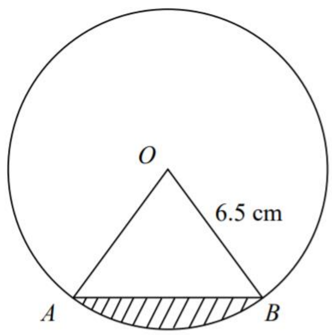
Practice Exam Paper G
Time: 2 Hours

P1 P2

1.



The figure shows the sector *AOB* of a circle, with centre *O* and radius 6.5 cm, and ∠*AOB* = 0.8 radians.

a. Calculate, in cm^2, the area of the sector *AOB* **(2)**

b. Show that the length of the chord *AB* is 5.06 cm, to 3 significant figures **(3)**

The segment *R*, shaded in the figure, is enclosed by the arc *AB* and the straight line *AB*

c. Calculate, in cm, the perimeter of *R*. **(2)**

(Total Marks: 7)

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2. The curve *C* has equation $y = \cos\left(x + \frac{\pi}{4}\right)$, $0 \le x \le 2\pi$

a. Sketch *C* **(2)**

b. Write down the exact coordinates of the points at which *C* meets the coordinate axes **(3)**

c. Solve, for x in the interval $0 \le x \le 2\pi$, $\cos\left(x + \frac{\pi}{4}\right) = 0.5$, giving your answers in terms of π **(4)**

(Total Marks: 9)

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3. A geometric series is $a + ar + ar^2 + \dots$

a. Prove e that the sum of the first *n* terms of this series is $S_n = \frac{a(1-r^n)}{1-r}$ **(4)**

The first and second terms of a geometric series *G* are 10 and 9 respectively

b. Find, to 3 significant figures, the sum of the first twenty terms of *G* **(3)**

c. Find the sum to infinity of *G* **(2)**

Another geometric series has its first term equal to its common ratio. The sum to infinity of this series is 10.

d. Find the exact value of the common ratio of this series **(3)**

(Total Marks: 12)

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4. The root of the equation $f(x) = 0$, where

$$f(x) = x + \ln 2x - 4$$

is to be estimated using the iterative formula $x_{n+1} = 4 - \ln 2x_n$, with $x_0 = 2.4$

a. Showing your values of x_1, x_2, x_3, …., obtain the value, to 3 decimal places, of the root **(4)**

b. By considering the change of sign of $f(x)$ in a suitable interval, justify the accuracy of your answer to part (a). **(2)**

(Total Marks: 6)

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5. The function f is defined by

$$f: x\ a\ |2x - a|$$

where a is a positive constant.

a. Sketch the graph of $y = f(x)$, showing the coordinates of the points where the graph cuts the axes. **(2)**

b. On a separate diagram, sketch the graph of $y = f(2x)$, showing the coordinates of the points where the graph cuts the axes. **(2)**

c. Given that a solution of the equation $f(x) = \frac{1}{2}x$ is $x = 4$, find the two possible values of a. **(4)**

(Total Marks: 8)

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6. The function f, defined for $x > 0$, is such that,

$$f'(x) = x^2 - 2 + \frac{1}{x^2}$$

a. Find the value of $f''(x)$ at $x = 4$. **(3)**

b. Given that $f(3) = 0$, find $f(x)$. **(4)**

c. Prove that f is an increasing function. **(3)**

(Total Marks: 10)

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7. $f(x) = \frac{2}{x-1} - \frac{6}{(x-1)(2x+1)}, x > 1$

a. Prove that $f(x) = \frac{4}{2x+1}$ **(4)**

b. Find the range of f **(2)**

c. Find $f^{-1}(x)$ **(3)**

d. Find the range of $f^{-1}(x)$ **(1)**

(Total Marks: 10)

8a. Expand $(1 - 3x)^{-2}$, $|\, x \,| < \frac{1}{3}$, in ascending powers of x up to and including the term in x^3 , simplifying each coefficient. **(4)**

b. Hence, or otherwise, show that for small x,

$$\left(\frac{2-x}{1-3x}\right)^2 \approx 4 + 20x + 85x^2 + 330x^3$$ **(3)**

(Total Marks: 7)

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9. $f(x) = \frac{7+3x+2x^2}{(1-2x)(1+x)^2}, |x| > \frac{1}{2}$

a. Express $f(x)$ in partial fractions. **(4)**

b. Show that,

$$\int_1^2 f(x)dx = p - \ln q$$

where p is rational and q is an integer. **(7)**

(Total Marks: 11)

