses the y-axis **(1)**

b. Show that C crosses the x-axis at $x = 2$ and find the x-coordinate of the other point where C crosses the x-axis. **(3)**

c. Find $\frac{dy}{dx}$ **(3)**

d. Hence find the exact coordinates of the turning points of C. **(5)**

(Total Marks: 12)

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6. The functions f and g are defined by,

$$f : x \to 2|x| + 3$$
$$g : x \to 3 - 4x$$

a. State the range of f. **(2)**

b. Find $fg(1)$. **(2)**

c. Find g^{-1}, the inverse function of g. **(2)**

d. Solve the equation,

$$gg(x) + [g(x)]^2 = 0$$ **(5)**

(Total Marks: 11)

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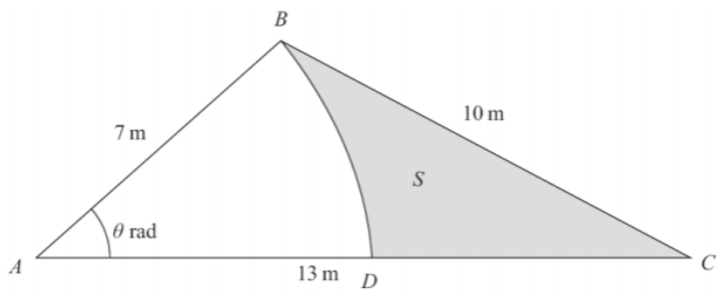
7. The figure shows the design for a triangular garden ABC where AB = 7 m, AC = 13 m and BC = 10 m.



Given that angle $BAC = \theta$ radians,

a. Show that, to 3 decimal places, $\theta = 0.865$ **(3)**

The point D lies on AC such that BD is an arc of the circle centre A, radius 7 m.

The shaded region S is bounded by the arc BD and the lines BC and DC. The shaded region S will be sown with grass seed, to make a lawned area.

Given that 50 g of grass seed are needed for each square metre of lawn,

b. Find the amount of grass seed needed, giving your answer to the nearest 10 g **(7)**

(Total Marks: 10)

8a. Express 2 cos 3x – 3 sin 3x in the form R cos (3x + α), where R and α are constants, $R > 0$ and $0 < \alpha < 2\pi$. Give your answers to 3 significant figures.

$f(x) = e^{2x} \cos 3x$ **(4)**

b. Show that $f'(x)$ can be written in the form

$f'(x) = Re^{2x} \cos (3x + \alpha)$

where R and α are the constants found in part (a) **(5)**

c. Hence, or otherwise, find the smallest positive value of x for which the curve with equation $y = f(x)$ has a turning point. **(3)**

(Total Marks: 12)

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9. $f(x) = \frac{5-8x}{(1+2x)(1-x)^2}$

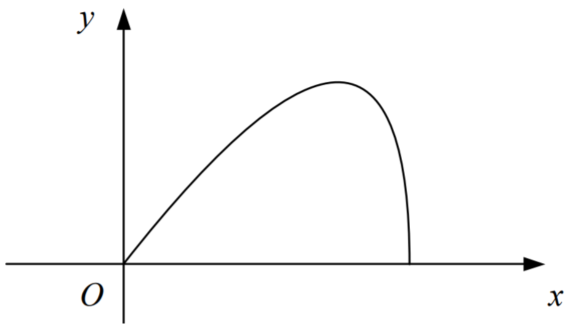
a. Express $f(x)$ in partial fractions. **(5)**

b. Find the series expansion of $f(x)$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(6)**

c. State the set of values of x for which your expansion is valid. **(1)**

(Total Marks: 12)

10.



