Practice Exam Paper E
Time: 2 Hours

P1 P2

1. $f(x) = 2x^3 - 5x^2 + x + 2$

a. Show that $(x - 2)$ is a factor of $f(x)$ **(2)**

b. Fully factorise, $f(x)$ **(4)**

c. Solve the equation $f(x) = 0$ **(1)**

Find the values of θ in the interval $0 \leq \theta \leq 2\pi$ for which,

$$2 \sin^3 \theta - 5 \sin^2 \theta + \sin \theta + 2 = 0$$

giving your answers in terms of π **(4)**

(Total Marks: 11)

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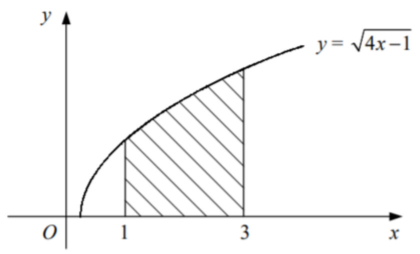
2. The figure shows the curve with equation $y = \sqrt{4x - 1}$



Use the trapezium rule with five equally-spaced ordinates to estimate the area of the shaded region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 3$. **(4)**

(Total Marks: 4)

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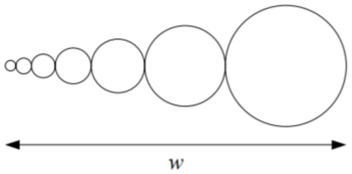
3. The figure shows part of a design being produced by a computer program.



The program draws a series of circles with each one touching the previous one and such that their centres lie on a horizontal straight line.

The radii of the circles form a geometric sequence with first term 1 mm and second term 1.5 mm. The width of the design is w as shown.

a. Find the radius of the fourth circle to be drawn **(2)**

b. Show that when eight circles have been drawn, $w = 98.5$ mm to 3 significant figures. **(4)**

c. Find the total area of the design in square centimetres when ten circles have been drawn. **(5)**

(Total Marks: 11)

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4. Given that $\cos x = \sqrt{3} - 1$, find the value of $\cos 2x$ in the form $a + b\sqrt{3}$, where a and b are integers. **(3)**

b. Given that,

$$2 \cos (y + 30)° = \sqrt{3} \sin (y - 30)°,$$

find the value of $\tan y$ in the form $k\sqrt{3}$ where k is a rational constant **(5)**

(Total Marks: 8)

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5. $f(x) = \frac{x^4+x^3-13x^2+26x-17}{x^2-3x+3}$

a. Find the values of the constants A, B, C and D such that

$$f(x) = x^2 + Ax + B + \frac{Cx+D}{x^2-3x+3}$$ **(4)**

The point P on the curve $y = f(x)$ has x-coordinate 1.

b. Show that the normal to the curve $y = f(x)$ at P has the equation

$$x + 5y + 9 = 0$$ **(6)**

(Total Marks: 10)

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6. $f(x) = 5 + e^{2x + 3}$

a. State the range of f **(1)**

b. Find an expression for $f^{-1}(x)$ and state its domain **(4)**

c. Solve the equation $f(x) = 7$ **(2)**

d. Find an equation for the tangent to the curve $y = f(x)$ at the point where $y = 7$. **(4)**

(Total Marks: 11)

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7. The functions f and g are defined by,

$$f : x \rightarrow |2x - 5|$$

$$g : x \rightarrow \ln (x + 3)$$

a. State the range of f **(1)**

b. Evaluate $fg(-2)$. **(2)**

c. Solve the equation $fg(x) = 3$, giving your answers in exact form. **(5)**

d. Show that the equation $f(x) = g(x)$ has a root, α, in the interval $[3, 4]$. **(2)**

e. Use the iteration formula,

$$x_{n+1} = \frac{1}{2}[5 + \ln(x_n + 3)]$$

with $x_0 = 3$, to find x_1, x_2, x_3 and x_4, giving your answers to 4 significant figures **(3)**

f. Show that your answer for x_4 is the value of α correct to 4 significant figures **(2)**

(Total Marks: 15)