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10. A curve has the equation,

$$x^2 - 4xy + 2y^2 = 1.$$

a. Find an expression for $\frac{dx}{dy}$ in its simplest form in terms of x and y. **(5)**

b. Show that the tangent to the curve at the point P (1, 2) has the equation

$$3x - 2y + 1 = 0$$ **(3)**

The tangent to the curve at the point Q is parallel to the tangent at P.

c. Find the coordinates of Q **(4)**

(Total Marks: 12)

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11. The rate of increase in the number of bacteria in a culture, N, at time t hours is proportional to N.

a. Write down a differential equation connecting N and t **(1)**

Given that initially there are N_0 bacteria present in a culture

b. Show that $N = N_0e^{kt}$, where k is a positive constant **(6)**

Given also that the number of bacteria present doubles every six hours,

c. Find the value of k, **(3)**

d. Find how long it takes for the number of bacteria to increase by a factor of ten, giving your answer to the nearest minute. **(3)**

(Total Marks: 13)

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12. A curve has parametric equations

$$x = \sec\theta + \tan\theta,$$
$$y = \operatorname{cosec}\theta + \cot\theta,$$
$$0 < \theta < \frac{\pi}{2}$$

a. Show that $x + \frac{1}{x} = 2\sec\theta$. **(5)**

Given that $y + \frac{1}{y} = 2\operatorname{cosec}\theta$

b. Find a cartesian equation for the curve. **(3)**

c. Show that $\frac{dx}{d\theta} = \frac{1}{2}(x^2 + 1)$ **(3)**

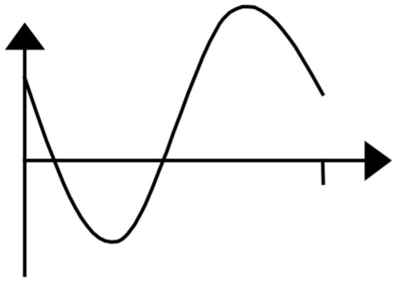
d. Find an expression for $\frac{dy}{dx}$ in terms of x and y **(4)**

(Total Marks: 15)

Total Marks: 120

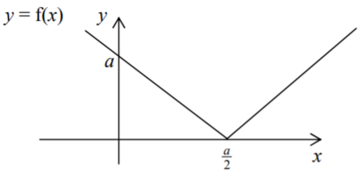
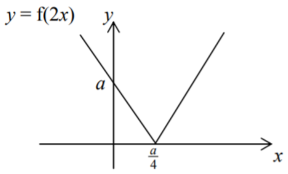
Mark Scheme

1a	$\frac{1}{2}r^2\theta = \frac{1}{2} \times 6.5^2 \times 0.8 = 16.9$	**M1** **A1**
1b	$\sin 0.4 = \frac{x}{6.5}$ $x = 6.5 \sin 0.4$	**M1** **A1**
	$AB = 2x = 5.06$	**A1**
1c	$r\theta + 5.06 = (6.5 \times 0.8) + 5.06 = 10.26$	**M1** **A1**

2a		**B1** **Shape** **B1** **Position**
2b	$(0, \frac{1}{\sqrt{2}})$ $(\frac{\pi}{4}, 0)$ $(\frac{5\pi}{4}, 0)$	**B3**
2c	$(x + \frac{\pi}{4}) = \frac{\pi}{3}$	**B1**
	Other value: $(2\pi - \frac{\pi}{3}) = \frac{5\pi}{3}$	**M1**
	Subtract, $\frac{\pi}{4}$ $x = \frac{\pi}{12}$ $x = \frac{17\pi}{12}$	**M1** **A1**

3a	$(S =)\ a + ar + \ldots + ar^{n-1}$	**B1**
	$(rS =)\ ar + ar^2 + \ldots + ar^n$	**M1**
	$S(1 - r) = a(1 - r^n)$ $S = \frac{a(1-r^n)}{1-r}$	**M1** **A1**
3b	$r = 0.9$	**B1**
	$S_{20} = \frac{10(1-0.9^{20})}{1-0.9} = 87.8$	**M1** **A1**
3c	Sum to infinity $= \frac{a}{1-r} = \frac{10}{1-0.9} = 100$	**M1** **A1**
3d	$\frac{a}{1-r} = \frac{r}{r-1} = 10$	**M1**
	$r = 10(1 - r)$ $r = \frac{10}{11}$	**M1** **A1**