The figure shows the curve with parametric equations

a. Find $\frac{dy}{dx}$ in terms of t **(3)**

b. Find, in exact form, the coordinates of the point where the tangent to the curve is parallel to the x-axis. **(3)**

c. Show that the region bounded by the curve and the x-axis has area 2 **(6)**

(Total Marks: 12)

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11. The curve C has the equation $ye^{-2x} = 2x + y^2$

a. Find the differentiation of the curve in terms of x and y **(5)**

The point P on C has coordinates (0, 1).

b. Find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**

(Total Marks: 9)

12a. Find $\int (4y+3)^{-\frac{1}{2}}\, dy$ **(2)**

b. Given that $y = 1.5$ at $x = -2$, solve the differential equation,

$$\frac{dy}{dx} = \frac{\sqrt{4y+3}}{x^2}$$

giving your answer in the form $y = f(x)$ **(6)**

(Total Marks: 8)

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13. Find,

a. $\int xe^x \ dx$ **(2)**

b. $\int x^2e^x \ dx$ **(3)**

(Total Marks: 5)

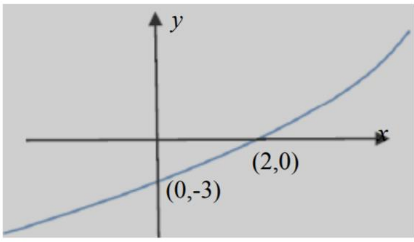
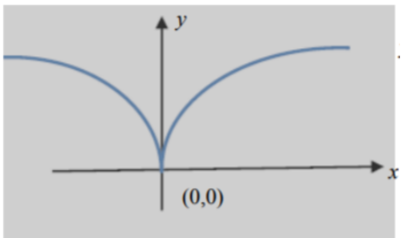
Total Marks: 120

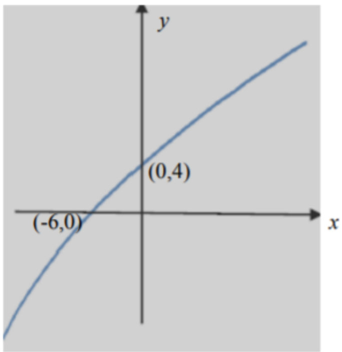
Mark Scheme

1a	3.84, 4.14, 4.58	**B1** **B1**
1b	$\frac{1}{2} \times 4 \times (3 + 4.58) + 2(3.47 + 3.84 + 4.14 + 4.39)$	**B1** **M1** **A1**
	= 7.852	**A1**

2a	Length *DEA* = 7(2.1) = 14.7	**M1** **A1**
2b	Angle *CBD* = $\pi - 2.1$	**M1**
	7 cos (π - 2.1)	**M1**
	7 sin (π - 2.1)	**M1**
	P = 7 cos (π - 2.1) + 7 sin (π - 2.1) + 7 + 14.7 = 31.2764…	**A1**

3a	$P = 80e^{\frac{t}{5}}$ $t = 0$ $P = 80(1) = 80$	**B1**
3b	$P = 1000$ $1000 = 80e^{\frac{t}{5}}$ $\frac{1000}{80} = e^{\frac{t}{5}}$	**M1**
	$t = 5\ln\left(\frac{1000}{80}\right)$ $t = 12.6286\ldots$	**A1**
3c	$\frac{dP}{dt} = 16e^{\frac{t}{5}}$	**M1** **A1**
3d	$50 = 16e^{\frac{t}{5}}$ $t = 5\ln\left(\frac{50}{16}\right) = 5.69717\ldots$	**M1**
	$P = 80e^{\frac{1}{5}\left(5\ln\left(\frac{50}{16}\right)\right)}$	**M1**
	$P = \frac{80(50)}{16} = 250$	**A1**

4a	*ff*(-3) = *f*(0) = 2	**M1** **A1**
4b	 Shape (0, -3) and (2, 0)	**B1** **B1**
4c	 Shape (0, 0)	**B1** **B1**

4d	 Shape (-6, 0) or (0, 4) (-6, 0) and (0, 4)	**B1** **B1** **B1**

5a	Either $y = 2$ or $(0, 2)$	**B1**
5b	When $x = 2$ $y = (8 - 10 + 2)e^{-2} = 0$	**B1**
	$(2x^2 - 5x + 2) = 0$ $(x - 2)(2x - 1) = 0$	**M1**
	$x = 2$ or $x = \frac{1}{2}$	**A1**
5c	$\frac{dy}{dx} = (4x - 5)e^{-x} - (2x^2 - 5x + 2)e^{-x}$	**M1** **A1** **A1**
5d	$(4x - 5)e^{-x} - (2x^2 - 5x + 2)e^{-x} = 0$	**M1**
	$2x^2 - 9x + 7 = 0$ $(2x - 7)(x - 1) = 0$	**M1**
	$x = \frac{7}{2}, x = 1$	**A1**
	When $x = \frac{7}{2}, y = 9e^{-\frac{7}{2}}$ When $x = 1$, $y = -e^{-1}$	**M1** **A1**