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8. Prove the identity $2 \cot 2x + \tan x \equiv \cot x$,

$$x \neq \frac{n}{2}\pi \ \pi, \ n \in Z.$$ **(5)**

8b. Solve, for $0 \leq x < \pi$, the equation $2 \cot 2x + \tan x = \operatorname{cosec}^2 x - 7$,

giving your answers to 2 decimal places **(6)**

(Total Marks: 11)

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9. A curve has the equation

$$x^3 + 2xy - y^2 + 24 = 0.$$

Show that the normal to the curve at the point $(2, -4)$ has the equation $y = 3x - 10$. **(8)**

(Total Marks: 8)

10a. Expand $(4 - x)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2, simplifying each coefficient. **(4)**

b. State the set of values of x for which your expansion is valid **(1)**

c. Use your expansion with $x = 0.01$ to find the value of $\sqrt{399}$, giving your answer to 9 significant figures. **(4)**

(Total Marks: 9)

11a. Use the substitution $x = 2 \sin u$ to evaluate,

$$\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}} \, dx$$ **(5)**

b. Use integration by parts to evaluate,

$$\int_0^{\frac{\pi}{2}} x \cos x \ dx$$ **(6)**

(Total Marks: 11)

12a. Sketch on the same diagram the graphs of $y = |x| - a$ and $y = |3x + 5a|$, where a is a positive constant.

Show on your diagram the coordinates of any points where each graph meets the coordinate axes **(6)**

b. Solve the equation, $|x| - a = |3x + 5a|$ **(5)**

(Total Marks: 11)

Total Marks: 120

Mark Scheme

1a	$f(2) = 16 - 20 + 2 + 2 = 0 \therefore (x - 2)$ is a factor	**M1** **A1**
1b	$\begin{array}{r} 2x^2 - x - 1 \\ x-2 \overline{) 2x^3 - 5x^2 + x + 2} \\ \underline{2x^3 - 4x^2} \\ -x^2 + x \\ \underline{-x^2 + 2x} \\ -x + 2 \\ \underline{-x + 2} \end{array}$	**M1** **A1**
	$f(x) = (x - 2)(2x^2 - x - 1) = (x - 2)(2x + 1)(x - 1)$	**M1** **A1**
1c	$x = -\frac{1}{2}, 1, 2$	**B1**
1d	sin θ = 2, (no solutions) $\sin \theta = -\frac{1}{2}$, sin θ = 1	**M1**
	$\theta = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$ or $\frac{\pi}{2}$	**M1** **B1**
	$\theta = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$	**A1**

2	**x**: 1, 1.5, 2, 2.5, 3 $\sqrt{4x-1}$: $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, 3, $\sqrt{11}$	**M1**
	area $= \frac{1}{2} \times 0.5 \times [\sqrt{3} + \sqrt{11} + 2(\sqrt{5} + \sqrt{7} + 3)$	**B1** **M1**
	= 5.20 (3 s.f)	**A1**

x	1	1.5	2	2.5	3
$\sqrt{4x-1}$	$\sqrt{3}$	$\sqrt{5}$	$\sqrt{7}$	3	$\sqrt{11}$

3a	$r = 15$ $u_4 = 1 \text{ x } (1.5)^3 = 3.375$ mm	**M1** **A1**
3b	$w = 2 \text{ x } S_8$ GP: $a = 1$, $r = 1.5$	**M1**
	$= 2 \text{ x } \frac{1[(1.5)^8 - 1]}{1.5 - 1}$	**M1** **A1**
	= 98.516 = 98.5 mm	**A1**
3c	Areas form GP, $a = \pi \times 1^2 = \pi$ $r = (1.5)^2 = 2.25$	**B2**
	Total area $= \frac{\pi[(2.25)^{10} - 1]}{2.25 - 1} = 8354.8 \text{ mm}^2$	**M1** **A1**
	$= \frac{8354.8}{10^2}$ $= 83.5 \text{ cm}^2$ (3 s.f)	**A1**