4a	$f(x) = x + \ln 2x - 4$ $x_{n+1} = 4 - \ln 2x$ $x_0 = 2.4$ $x_1 = 2.431\ldots$ $x_2 = 2.418\ldots$ $x_3 = 2.422\ \ldots$	**M1** **M1**
	Root = 2.42	**A2**
4b	Choosing an appropriate interval e.g. [2.4215, 2.4225]	**M1**
	Establishing change of sign + Conclusion	**M1**

5a		**B1** **B1**
5b		**B1** **B1**
5c	$-(2x - a) = \frac{1}{2}x$ When $x = 4$, $a - 8 = 2$ $a = 10$	**M1** **A1**
	$2x - a = \frac{1}{2}x$ When $x = 4$ $8 - a = 2$ $a = 6$	**M1** **A1**

6a	$f''(x) = 2x - 2x - 3$	**M1** **A1**
	$= 8 - \frac{6}{24} = 7\frac{31}{32}$	**A1**
6b	$f(x) = \frac{1}{3}x^3 - 2x - \frac{1}{x}$	**M1** **A1**
	$0 = 9 - 6 - \frac{1}{3} + C$ $C = -\frac{8}{3}$	**M1** **A1**
6c	$f'(x) > 0$ needed, or $f'(x) \geq 0$, or "as x increases, $f(x)$ increases"	**B1**
	$f(x) = (x - \frac{1}{x})^2$, > 0 always, or ≥ 0 always	**M1** **A1**

7a	$f(x) = \frac{2(2x+1)-6}{(x-1)(2x+1)} = \frac{4x-4}{(x-1)(2x-1)}$	**M1** **A1**
	$f(x) = \frac{4(x-1)}{(x-1)(2x-1)} = \frac{4}{(2x+1)}$	**M1** **A1**
7b	$0 < f < \frac{4}{3}$	**B1** **B1**
7c	$y = \frac{4}{2x+1}$	**M1**
	$x = \frac{4-y}{2y}$	**M1**
	$f^{-1}(x) = \frac{4-x}{2x}$	**A1**
7d	Range of f^{-1} = domain of f ∴ $f^{-1} > 1$ or $y > 1$ or > 1	**B1**

8a	$(1 - 3x)^{-2} = 1 + (-2)(-3x) + \frac{(-2)(-3)}{2}(-3x)^2 + \frac{(-2)(-3)(-4)}{3\times2}(3x)^3 + \ldots$	**M1**
	$= 1 + 6x + 27x^2 + 108x^3$	**A3**
8b	$\left(\frac{2-x}{1-3x}\right)^2 = (2 - x)^2 (1 - 3x)^{-2} = (4 - 4x + x^2)(1 + 6x + 27x^2 + 108x^3 + \ldots)$	**M1**
	$= 4 + 24x + 108x^2 + 432x^3 - 4x - 24x^2 - 108x^3 + x^2 + 6x^3 + \ldots$	**A1**
	For small values of x,	**A1**

	$\left(\frac{2-x}{1-3x}\right)^2 = 4 + 20x + 85x^2 + 330x^3$	

9a	$\frac{7+3x+2x^2}{(1-2x)(1+x)^2} \equiv \frac{A}{1-2x} + \frac{B}{1+x} + \frac{C}{(1+x)^2}$ $7 + 3x + 2x^2 \equiv A(1+x)^2 + B(1-2x)(1+x) + C(1-2x)$ $x = \frac{1}{2}$ $9 = \frac{9}{4}A$ $A = 4$	**B1**
	$x = -1 \Rightarrow 6 = 3C \Rightarrow C = 2$	**B1**
	coeffs $x^2 \Rightarrow 2 = A - 2B \Rightarrow B = 1$	**M1**
	Therefore, $f(x) = \frac{4}{1-2x} + \frac{1}{1+x} + \frac{2}{(1+x)^2}$	**A1**
9b	$\int_1^2 \left(\frac{4}{1-2x} + \frac{1}{1+x} + \frac{2}{(1+x)^2}\right) dx$ $= [-2\ln\|1-2x\| + \ln\|1+x\| - 2(1+x)^{-1}]_1^2$	**M1** **A3**
	$= (-2\ln 3 + \ln 3 - \frac{2}{3}) - (0 + \ln 2 - 1)$	**M1**
	$= -\ln 3 - \ln 2 + \frac{1}{3}$ $= \frac{1}{3} - \ln 6$	**M1** **A1**