6a	$f(x) \geq 3$	**M1** **A1**
6b	An attempt to find $2\|3 - 4x\| + 3$ $x = 1$	**M1**
	Correct answer $fg(1) = 5$	**A1**
6c	$y = 3 - 4x$ $4x = 3 - y$ $x = \frac{3-y}{4}$	**M1**
	$g^{-1}(x) = \frac{3-x}{4}$	**A1**
6d	$[g(x)]^2 = (3 - 4x)^2$	**B1**
	$gg(x) = 3 - 4(3 - 4x)$	**M1**
	$-8 + 16x + 9 - 24x + 16x^2 = 0$ $16x^2 - 8x = 0$	**A1**
	$8x(2x - 1) = 0$ $x = 0$ $x = 0.5$	**M1** **A1**

7a	$10^2 = 7^2 + 13^2 - 2 \times 7 \times 13 \cos\theta$	**M1**
	$\cos\theta = \frac{59}{91}$	**A1**
	$\theta = 0.8653 \ldots = 0.865$	**A1**
7b	Area triangle $ABC = \frac{1}{2} \times 13 \times 7 \sin 0.865$	**M1**
	Area of sector $ABD = \frac{1}{2} \times 7^2 \times 0.865$	**M1**
	$= 34.6$	**A1**

	Area of $S = \frac{1}{2} \times 13 \times 7 \sin 0.865 - \frac{1}{2} \times 7^2 \times 0.865 = 13.4$	**M1** **A1**
	Amount of seed = 13.4 x 50 = 670g	**M1** **A1**

8a	$R^2 = 2^2 + 3^2$	**M1**
	$R = \sqrt{113}$	**A1**
	$\tan \alpha = \frac{3}{2}$	**M1**
	$\alpha = 0.983$	**A1**
8b	$f'(x) = 2e^{2x} \cos 3x - 3e^{2x} \sin 3x$	**M1** **A1** **A1**
	$= e^{2x} (2 \cos 3x - 3 \sin 3x)$	**M1**
	$= e^{2x} (R \cos (3x + \alpha)$	**A1**
8c	$f'(x) = 0$ $\cos (3x + \alpha) = 0$	**M1**
	$3x + \alpha = \frac{\pi}{2}$	**M1**
	$x = 0.196\ldots$	**A1**

9a	$\frac{5-8x}{(1+2x)(1-x)^2} = \frac{A}{1+2x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$ $5 - 8x = A(1-x)^2 + B(1+2x)(1-x) + C(1+2x)(1-x)$	**M1**
	$x = -\frac{1}{2}$ $9 = \frac{9}{4}A$ $A = 4$	**A1**
	$x = 1$ $-3 = 3C$ $C = -1$	**A1**
	Coefficients of x^2 $0 = A - 2B$ $B = 2$	**M1** **A1**
b	$f(x) = 4(1+2x)^{-1} + 2(1-x)^{-1} - (1-x)^{-2}$ $(1+2x)^{-1} = 1 + (-1)(2x) + \frac{(-1)(-2)}{2}(2x)^2 + \frac{(-1)(-2)(-3)}{3\times2}(2x)^3 + \cdots$	**M1**
	$= 1 - 2x + 4x^2 - 8x^3 + \ldots$	**A1**
	$(1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots$	**B1**
	$(1-x)^{-2} = 1 + (-2)(-x) + \frac{(-2)(-3)}{2}(-x)^2 + \frac{(-2)(-3)(-4)}{3\times2}(-x)^3 + \cdots$ $= 1 + 2x + 3x^2 + 4x^3 + \ldots$	**A1**
	$f(x) = 4(1 - 2x + 4x^2 - 8x^3) + 2(1 + x + x^2 + x^3) - (1 + 2x + 3x^2 + 4x^3)$	**M1**
	$= 5 - 8x + 15x^2 - 34x^3\ldots$	**A1**
9c	$\lvert x \rvert < \frac{1}{2}$	**A1**