4a	$\cos^2 x = (\sqrt{3} - 1)^2 = 3 - 2\sqrt{3} + 1 = 4 - 2\sqrt{3}$	**M1**
	$\cos 2x = 2\cos^2 x - 1 = 2(4 - 2\sqrt{3}) - 1 = 7 - 4\sqrt{3}$	**M1** **A1**
4b	$2(\cos y \cos 30 - \sin y \sin 30) = \sqrt{3} (\sin y \cos 30 - \cos y \sin 30)$	**M1** **A1**
	$\sqrt{3} \cos y - \sin y = \frac{3}{2} \sin y - \frac{1}{2} \sqrt{3} \cos y$	**B1**
	$\frac{3}{2}\sqrt{3} \cos y = \frac{5}{2} \sin y$ $\tan y = \frac{3}{2}\sqrt{3} \div \frac{5}{2} = \frac{3}{5}\sqrt{3}$	**M1** **A1**

5a	$\begin{array}{r} x^2 + 4x - 4 \\ x^2 - 3x + 3 \overline{) x^4 + x^3 - 13x^2 + 26x - 17} \\ \underline{x^4 - 3x^3 + 3x^2} \\ 4x^3 - 16x^2 + 26x \\ \underline{4x^3 - 12x^2 + 12x} \\ -4x^2 + 14x - 17 \\ \underline{-4x^2 + 12x - 12} \\ 2x - 5 \end{array}$	**M1**
	$f(x) = x^2 + 4x - 4 + \frac{2x+5}{x^2-3x+3}$ $A = 4, B = -4, C = 2, D = -5$	**A3**
5b	$f'(x) = 2x + 4 + \frac{2\times(x^2-3x+3)-(2x-5)\times(2x-3)}{(x^2-2x+3)^2}$	**M1** **A2**
	$x = 1$, $y = -2$, gradient = 5 Gradient of normal = $-\frac{1}{5}$	**M1**
	$y + 2 = -\frac{1}{5}(x-1)$	**M1**
	$5y + 10 = -x + 1$ $x + 5y + 9 = 0$	**A1**

6a	$f(x) > 5$	**B1**
6b	$y = 5 + e^{2x-3}$ $2x - 3 = \ln(y-5)$	**M1**
	$x = \frac{1}{2}[3 + \ln(y-5)]$	**M1**
	$f^{-1}(x) = \frac{1}{2}[3 + \ln(x-5)]$, $x \in \mathrm{R}$, $x > 5$	**A2**
6c	$x = f^{-1}(7) = \frac{1}{2}(3 + \ln 2)$	**M1** **A1**
6d	$f'(x) = 2e^{2x-3}$	**M1**
	Gradient = 4	**A1**
	$y - 7 = 4[x - \frac{1}{2}(3 + \ln 2)]$	**M1** **A1**

7a	$f(x) \geq 0$	**B1**
7b	$= f(0) = 5$	**M1** **A1**
7c	$fg(x) = f[\ln(x+3)] = \lvert 2\ln(x+3) - 5 \rvert$	**M1**
	$\lvert 2\ln(x+3) - 5 \rvert = 3$ $2\ln(x+3) = 2, 8$	**M1**
	$\ln(x+3) = 1, 4$	**A1**
	$x = e - 3, e^4 - 3$	**M1** **A1**
7d	let $h(x) = f(x) - g(x)$ $h(3)$ $= -0.79$, $f(4) = 1.1$	**M1**
	sign change, $h(x)$ continuous ∴ root	**A1**
7e	$x_1 = 3.396$, $x_2 = 3.428$, $x_3 = 3.430$, $x_4 = 3.431$	**M1** **A2**
7f	$h(3.4305) = -0.000052$, $f(3.4315) = 0.0018$	**M1**
	sign change, $h(x)$ continuous ∴ root ∴ $\alpha = x_4$ to 4sf	**A1**

8a	LHS = $\frac{2\cos 2x}{\sin 2x} + \frac{\sin x}{\cos x}$	**M1**
	$= \frac{\cos 2x}{\sin x \cos x} + \frac{\sin x}{\cos x}$	**M1**
	$= \frac{\cos 2x + \sin^2 x}{\sin x \cos x}$	**A1**
	$= \frac{(\cos^2 x - \sin^2 x) + \sin^2 x}{\sin x \cos x}$	**M1**
	$= \frac{\cos^2 x}{\sin x \cos x}$	**A1**