10a	$2x - 4y - 4x\frac{dy}{dx} + 4y\frac{dy}{dx} = 0$	**M1** **A2**
	$\frac{dy}{dx} = \frac{2x-4y}{4x-4y} = \frac{x-2y}{2x-2y}$	**M1** **A1**
10b	Gradient $= \frac{3}{2}$	**M1**
	$y - 2 = \frac{3}{2}(x-1)$	**M1**
	$2y - 4 = 3x - 3$ $3x - 2y + 1 = 0$	**A1**
10c	$\frac{x-2y}{2x-2y} = \frac{3}{2}$	**M1**
	$2(x - 2y) = 3(2x - 2y)$ $y = 2x$	**A1**
	Sub $\Rightarrow x^2 - 8x^2 + 8x^2 = 1$	**M1**
	$x^2 = 1$, $x = 1$ (at P) or -1 Q (-1, -2)	**A1**

11a	$\frac{dN}{dt} = kN$	**B1**
11b	$\int \frac{1}{N} dN = \int k\, dt$	**M1**
	$\ln\|N\| = kt + c$	**M1 A1**
	$t = 0$, $N = N_0$ $\ln\|N_0\| = c$	**M1**
	$\ln\|N\| = kt + \ln\|N_0\|$ $\ln\left\|\frac{N}{N_0}\right\| = kt$	**M1**
	$\frac{N}{N_0} = e^{kt}$ $N = N_0 e^{kt}$	**A1**
11c	$2N_0 = N_0 e^{6k}$	**M1**
	$k = \frac{1}{6}\ln 2 = 0.116$	**M1** **A1**
11d	$10N_0 = N_0 e^{0.1155t}$	**M1**
	$t = \frac{1}{0.1155}\ln 10 = 19.932\ hours = 19\ hours\ 56\ mins$	**M1** **A1**

12a	$x + \frac{1}{x} = \sec\theta + \tan\theta + \frac{1}{\sec\theta\tan\theta} = \frac{(\sec\theta+\tan\theta)^2+1}{\sec\theta\tan\theta}$	**M1**
	$= \frac{\sec^2\theta + 2\sec\theta\tan\theta + \tan^2\theta + 1}{\sec\theta\tan\theta} = \frac{2\sec^2\theta + 2\sec\theta\tan\theta}{\sec\theta\tan\theta}$	**M1** **A1**
	$= \frac{2\sec\theta(\sec\theta+\tan\theta)}{\sec\theta\tan\theta} = 2\sec\theta$	**M1** **A1**
12b	$\frac{x^2+1}{x} = \frac{2}{\cos\theta}$ $\cos\theta = \frac{2x}{x^2+1}$	**M1**
	$\frac{y^2+1}{y} = \frac{2}{\sin\theta}$ $\cos\theta = \frac{2y}{y^2+1}$ $\frac{4x^2}{(x^2+1)^2} + \frac{4y^2}{(y^2+1)^2}$	**M1** **A1**
12c	$\frac{dx}{d\theta} = \sec\theta\tan\theta + \sec^2\theta$	**M1**
	$= \sec\theta(\tan\theta + \sec\theta) = \frac{x^2+1}{2x} \times x = \frac{1}{2}(x^2+1)$	**M1** **A1**
12d	$\frac{dy}{d\theta} = -cosec\,\theta\cot\theta - \mathrm{cosec}^2\theta$	**M1**
	$= -\mathrm{cosec}\,\theta(\cot\theta + \mathrm{cosec}\,\theta) = -\frac{y^2+1}{2y} \times y = -\frac{1}{2}(y^2+1)$	**A1**
	$\frac{dy}{dx} = -\frac{y^2+1}{x^2+1}$	**M1** **A1**

Topic List

Q1	Radians
Q2	Trig graphs
Q3	Geometric series
Q4	Numerical methods
Q5	Modulus Function
Q6	Differentiation and functions
Q7	Functions
Q8	Binomial expansion
Q9	Partial fractions and integration
Q10	Implicit differentiation
Q11	Differential equations
Q12	Parametric equations