10a	$\frac{dx}{dt} = 1 + \cos t$ $\frac{dy}{dt} = \cos t$	**M1**
	$\frac{dy}{dx} = \frac{\cos t}{1+\cos t}$	**M1** **A1**
10b	$\frac{\cos t}{1+\cos t} = 0$ $\cos t = 0$ $t = \frac{\pi}{2}$	**M1** **A1**
	$(\frac{\pi}{2} + 1, 1)$	**A1**
10c	$= \int_{2}^{\pi} \sin t \times (1 + \cos t)\, dt$	**M1**

	$= \int_0^{\pi} (\sin t + ½ \sin 2t)$ dt	**A1**
	$= [- \cos t - 1\ 4 \cos 2t]_0^{\pi}$	**M1** **A1**
	$\left(1 - \frac{1}{4}\right) - \left(-1 - \frac{1}{4}\right) = 2$	**M1** **A1**

11a	$e^{-2x} \frac{dy}{dx} - 2ye^{-2x} = 2 + 2y \frac{dy}{dx}$	**M1** **A1**
	$\frac{d}{dx}(y\, e^{-2x}) = e^{-2x} \frac{dy}{dx} - 2ye^{-2x}$	**B1**
	$(e^{-2x} - 2y) \frac{dy}{dx} = 2 + 2y\, e^{-2x}$	**M1**
	$\frac{dy}{dx} = \frac{2+2y\, e^{-2x}}{e^{-2x} - 2y}$	**A1**
11b	At P, $\frac{dy}{dx} = \frac{2+2e^0}{e^0 - 2} = -4$	**M1**
	Gradient of normal = $\frac{1}{4}$	**M1**
	$y - 1 = \frac{1}{4}(x - 0)$	**M1**
	$x - 4y + 4 = 0$	**A1**

12a	$\int (4y+3)^{-\frac{1}{2}}\, dx = \frac{(4y+3)^{\frac{1}{2}}}{4\left(\frac{1}{2}\right)} + c$ $= \frac{1}{2}(4y+3)^{\frac{1}{2}} + c$	**M1** **A1**
12b	$\int \frac{1}{\sqrt{4y+3}}\, dy = \int \frac{1}{x^2}\, dx$	**B1**
	$\int (4y+3)^{-\frac{1}{2}}\, dy = \int x^{-2}\, dx$ $\frac{1}{2}(4y+3)^{\frac{1}{2}} = -\frac{1}{x} + c$	**M1**
	Using (-2, 1.5) $\frac{1}{2}(4 \times 1.5 + 3)^{\frac{1}{2}} = -\frac{1}{-2} + c$	**M1**
	$c = 1$	**A1**
	$\frac{1}{2}(4 \times 1.5 + 3)^{\frac{1}{2}} = -\frac{1}{-2} + 1$ $(4y+3)^{\frac{1}{2}} = 2 - \frac{2}{x}$	**M1**
	$y = \frac{1}{4}\left(2 - \frac{2}{x}\right)^2 - \frac{3}{4}$	**A1**

13a	$\int xe^x dx = xe^x - \int e^x\, dx$	**M1**
	$= xe^x - e^x + c$	**A1**
13b	$\int x^2 e^x dx = x^2 e^x - \int e^x 2x\, dx$	**M1** **A1**
	$= x^2 e^x - 2 \int e^x x\, dx$ $= x^2 e^x - 2(xe^x - e^x) + c$	**A1**

Topic List

Q1	Trapezium rule
Q2	Radians
Q3	Maxima and minima
Q4	Graph sketching
Q5	Differentiation
Q6	Functions
Q7	Radians
Q8	Trig modelling
Q9	Partial fractions and binomial expansion
Q10	Parametric equations
Q11	Implicit differentiation
Q12	Differential equations
Q13	Integration by parts