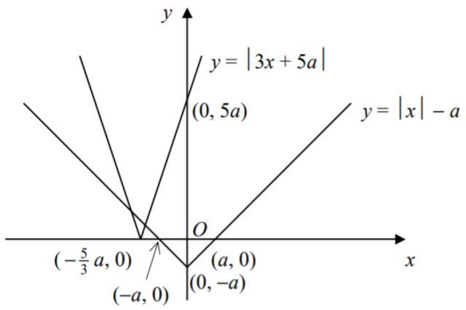
	$= \frac{\cos x}{\sin x}$ $= \cot x$	
8b	$\cot x = \text{cosec}^2 x - 7$ $\cot x = 1 + \cot^2 x - 7$	**M1**
	$\cot^2 x - \cot x - 6 = 0$ $(\cot x + 2)(\cot x - 3) = 0$	**M1**
	$\cot x = -2$ $\cot x = 3$	**A1**
	$\tan x = -\frac{1}{2}$ $\tan x = \frac{1}{3}$	**M1**
	$x = \pi - 0.4636$ or 0.32 $x = 0.32, 2.68$	**A2**

9	$3x^2 + 2y + 2x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$	**M1** **A2**
	$(2, -4) \Rightarrow 12 - 8 + 4\frac{dy}{dx} + 8\frac{dy}{dx} = 0$ $\frac{dy}{dx} = -\frac{1}{3}$	**M1** **A1**
	Gradient of normal = 3	**M1**
	$y + 4 = 3(x - 2)$	**M1**
	$y = 3x - 10$	**A1**

10a	$4^{\frac{1}{2}}(1 - \frac{1}{4}x)^{\frac{1}{2}} = 2\left(1 - \frac{1}{4}x\right)^{\frac{1}{2}}$	**B1**
	$= 2[1 + \frac{1}{2}\left(-\frac{1}{4}x\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}\left(-\frac{1}{4}x\right)^2 + \cdots]$ $= 2 - \frac{1}{4}x - \frac{1}{64}x^2$	**M1** **A2**
10b	$\lvert x \rvert < 4$	**B1**
10c	x = 0.01 $(4 - x)^{\frac{1}{2}} = \sqrt{3.99} = \sqrt{\frac{399}{100}} = \frac{1}{10}\sqrt{399}$	**M1**
	x = 0.01 $(4 - x)^{\frac{1}{2}} = 2 - \frac{1}{400} - \frac{1}{640000} = 1.997498438$	**M1**
	$\sqrt{399} = 10 \times 1.997498438 = 19.974\ldots$	**M1** **A1**

11a	$x = 2\sin u$ $\frac{dx}{du} = 2\cos u$	**M1**
	$x = 0, u = 0$ $x = \sqrt{3}, u = \frac{\pi}{3}$	**B1**
	$I = \int_0^{\frac{\pi}{3}} \frac{1}{2\cos u} \times 2\cos u\, du = \int_0^{\frac{\pi}{3}} 1\, du$	**A1**
	$[u]_0^{\frac{\pi}{3}} = \frac{\pi}{3} - 0 = \frac{\pi}{3}$	**M1** **A1**
11b	$u = x$ $u' = 1$ $v' = \cos x$ $v = \sin x$	**M1**
	$I = [x \sin x]_0^{\frac{\pi}{2}} - \int \sin x \; dx$	**A2**
	$= [x \sin x + \cos x]_0^{\frac{\pi}{2}}$	**M1**

	$= (\frac{\pi}{2} + 0) - (0 + 1) = \frac{\pi}{2} - 1$	**M1** **A1**

12a		**B3** **B3**
12b	$x - a = 3x + 5a$ $x = -\frac{3}{2}a$	**M1** **A1**
	$-x - a = -(3x + 5a)$ $x = -2a$	**M1** **A1**
	$x = -2a$ $y = -\frac{3}{2}a$	**A1**

Topic List

Q1	Factor theorem, trig
Q2	Trapezium rule
Q3	Geometric series
Q4	Trigonometry
Q5	Differentiation
Q6	Functions
Q7	Functions, exponentials and logs, numerical methods
Q8	Proving trig and solving trig. equations
Q9	Implicit differentiation
Q10	Binomial expansion
Q11	Integration by substitution and integration by parts
Q12	Sketching and solving modulus functions